746

PROCEEDINGS

OF

THE PHYSICAL SOCIETY

From December 1926 to August 1927

VOL. XXXIX.

LONDON:
FLEETWAY PRESS LTD.,
3-9, DANE STREET, HIGH HOLBORN, W.C.1
MDCCCCXXVII.

10635

LONDON:
FLEETWAY PRESS LTD.,
DANE ST., HIGH HOLBORN, W.C.1

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY.

SESSION 1926-1927.

The meetings were held at the Imperial College of Science, the President being in the Chair except where a statement to the contrary occurs.

October 22, 1926.

A DEMONSTRATION of Selenium Cells was given by Professor Thirring.

The following Papers were read :—

1. "The Corrosion Products and Mechanical Properties of Certain Light Aluminium Alloys, as affected by Atmospheric Exposure," by Prof. ERNEST WILSON.

2. "The Distribution of Intensity in a Positive Ray Spectral Line (Part II.)," by M. C. JOHNSON, M.A., M.Sc.

November 12, 1926.

The following Papers were read :—

1. "A Rapid Bolometer made by Sputtering on Thin Films," by H. DEWHURST, A.R.C.S., D.I.C.

2. "A Hygrometer Employing Glycerine," by EZER GRIFFITHS, D.Sc., F.R.S., and J. H. AWBERY, B.A., B.Sc.

3. "The Effect of Working on the Physical Properties of Tungsten," by J. W. AVERY, B.Sc., D.I.C., and C. J. SMITHELLS, D.Sc.

November 26, 1926.

The following Papers were read :—

1. "Electro-Endosmosis and Electrolytic Water Transport," by H. C. HEPBURN, B.Sc.

2. "The Input Impedances of Thermionic Valves at Low Frequencies," by L. HARTSHORN, A.R.C.S., B.Sc., D.I.C.

December 10, 1926.

The following Papers were read :—

1. "The Principle Governing the Distribution of Current in a System of Linear Conductors," by FRANK WENNER, Ph.D.

2. "A Capacitance Bridge of Wide Range, and a New Inductometer," by ALBERT CAMPBELL, M.A.

A DEMONSTRATION of "The Behaviour of Bodies with Non-Conducting Surfaces in Electrostatic Fields" was given by Mr. L. G. VEDY.

January 4, 5 and 6, 1927.

The Annual Exhibition of Apparatus was held by the Physical Society of London and the Optical Society from 3-6 p.m. and from 7-10 p.m. each day.

Discourses were given as follows :—

January 4.

A Lecture on Light and Electricity, as it might have been given in 1709, with the apparatus of the time, by Professor F. N. DA C. ANDRADE.

January 5.

"Progress in Electrical Instrument Design and Construction," by Dr. C. V. DRYSDALE.

January 6.

"Television," by Mr. J. L. BAIRD.

January 28, 1927.

The following Papers were read :—

1. "Electrical Polarisation in Selenium Cells and the Effects of Desiccation," by Prof. A. O. RANKINE, D.Sc., and J. W. AVERY.

2. "Synchronous Alternating-current Motors and Mechanical Vibrating Systems Maintained by Thermionic Valves at the Frequency of the Vibrating Systems," by T. G. HODGKINSON, A.M.I.E.E.

3. "On the Spectrum of Ionised Tin," by K. R. RAO, M.A.

4. "On the Spectrum of Doubly Ionised Gallium and Indium," by K. R. RAO, M.A.

A DEMONSTRATION illustrating some Physical Problems connected with Apple Transport was given by Dr. EZER GRIFFITHS, F.R.S.

February 11, 1927.

Annual General Meeting.

GENERAL BUSINESS.

The Report of the Council and that of the Treasurer were presented and unanimously adopted.

REPORT OF THE COUNCIL.

During the year fourteen ordinary Science Meetings have been held at the Imperial College of Science The Director of the National Physical Laboratory kindly invited the Society to visit the Laboratory on May 22, but this visit had to be cancelled owing to difficulties arising from the industrial situation in the country For the same reason the Council did not arrange a provincial meeting, like those successfully held in 1924 and 1925 ; but a visit to Bristol will probably be made in 1927.

At the Science Meetings 32 Papers were presented and 12 Demonstrations were given.

Prof. Charles Fabry, who was elected an Honorary Fellow of the Society at the Annual General Meeting, delivered the Eleventh Guthrie Lecture on April 23, the subject being "The Absorption of Radiation by the Upper Atmosphere." About 105 Fellows and Visitors were present.

The average attendance at the meetings of the Society was 54.

The Sixteenth Annual Exhibition, arranged jointly by the Physical and Optical Societies, was held, through the courtesy of the Governing Body, at the Imperial College, on January 5, 6 and 7. Trade exhibits were arranged by seventy-two firms, and the newly instituted Research and Experimental Section contained sixty-one exhibits contributed from twenty sources. Discourses were given by Mr. J. E. Barnard, F.R.S., on "The Search for Ultra-Microscopic Organisms"; by Professor A. F. C. Pollard on "The Mechanical Design of Instruments"; and by Major W. S. Tucker on "Electrical Listening." On the third day the general public were admitted and the total attendance at the Exhibition is estimated at 3,500.

The extension of the scope of the Exhibition, as indicated in the Annual Report for 1925, has been much appreciated.

Dr. D. Owen and Dr. J. H. Vincent have been appointed representatives of the Society on the Board of the Institute of Physics, and Mr. J. Guild and Dr. D. Owen on the Science Abstracts Committee. Mr. F. J. W. Whipple has been reappointed as the Society's representative on the Geophysical Committee of the Royal Astronomical Society, and Prof. E. A. Owen has been appointed to represent the Society on the International X-Ray Unit Committee.

Prof. O. W. Richardson and Mr. F. E. Smith represented the Society at the Coming-of-Age Celebrations of Sheffield University on July 1 and 2, and conveyed to the University an Address of Congratulation on behalf of the Society.

The Council has awarded the Fourth Duddell Medal to Mr. F. Twyman, F.R.S. This will be presented at the Annual General Meeting.

Owing to the rapid growth of the subject, Prof. A. Fowler has not found it possible to prepare the supplement promised for the new edition of his Report on Series in Line Spectra. The Council has therefore arranged for the original Report to be reprinted, together with a new index.

The new agreement, mentioned in the last Annual Report, between the Society and the Institution of Electrical Engineers in respect of "Science Abstracts" has been signed and sealed. The agreement is regarded by the Council as satisfactory.

As a consequence of the deliberations indicated in the last Annual Report concerning the question of changing the name of the Society, the Council has decided to take no formal action in the matter, but to adopt procedure similar to that of the Royal Society in not giving prominence to the words "of London." Commencing with Volume 39 of the Proceedings the words "of London" are to be omitted from all publications and printed papers except where legally necessary on account of the name of the Society remaining "The Physical Society of London," according to the Memorandum of Association.

The Council has entered into negotiations with the Institute of Physics for the purpose of arranging for the routine business of the Society to be performed by the Institute through the agency of the Secretary of the Institute at its new offices at 1, Lowther Gardens, South Kensington.

The Society has sustained a great loss in the death of Mr. W. R. Cooper (elected 1895), who, until recently, was Treasurer of the Society, and had also filled other important offices. The Council on March 26 passed unanimously a resolution of appreciation, which was afterwards adopted by the General Meeting.*

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* Proceedings, Vol. 38, p. xvii.

It is necessary also to record with regret the deaths of Sir David Salamons (elected 1876), Mr. J. E. Judson (elected 1877), Mr. R. H. Solly (elected 1878), Prof. W. J. Lewis (elected 1882), Sir Henry Mance (elected 1890), Mr. W. R. Pidgeon (elected 1891), and Dr. S. G. Mostyn (elected 1896), all Life Fellows of the Society; also of Mr. R. W. Forsyth (a former Recorder of the Society, elected 1898), Mr. J. S. Percy (elected 1918) and Mr. F. S. Spiers (elected 1926).

The number of Honorary Fellows on the Roll on December 31, 1926, was 12, the maximum permissible number. At the same date Ordinary Fellows numbered 666 and Students 12. The changes in the Membership of the Society during the year are shown in the appended table :—

——	Total Dec. 31, 1925.	Changes during 1926.			Total Dec. 31, 1926.
Honorary Fellows...	11	Elected		1	
		Net increase		1	12
Ordinary Fellows ...	640	Elected		38	
		Student transfers		9	
				47	
		Deceased	11		
		Resigned or lapsed ...	10		
				—21	
		Net increase		26	666
Students	16	Elected		5	
		Transferred to Fellowship...		9	
		Net decrease		4	12
Total Membership	667	Net increase		23	690

REPORT OF THE TREASURER.

The Expenditure during the past year exceeded the Income by £1 6s. 1d. This is due to the increased cost of the Proceedings. The number of pages in Volume 38 is greater than that in Volume 37 by 139, and the cost of the Proceedings in 1926 was £161 5s. greater than in the preceding year.

The Council is again indebted to the Council of the Royal Society for a grant of £100 towards the cost of publications.

The investments have been valued at market prices through the courtesy of the Manager of the Charing Cross Branch of the Westminster Bank. Owing to the rise in value of securities during the year, the market value of the Society's investments (including the Trust Funds) increased during the twelve months by £11.

INCOME AND EXPENDITURE ACCOUNT.

From January 1st to December 31st, 1926.

1925. £ s. d.	Expenditure.	£ s. d.	£ s. d.
	To		
513 0 0	"Science Abstracts" (Institution of Electrical Engineers, 1926)		434 0 0
	Ordinary Publications:—		
892 1 1	"Proceedings"	1,053 6 1	
70 5 0	"Bulletin"	73 3 3	
102 8 7	General	92 17 10	
			1,219 7 2
93 1 2	Postage of Publications		115 9 4
62 1 1	Reporting	61 12 3	
29 6 3	Refreshments and Expenses of Ordinary Meetings	37 12 0	
203 14 0	Expenses of Exhibition held jointly with the Optical Society	369 12 7	
28 0 10	Secretary and Treasurer's Expenses	23 16 9	
8 5 1	Periodicals and Library	8 8 4	
1 15 0	Insurance	1 15 0	
2 2 0	*Registered Address.*		
0 5 2	Cheque Books and Charges	0 15 7	
20 15 0	Accountancy Charges and Clerical Assistance	41 15 6	
2 15 9	Sundry Expenses	1 3 4	
			546 11 4
20 0 0	Guthrie Lecture (Honorarium)		20 0 0
10 10 0	*Donation to the 1926 Optical Convention.*		
5 5 0	*Lorentz Memorial.*		
	Participation in "Coming of Age" Celebrations, Sheffield University		10 6 0
	Optical Society—Exhibition Account		21 0 0
90 7 1	*Excess of Income over Expenditure.*		
£2,155 18 1			£2,366 13 10

1925. £ s. d.	Income.	£ s. d.	£ s. d.
	By		
954 0 10	Subscriptions by Fellows*	1,063 3 11	
9 9 0	,, (Voluntary†)	9 9 0	
8 18 6	,, by Students	8 18 6	
42 11 0	,, for "Science Abstracts" and Advance Proofs	52 18 6	
1,014 19 4			1,134 9 11
605 6 10	Sale of Publications (Fleetway Press, Ltd.)		484 10 8
	Dividends:—		
	London Midland & Scottish Railway	44 15 4	
	Metropolitan Board of Works	7 0 0	
	Lancaster Corporation Stock	9 12 0	
	New South Wales Stock	10 3 4	
	Southern Railway	28 14 8	
	London & North Eastern Railway	16 0 0	
	India 3½%	17 10 0	
	4% Funding Loan	20 16 0	
	5% War Loan	35 0 0	
		189 11 4	
	Less: Transferred to Duddell Memorial Trust Fund	20 0 0	
162 10 7			169 11 4
40 0 0	Income Tax repayable one year to December 31st, 1926		29 10 11
9 19 8	Interest on Deposit Account		12 16 11
4 5 10	*From Optical Society for Share of Exhibition Expenses.*		
193 2 6	Exhibitors' Payments on account of Exhibition Catalogue		434 8 0
100 0 0	Royal Society Grant for Publications		100 0 0
25 13 4	*Royal Meteorological Society.*		
	Excess of Expenditure over Income, 1926		1 6 1
£2,155 18 1			£2,366 13 10

ROBERT S. WHIPPLE, *Honorary Treasurer*

Audited and found correct,

T. MATHER
G. H. BAILLIE } *Honorary Auditors.*

February 4th, 1927.

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* Sixty-one Fellows paid reduced subscriptions by the arrangement with the Institute of Physics, the total rebate amounting to £27 12s. 0d.

† "Voluntary Subscriptions" are subscriptions paid by Fellows who compounded for the low sum of £10.

BALANCE SHEET AT DECEMBER 31st, 1926.

LIABILITIES.	£	s.	d.	£	s.	d.
Accumulated Fund:—						
Balance as per last account	1,836	7	0			
Entrance Fees, 1926	38	17	0			
	1,875	4	0			
Add Increased Value of Investments	5	0	0			
	1,880	4	0			
Less Excess of Expenditure over Income, 1926	1	6	1			
				1,878	17	11
Life Compositions:—						
Balance as per last Account	2,205	10	0			
Added during the year	84	0	0			
				2,289	10	0
Duddell Memorial Trust Fund as per last Account	428	12	9			
Add Increased Value of Investments	1	0	0			
				429	12	9
W. F. Stanley Trust Fund as per last Account	423	0	0			
Add Increased Value of Investments	5	0	0			
				428	0	0
Sundry Creditors				253	18	0
Subscriptions paid in advance				38	15	10
				£5,318	14	6

ASSETS	£	s.	d.	£	s.	d
Investments at Market Value on date:—						
£399 London Midland & Scottish Railway 4 per cent. Debenture Stock	319	0	0			
£1,000 London Midland and Scottish Railway 4 per cent. Preference Stock	740	0	0			
£200 Metropolitan Board of Works 3½ per cent. Stock	192	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	276	0	0			
£254 2s. 9d. New South Wales 5 per cent. 1935-55	240	0	0			
£300 Southern Railway Preferred Ordinary Stock	234	0	0			
£442 Southern Railway Deferred Ordinary Stock	194	0	0			
£150 Southern Railway 5 per cent. Debenture Stock	150	0	0			
£500 London & North Eastern Railway 4 per cent. Debenture Stock	377	0	0			
£500 India 3½ per cent. Stock	355	0	0			
£650 4% Funding Loan, 1960-90	552	0	0			
£300 5% War Loan 1929/47 inscribed, "A" Account	302	0	0			
£400 5% War Loan 1929/47 inscribed, "B" Account (Duddell Memorial Fund)	403	0	0			
				4,334	0	0
Stock of Publications (as per Treasurer's valuation)				400	0	0
Subscriptions due	36	10	3			
Due from Fleetway Press (Contra)	179	17	0			
Due from Customs and Excise for Income Tax Reclaim	27	10	10			
Paid on Account 1927 Exhibition	20	0	0			
				263	18	1
Cash at Bankers	317	13	7			
Cash in hand (Treasurer)	0	8	10			
Cash in hand (Secretary)	2	14	0			
				320	16	5
				£5,318	14	6

Audited and found correct.

ROBERT S. WHIPPLE, *Honorary Treasurer.*

February 4th, 1927.

T. MATHER
G. H. BAILLIE } *Honorary Auditors.*

Note.—The value of the Society's library has not been brought into the Balance Sheet.

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1926.

	£	s.	d.
128 Fellows paid £10	1,280	0	0
3 Fellows paid £15	45	0	0
1 Fellow paid £20	20	0	0
1 Fellow paid £20 10s.	20	10	0
14 Fellows paid £21	294	0	0
20 Fellows paid £31 10s.	630	0	0
167	£2,289	10	0

W. F. STANLEY TRUST FUND (FOR THE "BULLETIN").

	£	s.	d.		£	s.	d.
£300 Southern Railway Preferred Ordinary Stock	234	0	0	Carried to Balance Sheet	428	0	0
£442 Southern Railway Deferred Ordinary Stock	194	0	0				
	£428	0	0		£428	0	0

DUDDELL MEMORIAL TRUST FUND.

CAPITAL.

	£	s.	d.		£	s.	d.
£400 War Loan 5% 1929/47 inscribed Stock	403	0	0	Carried to Balance Sheet	403	0	0

REVENUE.

	£	s.	d.		£	s.	d.
Balance at 31st December, 1925	26	12	9	Honorarium to Medallist	20	0	0
Dividends	20	0	0	Balance carried to Balance Sheet	26	12	9
	£46	12	9		£46	12	9

Audited and found correct,

T. MATHER
G. H. BAILLIE } *Honorary Auditors.*

ROBERT S. WHIPPLE, *Honorary Treasurer.*

February 4th, 1927.

ELECTION OF OFFICERS AND COUNCIL.

The following Officers and Members of Council were elected for the year 1927-1928 :—

President.—Prof. O. W. Richardson, M.A., D.Sc., F.R.S.

Vice-Presidents (who have filled the office of President).—Sir Oliver J. Lodge, D.Sc., F.R.S., Sir Richard Glazebrook, K.C.B., D.Sc., F.R.S., C. Chree, Sc.D., LL.D., F.R.S., Prof. H. L. Callendar, M.A., LL.D., F.R.S., Sir Arthur Schuster, Ph.D., Sc.D., F.R.S., Sir J. J. Thomson, O.M., D.Sc., F.R.S., Prof. C. Vernon Boys, F.R.S., Prof. C. H. Lees, D.Sc., F.R.S., Prof. Sir W. H. Bragg, K.B.E., M.A., F.R.S., Alexander Russell, M.A., D.Sc., F.R.S., F. E. Smith, C.B.E., F.R.S.

Vice-Presidents.—E. H. Rayner, M.A., Sc.D., Prof. E. A. Owen, M.A., D.Sc., D. Owen B.A., D.Sc., Prof. F. L. Hopwood, D.Sc.

Secretaries.—Prof. A. O. Rankine, O.B.E., D.Sc., Imperial College of Science and Technology; J. Guild, A.R.C.S., D.I.C., National Physical Laboratory, Teddington, Middlesex.

Foreign Secretary.—Sir Arthur Schuster, Ph.D., Sc.D., F.R.S.

Treasurer.—R. S. Whipple, 45, Grosvenor Place, S.W.1.

Librarian.—J. H. Brinkworth, M.Sc., A.R.C.S., Imperial College of Science and Technology.

Ordinary Members of Council.—R. W. Paul, Prof. A. M. Tyndall, D.Sc., T. Smith, B.A., A. Ferguson, M.A., D.Sc., J. S. G. Thomas, D.Sc., D. W. Dye, B.Sc., Sir Richard Paget, Bart., Prof. E. N. da C. Andrade, D.Sc., Ezer Griffiths, D.Sc., F.R.S., A. B. Wood, D.Sc.

PRESENTATION OF THE DUDDELL MEDAL.

The PRESIDENT, on behalf of the Council, presented the Duddell Memorial Medal for 1927 to Mr. F. TWYMAN, F.R.S., after making the following remarks :—

The Council of the Physical Society, at its meeting on December 10, 1926, awarded the Fourth Duddell Medal for meritorious work on scientific instruments and materials to Mr. F. Twyman, F.R.S. The firm of Adam Hilger, Ltd., of which Mr. Twyman has for many years been both managing and technical director, enjoys, and enjoys deservedly, a reputation for the production of optical instruments employed in physical research which is not approached by that of any other firm in this or any other country. To quite a remarkable extent the fundamental researches which have led to the formation of the current conceptions of the nature of matter have been carried out with the aid of instruments of the necessary high degree of precision constructed in the Hilger workshops. The production of these instruments has frequently involved the solution of problems which have only been met successfully through Mr. Twyman's persistence and resourcefulness.

In addition to the services rendered to the cause of pure science, Mr. Twyman has carried out notable work on a number of technical problems. Two may be specially mentioned. His investigations on the annealing of glass (which incidentally led to "Twyman's Law" on the influence of temperature upon the mobility of the melt) are of fundamental importance, and his instruments for controlling this operation have been of service to all branches of the glass-making industry. These investigations have been an important factor in securing home supplies of reliable glassware for scientific purposes. Moreover, during the war, they led to a notable increase in the output of optical glass, by substituting for the traditional routine a novel scheme of annealing, which, without any deterioration in the quality of the product, enabled the time occupied in this operation to be greatly reduced.

As another example of technical work, the extensive series of Hilger Interferometers may be mentioned. The Michelson type of interferometer has been modified and adapted to a large number of special uses of interest to the optical industry. By the use of these instruments accurate measurements can now be made of the defects of all manner of optical parts and instruments, whether these defects are due to faulty design, imperfect workmanship or defective material. It is characteristic of Mr. Twyman that these new instruments were immediately used not merely to measure the defects, but also as a means of removing them.

VOTES OF THANKS.

The following votes of thanks were carried by acclamation: To the Hon. Auditors (proposed by Prof. A. M. **TYNDALL**, seconded by Mr. J. **BRINKWORTH**), to the retiring Officers and Council (proposed by Dr. J. S. **ANDERSON**, seconded by Dr. **EZER GRIFFITHS**), to the Governors of the Imperial College of Science (proposed by Dr. J. **VINCENT**, seconded by Mr. R. W. **PAUL**).

ORDINARY MEETING FOLLOWING THE ANNUAL GENERAL MEETING.

The Presidential address was delivered by Prof. O. W. **RICHARDSON**, M.A., D.Sc., F.R.S., who took as his subject, "The Present State of Atomic Physics."

A vote of thanks to the Lecturer was proposed by Prof. **CH. FABRY**, seconded by Mr. F. **TWYMAN**, and carried by acclamation.

February 25, 1927.

The **TWELFTH GUTHRIE LECTURE** was given by Prof. Sir **ERNEST RUTHERFORD**, O.M., D.Sc., P.R.S., who took as his subject, "Atomic Nuclei and Their Transformations."

A vote of thanks to the Lecturer was proposed by Sir **RICHARD** T. **GLAZEBROOK**, F.R.S., seconded by Mr. J. H. **JEANS**, Sec. R.S., and carried by acclamation.

March 11, 1927.

The following Papers were read :—

1. "The Measurement of the Absorption Coefficients of Light Filters," by G. M. B. DOBSON, D.Sc., and I. O. GRIFFITH, M.A.

2. "A Comparison of the Behaviour in Thermal Diffusion of Nitrogen and Carbon Monoxide, and of Nitrous Oxide and Carbon Dioxide," by T. L. IBBS, M.C., Ph.D., and L. UNDERWOOD, M.Sc.

3. "The Relighting of a Neon Lamp when Momentarily Extinguished at Voltages below the Striking Potential," by R. R. NIMMO, M.Sc.

4. "The Electrification of Dust Clouds," by G. B. DEODHAR, M.Sc.

March 25, 1927.

E. H. RAYNER, M.A., D.Sc., in the Chair.

The following Papers were read :—

1. "Acoustical Experiments with a Mechanical Vibrator," by E. MALLETT, D.Sc.

2. "The Stationary Wave Method of Measuring Sound Absorption at Normal Incidence," by E. T. PARIS, D.Sc., F.Inst.P.

3. "A Ball and Tube Flowmeter Suitable for Pressure Circuits," by J. H. AWBERY, B.A., B.Sc., and EZER GRIFFITHS, D.Sc., F.R.S.

A DEMONSTRATION of the Astrolabe and some other Mediaeval Surveying and Navigational Instruments was given by ALLAN FERGUSON, M.A., D.Sc.

April 8, 1927.

D. OWEN, B.A., D.Sc., in the Chair.

The following Papers were read :—

1. "A Gas Analysis Instrument based on Sound-Velocity Measurement," by EZER GRIFFITHS, D.Sc., F.R.S.

2. "The Scattering of X-Rays and the 'J' Phenomenon," by B. L. WORSNOP, B.Sc.

3. "The Characteristics of Thermionic Rectifiers," by Prof. C. L. FORTESCUE.

May 13, 1927.

The following Papers were read :—

1. "The Theory of Luminescence in Radio-active Luminous Compounds," by J. W. T. WALSH, M.A.

2. "Distortion of Resonance Curves of Electrically Driven Tuning Forks," by E. MALLETT, D.Sc.

A DEMONSTRATION of an Electromagnetic Relay, with Separate Make and Break Points, for Temperature Regulations, was given by Mr. J. GUILD.

May 27, 1927.

The following Papers were read :—

1. "A Duplex Reversal Key with Mercury Contacts," by EZER GRIFFITHS, D.Sc., F.R.S., and EDGAR A. GRIFFITHS.

2. "The Measurement of the Inductances of Four-terminal Resistance Standards," by L. HARTSHORN, A.R.C.S., B.Sc., D.I.C.

3. "Magnetic Disturbances and Aurora as observed by the Australian Antarctic Expedition at Cape Denison in 1912 and 1913," by C. CHREE, Sc.D., LL.D., F.R.S.

4. "Series in the Spectrum of Trebly-ionised Tin (Sn IV)," by K. R. RAO, M.A.

A DEMONSTRATION of the Production of Splashes by Electric Discharge was given by Mr. G. L. ADDENBROOKE.

June 3, 1927.

D. OWEN, B.A., D.Sc., in the Chair.

A LECTURE on the Forthcoming Solar Eclipse was delivered by E. H. RAYNER, M.A., D.Sc.

June 10, 1927.

The following Papers were read :—

1. "The Latent Heat of Evaporation of Sulphur," by J. H. AWBERY, B.A., B.Sc.

2. "The Refraction and Dispersion of Carbon Tetrachloride," by H. LOWERY, M.Sc., F.Inst.P.

3. "Regularities in the Spectrum of Ionised Neon," by P. K. **Kichlu**, M.Sc.

A **Demonstration** of the Schönherr-Hessburgh Nitrogen Fixation Arc was given by Capt. G. I. **Finch**.

June 24, 1927.

The following Papers were read:—

1. "Newton's Law for the Emission of Heat in Carbon Dioxide," by **Sybil Marshall**, B.Sc., A.R.C.S., D.I.C.

2. "Some Experiments with Sound Waves of High Frequency," by S. R. **Humby**, M.C., M.A.

3. "Some Additional Refinements for Precision Balances," by J. J. **Manley**, M.A.

A **Demonstration** of the Use of a Magnetron to Indicate the Rise of Current in an Inductive Circuit, was given by D. **Owen**, B.A., D.Sc.

June 28 and 29, 1927.

An **Excursion** to Richmond, Yorkshire, was arranged for the purpose of witnessing the Solar Eclipse of June 29, 1927.

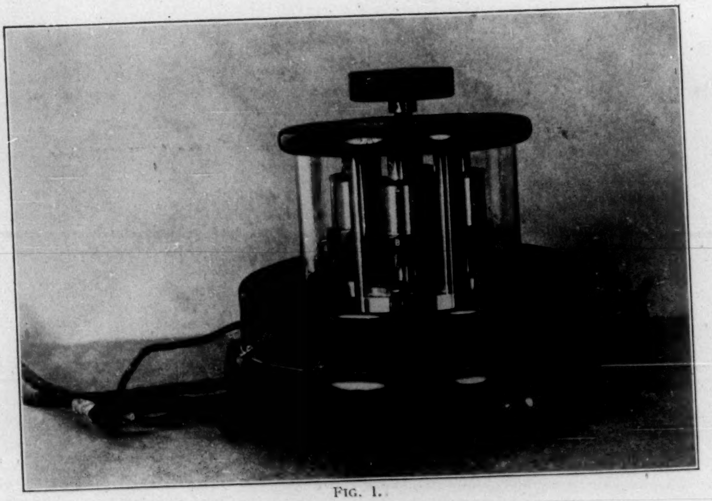

FIG. 1.

[*To face page* 375

I.—THE ABSORPTION OF RADIATION IN THE UPPER ATMOSPHERE.

The Eleventh Guthrie Lecture. Delivered April 23, 1926.

BY PROF. CHARLES FABRY.

(Translated from the Lecturer's manuscript by the Assistant Secretary.)

THE subject upon which I propose to address you is far from being a new one. It is, however, topical, for several new and interesting facts have been added to our knowledge of it in recent times, and since these discoveries have been followed with as much interest in England as in France, while I have myself contributed directly or indirectly to their study, it seemed to me that the question would be a suitable one for a lecture given by a French physicist before a British society.

LIMITATION OF THE SOLAR SPECTRUM.

The subject dates from the discovery of the limitation of the solar spectrum. Whatever the apparatus employed and whatever the height at which the observations are taken, it is impossible to detect in that spectrum any radiation of a wavelength less than about 2900 Å.U., and further investigations have shown that a similar limitation characterizes the spectra of all the stars. Moreover, since the limit shifts in the direction of increasing wavelength as the sun recedes from the zenith, the assumption naturally follows that the limitation is produced by absorption in the earth's atmosphere. A careful study of the question, made by Cornu[*] in 1881, led to the conclusion that the absorption is due to a permanent component of the atmosphere, and not to a very variable component such as water vapour or carbon dioxide; however, the absorption underwent slight variations from day to day. The nature of the absorbing component could not then be determined: As to its distribution in the atmosphere, Cornu did not rule out the lower strata: he even seems to have believed that the gas in question was uniformly distributed in altitude.

Almost at the same time, Hartley discovered that ozone has a strong absorption band in the ultra-violet, and attributed the limitation of the solar spectrum to this gas. In the absence of any numerical data his explanation remained hypothetical: chemical analysis of the lower atmosphere revealed the presence of ozone in such small quantities that its influence on the absorption remained doubtful.

In consequence of the researches of Miethe and Lehman[†] in 1909, doubt was thrown on the very existence of radiation of short wavelength in the energy emitted by the sun. These workers studied the solar spectrum at a series of successively higher altitudes, and established the fact that the limit at the ultra-violet end remained practically the same in all cases. Without denying the existence of atmospheric absorption (which is demonstrated by the fact that the limit of the spectrum shifts as the sun recedes from the zenith), they thought that there must also be strong absorption in the atmosphere of the sun, all radiation of wavelength less than 2900 being thereby completely cut off.

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[*] Cornu, Sur l'absorption atmosphérique des radiations ultra-violettes; Journal de Physique, Série 1, Vol. 10, p. 5 (1881). Sur la limite ultra-violette du spectre solaire, etc., Comptes Rendus de l'Académie des Sciences, Vol. 111, p. 941 (1890).

[†] Sitzungsberichte de Kön. Preus. Akad. der Wiss, Vol. 1, p. 268 (1909).

It was necessary, therefore, in order to carry further the study of this question, to introduce numerical data, instead of relying on merely qualitative methods. This work was begun in 1912 by M. Buisson and myself, and has since been continued by several other physicists.

ABSORPTIVE PROPERTIES OF OZONE.

Since ozone is supposed to be responsible for atmospheric absorption in the ultra-violet, it is necessary in the first place to know the absorptive properties of this gas. Its absorption of other parts of the spectrum also is interesting, for if the presence of ozone be demonstrated, all its absorption bands ought to be found in the solar spectrum.

The absorptive properties of ozone may be summarized as follows, the bands being arranged in order of increasing wavelength :—

(1). A very strong absorption band in the ultra-violet, the most intense part of which extends from 2300 to 3100, with a maximum at 2550. This band is continuous, without any structure. It may be referred to as the Hartley band.

(2). A complicated group of bands at the end of the preceding band, roughly between 3100 and 3500. These may be called the Huggins bands, since Huggins observed them for the first time in stellar spectra, although he was not aware of their origin.

(3). A group of bands discovered by Chappuis in 1882 and situated in the visible region, chiefly in the yellow and the red. The absorption in this case is much less marked than in the case of Group (2), being clearly observable only where a considerable depth of ozone is passed through.

(4). Bands in the infra-red, between 4μ and 7μ, about which scarcely anything is known.

Up to the present time, bands (1), (2) and (3) have been observed in the solar spectrum, and all of them have led to interesting results. The infra-red bands have not been observed.

THE HARTLEY BAND.

When I began, with M. Buisson, to concern myself with this question, adequate numerical data as to the absorption coefficients of ozone did not exist. We had first of all to measure these coefficients for the Hartley band.*

This determination was made by the method of photographic photometry. The source of light was a quartz mercury lamp, and the absorption was determined for all the mercury lines between 3300 and 2300. The coefficient of absorption a is defined by the equation

$$I = I_0 \cdot 10^{-ax}$$

where x is the depth in centimetres of pure gaseous ozone traversed by the radiation, while I_0, I, are the intensities of the incident and transmitted radiation respectively.

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* Fabry et Buisson, L'absorption de l'ultra-violet par l'ozone et la limite du spectre solaire, Journal de Physique, série 5, Vol. 3, p. 196 (1913).

We may also write

$$\alpha = 1/x \log_{10}(I_0/I)$$

The quantity $\log_{10}(I_0/I)$ may, as is usual in photographic technique, be called the *optical density* of the absorbent layer : the coefficient α may then be defined as the optical density of 1 cm. of pure gaseous ozone.

We have determined the values of α by a photographic method. For the purpose of reducing intensities as required diaphragms were placed over the objective of the spectrograph, while the degree of blackening of the plate was measured by means of our micro-photometer.* The result of these measurements is represented by the curve shown in Fig. 1, which gives the values of α as a function of the wavelength. It is easy to recognise the form of the absorption curve, which regularly occurs in broad bands, and there is a very pronounced maximum at $\lambda = 2550$. The maximum value of α reaches 123, an exceptionally high figure. A layer of pure gaseous ozone 25μ in thickness would reduce by one-half the intensity of radiation of wavelength 2550, and if the ozone could be reduced in volume so as to have the density of silver, the same result would be obtained with a layer $5m\mu$ in thickness. It may be said that, in this region, the absorption by ozone is metallic in character.

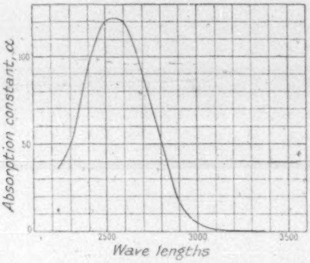

FIG. 1.

For the study of atmospheric absorption, the interesting part of the curve is that which extends to the right of the maximum, roughly between 2800 and 3200. Within this region, the values can be expressed by the following empirical formula :†

$$\log \alpha = 17{\cdot}58 - 0{\cdot}00564\lambda \quad \ldots\ldots\ldots\ldots \quad (1)$$

the wavelength λ being expressed in Ångströms.

I think it would be useful to make these measurements more complete in certain respects. At its long wavelength end the Hartley band joins the Huggins bands, of which we were not aware when our measurements were made : the curve descends in a less regular manner than is indicated by the preceding formula. Further, it would be interesting to find out for certain that the coefficients of absorption do not change with temperature and are not modified by the admixture of various gases with the ozone.

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* Revue d'Optique, Vol. 3, p. 1 (1924).

† I have had occasion to establish the fact that an analogous formula represents the descending part of the absorption curve with sufficient accuracy in the case of a great many broad bands.

THE MEASUREMENT OF THE ATMOSPHERIC COEFFICIENTS OF ABSORPTION.

In order to compare the atmospheric absorption with that of ozone and to verify the hypothesis put forward by Hartley, it is necessary to measure numerically the values of the atmospheric coefficient for radiations of various wavelengths, that is, to apply to the atmosphere the method which has just been explained as applied to ozone. If the two absorption curves coincide, we may feel certain that the absorption is due to ozone, and at the same time we shall have a measure of the quantity of this gas contained in the atmosphere.

The coefficient m of atmospheric absorption is defined by the equation

$$m=\log (I_0/I)$$

where I_0 is the intensity of the radiation before it passes through the atmosphere, and I is the intensity of the same radiation when it reaches the earth, the sun being at the zenith. As the quantity I_0 cannot be measured directly, the only way to measure m is to vary the depth of air passed through, by observing the sun at various heights above the horizon. If the sun's distance from the zenith be z, the depth of air passed through is multiplied by sec z, and the intensity I which reaches the earth is given by

$$\log I=\log I_0-m \sec z \qquad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \qquad (2)$$

The measurement of I for a series of values of z enables us to draw a graph in which the values of sec z are taken as abscissæ and those of log I as ordinates. The points obtained must lie on a straight line, the slope of which gives the coefficient m.

This is the well-known Bouguer-Langley method. It presents, however, certain special difficulties in the case we are considering, thus :—

(1). It is impossible to use the bolometric method, because the intensity of the radiation to be measured is too small after the solar spectrum has been weakened by atmospheric absorption.

(2). It is necessary to use for detecting the radiation an instrument giving very high definition, since the absorption varies very rapidly from point to point of the spectrum. A mean value taken over a short spectral range would have no meaning.

(3). The utmost care must be taken to eliminate from the radiation to be studied all that which comes from other parts of the spectrum. The latter is always in evidence on account of diffusion from the surfaces of the lenses and prisms.

The first two of the above considerations led us to employ the method of photometric photometry.

The third difficulty can be overcome in two different ways. Efforts may be made to eliminate the greater part of the undesired radiation by means of a suitably chosen absorbent : but the discovery of a suitable absorbent is a very difficult undertaking, and only a lucky chance can bring success in it. Cornu employed bromine vapour : Mr. Dobson has perfected this method, and has arrived at a really satisfactory result by the use of a filter formed of two parts, one containing bromine vapour and the other chlorine vapour.*

We have adopted another solution of the problem, consisting in the use of a double spectograph† in which only quartz is employed. The first spectroscope is formed of two 60-degree prisms, *P*, *P′* (Fig. 2), and two lenses, *L*, *L′*, having a focal length of about 1 metre. The spectrum is projected upon a horizontal slit *S*. This spectrum is contaminated by diffused light, the intensity of which is not negligible, on account of the enormous intensity in the solar radiation of the visible light and of the adjacent end of the ultra-violet spectrum. The second spectroscope gives a dispersion at right angles to the first, and purifies the spectrum; it is formed of the two prisms *p*, *p′* and the lenses *l*, *l′*. Finally the spectrum, completely purified, is projected on to a photographic plate at *T*. In this way it is practicable to make exposures up to one hour for the solar spectrum without the use of any screen.

The intensity falls off extremely rapidly towards the shorter wavelengths at the extremity of the solar spectrum. The normal exposure with slow plates is one minute for wavelengths in the neighbourhood of 2936; but with this exposure the plate is very much over-exposed for $\lambda=2975$, and for $\lambda=3100$ the normal exposure is 1/100 of a second. It would be impossible, therefore, to obtain the whole of the range with which we are concerned in a single exposure. This difficulty is overcome by covering the slit *S* with filters, which weaken a part of the spectrum. The region $\lambda<2990$ is left untouched; the region lying between 2990 and 3060 is reduced to 1/100, and the region lying between 3060 and 3200 is reduced to 1/2000.

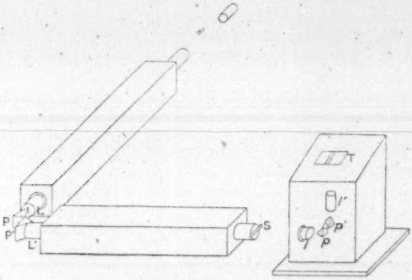

FIG. 2.

A series of exposures is made on the same plate at different hours of the day. At noon additional exposures are made with intensities reduced in known ratios by means of diaphragms, to enable the curve showing the rate of darkening of the plate to be traced, so that it may be possible to infer the intensities of the incident radiation from the corresponding densities on the plate. Measurements with the microphotometer (which take a long time and are very tedious) then make it possible to determine, for each wavelength, the intensity-ratios which obtain at different hours of the day.

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* The work of Messrs. Dobson and his collaborators has been described in several publications, of which the following is the latest and most complete: G. M. B. Dobson and D. N. Harrison, "Measurements of the Amount of Ozone in the Earth's Atmosphere," Proc. Roy. Soc., A, Vol. 110, p. 660 (1926).

† Ch. Fabry et H. Buisson, Etude de l'extrémité ultra-violette du spectre solaire, Journal de Physique, série 6, Vol. 2, p. 197 (1921).

Atmospheric absorption must be attributable to three causes :

(1). The principal absorption, which increases rapidly with diminishing wavelength, is due to the gas which we are seeking to identify with ozone.

(2). Molecular scattering in the air (Rayleigh's phenomenon) produces an absorption which can be exactly calculated for any given wavelength and solar elevation.

(3). Absorption is produced by the fog and dust contained in the air. This is practically independent of the wavelength over the small range which we are studying, but may vary in an unknown manner from hour to hour.

For a wavelength λ, let m be the coefficient of absorption, due to the unknown gas, and β that due to molecular scattering when the sun is at the zenith. If I_0 be the intensity of the radiation of wavelength λ before entering the atmosphere, the intensity I at the surface of the earth when the sun makes an angle z with the zenith will be given by

$$\log I = \log I_0 - (m+\beta) \sec z + \delta,$$

where δ is a number independent of λ, but varying irregularly during the day ; it represents absorption by fog and dust.

For a different wavelength λ' we shall have, at the same moment

$$\log I' = \log I_0' - (m'+\beta') \sec z + \delta.$$

Eliminating δ between these equations we have

$$\log I - \log I' = K - (m - m' + \beta - \beta') \sec z,$$

where K is a quantity independent of the time of day.

We take as λ' the longest wavelength in the region to be studied, so that $\lambda' = 3143$, in which case the principal absorption coefficient m' is small. For each of the other wavelengths we draw a graph, taking as abscissæ the values of sec z, and as ordinates those of $\log I - \log I'$. The curve obtained in this way must be a straight line, the slope of which gives the value of $(m - m' + \beta - \beta')$.

Fig. 3 shows a curve of this kind based on observations made on June 7, 1920. It will be seen that for each wavelength the points lie on a straight line, and that the slope of this line increases rapidly as the wavelength diminishes. This slope having been measured for each wavelength, the values of the quantity $(m - m' + \beta - \beta')$ are known ; further, the values of β and β' are known, so that the values of $m - m'$ can be calculated.

These results can then be compared with the absorption coefficients of ozone. Let us suppose that this gas is responsible for the atmospheric absorption, and let α and α' be the values of its absorption coefficient for wavelengths λ and λ' ; these values are given by equation (1). If x denote the total depth, expressed in centimetres, of pure gaseous ozone equivalent to the ozone distributed through the atmosphere, we have

$$m-m'=(a-a')x, \text{ or } x=\frac{m-m'}{a-a'}$$

If by means of this equation we calculate the value of x from data for a series of different wavelengths, we ought to obtain the same result in each case. The following table gives the figures for the data obtained on June 7, 1920. The column marked P gives the slope measured directly on Fig. 3, and the remaining columns contain quantities which have already been explained.

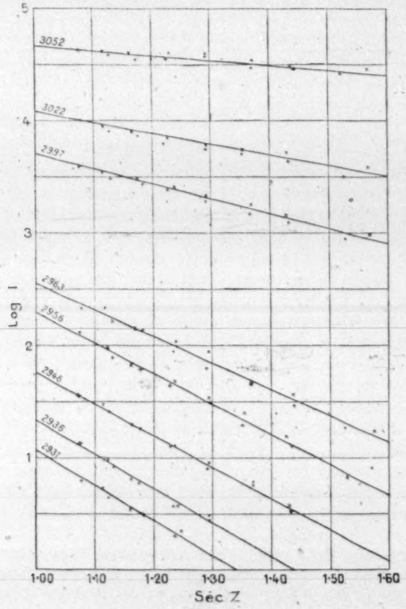

FIG. 3.

λ	β	β—β′	α	α − α′	P	$m-m'$	x
2931	0·47	0·11	11·2	10·5	3·15	3·04	0·290
2936	0·46	0·10	10·5	9·8	3·16	3·06	0·312
2946	0·46	0·10	9·3	8·6	2·78	2·68	0·312
2956	0·45	0·09	8·1	7·4	2·66	2·57	0·347
2963	0·45	0·09	7·4	6·7	2·37	2·28	0·340
2997	0·43	0·07	4·7	4·0	1·32	1·25	0·309
3022	0·42	0·06	3·4	2·7	0·96	0·90	0·333
3052	0·40	0·04	2·3	1·6	0·44	0·40	0·250
3143	0·36	0·00	0·7	0	0	0	...

It will be seen that the value found for x is practically constant,* and that the total thickness of the layer of ozone is a little more than 3 mm.

The hypothesis with regard to ozone has thus been completely confirmed. In other ways, also, it explains the facts well, and in particular it explains why it is quite impossible to observe radiation of short wavelength through the atmosphere. The shortest wavelength that it has been possible to record is 2885; the coefficient of absorption for 2898 (taking account of both ozone absorption and molecular scattering) would be 6·36—that is to say, the atmosphere allows one two-millionth part of the incident energy to pass through it when the sun is at the zenith. The importance of avoiding all extraneous illumination when recording such feeble intensities will be readily appreciated. It is probably out of the question to carry the investigation beyond this wavelength; for $\lambda=2800$, the absorption by atmospheric ozone reduces the intensity to 10^{-16} of its original value.

THE HUGGINS BANDS.

These bands were observed by Huggins in the spectrum of Sirius, but he did not discover their terrestrial origin. The true explanation of their occurrence is due to Fowler and Strutt,† who showed that they are also observable in the spectrum of the sun when the latter is near the horizon. It is more difficult, however, to detect them in the case of the sun than in the case of the whiter stars, because in the neighbourhood of 3300, where these bands occur, the solar spectrum is filled with Fraunhofer lines, which render the ozone bands less clearly apparent.

These bands are entirely characteristic of ozone, and would be sufficient of themselves to demonstrate the presence of ozone in the atmosphere. They are not observed when the sun (or a star) is near the zenith, because the absorption is not then sufficiently intense, and this fact is in good accordance with measurements made in the laboratory of the absorption coefficients for the Huggins bands. A depth of atmosphere three or four times as great as that at the zenith renders the bands clearly visible.

An interesting method for observing the Huggins bands in the light which reaches us from the sun has recently been announced by Cabannes and Dufay.‡ It consists in photographing the light which comes zenithally from the sky when the sun is very near the horizon, or even during early twilight: in this way it is possible to examine the light which has not passed obliquely through the lower strata of the atmosphere. Now although, as is indicated below, there is strong absorption by ozone in the upper atmosphere, nevertheless, in this case, radiation of the ultra-violet range considered occurs in far greater proportion than in the case of radiation transmitted mainly through the lower atmospheric strata. The reason why the proportion is greater than for direct solar illumination is that the radiation coming from the sky has been scattered in accordance with the λ^{-4} law.

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* Little importance should be attached to the value of x deduced from measurements of radiation of wavelength 3052, which is absorbed to such a slight extent that the resulting value of x is the quotient of two quantities too small to give a very precise result. The observations on 2931, also, are not very trustworthy; in this case the atmospheric absorption is too strong and the radiation too weak for accurate measurements to be made.

† Proc. Roy. Soc., A, Vol. 93, p. 577 (1917).

‡ Mesure de l'altitude de la couche d'ozone dans l'atmosphère, Comptes-Rendus de l'Académie des Sciences, Vol. 181, p. 302; 24 août (1925). Also Journal de Physique (1927).

This method of observation has led to the discovery of several new bands, the wavelengths in which lie between 3445 and 3608. It is possible that these also belong to ozone, but this is not yet certain.

CHAPPUIS BANDS.

These bands, which occur in the visible spectrum, chiefly in the red and yellow regions, can only be observed with a great depth of ozone. Chappuis* used a tube 4·50 metres long, with an ozone concentration which was not measured, but was as strong as possible. Under these conditions the gas is blue in colour.

The coefficients of absorption have been determined in my laboratory by M. Colange: Fig. 4 gives the absorption curve. The maximum value of the coefficient is that for 6000—namely, 0·05; with this should be compared the value 123 for $\lambda=2550$. A layer of ozone 3·5 mm. deep would, when the sun is at the zenith, absorb only 4 per cent. of radiation of wavelength 6000.

The Chappuis bands have never been observed directly in the solar spectrum. I have often looked for them in the spectrum of the setting sun, but have never found them. This is explained by the slightness of the difference between the maxima and minima of absorption (*see* the curve in Fig. 4), and also by the fact that in the neighbourhood of these bands the spectrum is filled with lines due to water vapour, which assume great importance when the sun is low.

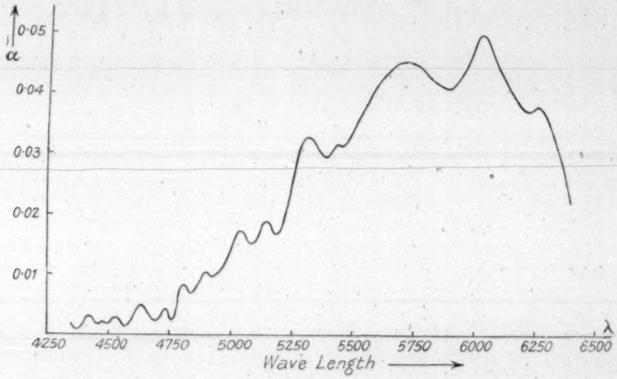

FIG. 4.

However, the atmospheric absorption due to the Chappuis bands has recently been brought out by Cabannes and Dufay,† by inference from data which have long been known. The atmospheric coefficients of absorption were determined very exactly by Fowle from his observations at Mount Wilson.‡ These data cover the infra-red, the visible spectrum, and the beginning of the ultra-violet ; they have been corrected by allowing for the effect of water vapour. Fowle remarked that molecular scattering explains the greater part of the atmospheric absorption which has been corrected in this way, but that there is a residual selective absorption chiefly in the red and the yellow. Cabannes and Dufay have shown that this residual absorption is that of ozone in the Chappuis bands. They employed a diagram due to Fowle ; since the absorption coefficient for molecular scattering varies as λ^{-4}, a graph is drawn with values of λ^{-4} as abscissæ and values of the absorption coefficient as ordinates,* and in this way the curve shown in Fig. 5 is obtained.

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* Chappuis, Annales de l'Ecole Normale Supérieure, Vol. 11, p. 137 (1882), and Journal de Physique, série 2, Vol. 1, p. 494 (1882).

† Société Française de Physique, 2 juillet (1926). Journal de Physique, Sept. (1926).

‡ Astrophysical Journal, Vol. 38, p. 392 (1913), and Vol. 40, p. 435 (1914).

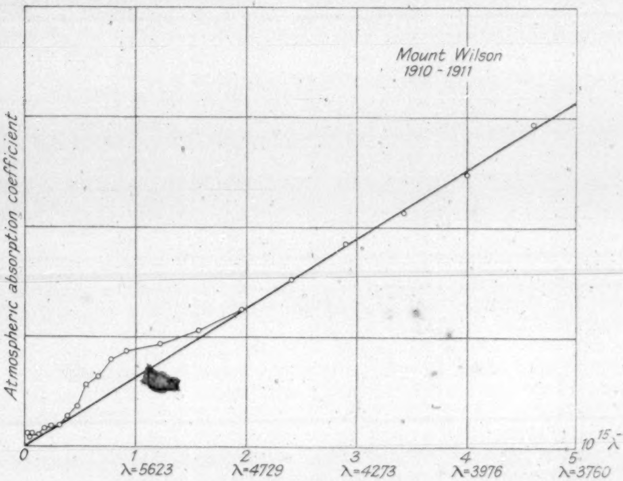

FIG. 5.

Throughout the greater part of the spectrum the points lie on a straight line, but at one place the curve breaks clearly away from the line, and it is exactly in this region that the Chappuis bands occur. The differences between the ordinates of the curve and those of the straight line must represent the coefficient of absorption of atmospheric ozone. In order to see whether this is really the case, the ordinate differences obtained in this way are used to calculate the values of the absorption coefficients of ozone (for the standard depth of 1 cm.) on the assumption that the depth of the atmospheric layer of ozone is 0·32 cm. The values so found are shown as a function of the wavelength in Fig. 6, where each individual point is deduced from one of Fowle's results, while the continuous curve is that of Colange (Fig. 4). The points lie on the curve with a precision which is truly remarkable, when it is considered that the starting point was the very difficult measurements of atmospheric absorption made by Fowle, and that the absorption by ozone constitutes only a small part of the absorption so measured.

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* In defining the absorption coefficient, Fowle employs Napierian instead of ordinary logarithms.

By treating in the same way the absorption measurements made at Mount Wilson in various years, Cabannes and Dufay believe that they can show variation in the quantity of ozone present in the atmosphere.

INFRA-RED BANDS.

These bands have never been observed in the solar spectrum; they probably lie in a region where the latter cannot be observed. It has, however, been supposed that they play a certain part in meteorological phenomena by absorbing a part of the radiation given out by the earth into space. No numerical value of the absorption coefficients is known, and it is impossible to say whether the supposed effect is real or negligible.

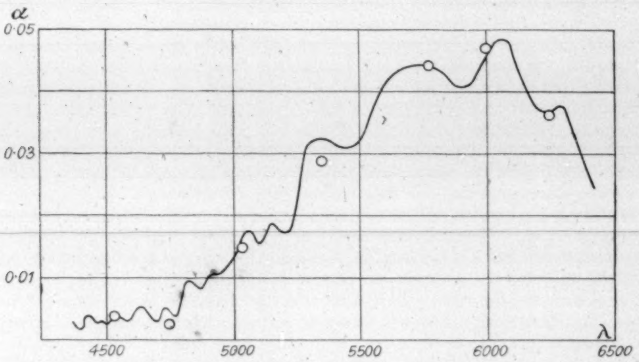

FIG. 6.

LOCATION AND ORIGIN OF ATMOSPHERIC OZONE.

The preceding observations give no information as to the distribution of ozone in the atmosphere. If the layer of 3 mm. were uniformly distributed, the proportion of ozone would be 4×10^{-7} by volume, or 60 mg. per 100 kg. of air by weight.

The chemical estimation of very small quantities of ozone appears to be a very uncertain operation. The most reliable measurements give proportions of this gas very much lower than the figures I have just mentioned. Lespieau* finds, at the surface of the earth, 3 mg. of ozone per 100 kg. of air; he finds the same proportion at the top of Mount Blanc, at a height of 4,800 metres.

An optical method of estimating the ozone in accessible layers of air was suggested in 1912 by M. Buisson and myself. It was to consist in studying the absorption in these layers with a source of light rich in ultra-violet rays, such as a mercury lamp. The experiment was carried out in 1918 by R. J. Strutt,[*] who found that the mercury line 2536 could be transmitted through 6·5 km. of air. If the ozone were uniformly distributed through the atmosphere, a layer of this depth would only transmit 10^{-33} of the radiation in question. It would be interesting to repeat this experiment in various places and to calculate from the absorption the proportion of ozone present in each case.

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* Lespieau, Bulletin de la Société chimique de Paris, série 3, Vol. 35, p. 616 (1906).

It is certain, then, that the quantity of ozone disclosed by the study of the solar spectrum is not furnished by the lower atmosphere, or even by the part of the atmosphere which is accessible on a mountain. The same conclusion follows from the fact that the limits of the solar spectrum do not alter appreciably when observations are taken at the greatest accessible altitudes. The inference is, therefore, forced upon us that the ozone occurs principally at extremely high altitudes. A theory as to the manner in which it is formed may be propounded as follows :—

Radiation of very short wavelength ($\lambda < 1850$) acts upon oxygen, transforming it into ozone ; at the same time this radiation is strongly absorbed by the air. It is probable that such radiation exists in the solar emission ; it must produce ozone in the earth's atmosphere, but only in the highest part, because it cannot reach the lower strata. Radiation of $\lambda = 1850$ is completely absorbed by 10 metres of air at atmospheric pressure, and could scarcely penetrate lower than the stratum 40 km. above the earth. On the other hand, radiation lying between $\lambda = 2000$ and $\lambda = 3000$ decomposes ozone, and between these two inverse actions a state of equilibrium must be established, the quantity of ozone depending on the relative intensities of the two ranges of radiation. We shall come back a little later on to the question of variations in the quantity of ozone. It would be extremely interesting to be able to determine the height at which the greater part of the gas is to be found. Experiments at the top of a mountain and even experiments with balloons have hitherto given no result. Together with M. Buisson, however, I have suggested the following method for attacking this problem :—

The law (equation (2)), according to which sec z represents the falling off of intensity as the zenithal distance increases, neglects the curvature of the earth. It remains accurate so long as the zenithal distance is not great, but it is no longer exact when the sun approaches the horizon. The upper strata are then traversed less obliquely than the lower strata, and the falling off is slower than is indicated by the equation. The divergence from equation (2) increases with the height at which the absorbing layer is located. In other words, if we draw the graph giving log 1 in terms of sec z we shall obtain a straight line if the absorbing gas be at a low altitude, the graph departing from linearity in an increasing degree as the altitude of the gas increases.

This method has recently been carried into effect by MM. Cabannes and Dufay,[†] by measuring the Huggins bands in the light scattered from the sky in a zenithal direction, with the result that the ozone is found to occur at a height of some 50 kilometres. This result can only be approximate, for the ozone is certainly not restricted to a shallow layer, but distributed over a considerable depth above a certain altitude. The conclusion is reached, however, that the absorbing layer lies very much higher than the altitudes which can be reached by means at our disposal.

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[*] Proc. Roy. Soc., A, Vol. 94, p. 260 (1918).

[†] Comptes-Rendus, loc.cit.

THE RADIATION EMITTED BY THE SUN.

It is only by extrapolation that we can obtain any information as to the radiation emitted by the sun for the range of wavelengths below 2885, of which no trace reaches us. The hypothesis has been put forward by Miethe and Lehmann that this radiation does not exist, in consequence of absorption in the solar atmosphere. According to these authors the range in the neighbourhood of 2900 is also greatly weakened by such absorption.

This question was studied by M. Buisson and myself after investigating the laws of absorption by the earth's atmosphere. By comparison with the radiation from the crater of the electric arc, assumed to be identical with the radiation of a black body at 3750 *K*, we obtained the spectral intensity curve for the solar spectrum for the range 3150-2898, applying to the observed values a correction for atmospheric absorption. In consequence of the Fraunhofer lines there are fluctuations from point to point in the actual solar spectrum, but there is no such marked discrepancy as would be expected if the solar atmosphere suppressed the short-wave length range.

The same conclusion is reached on comparing the spectrum of radiation coming from the centre with that coming from the circumference of the sun. If the solar atmosphere contained a layer sufficiently absorbent to eliminate the range $\lambda<2880$, such absorption would be much more marked at the edge of the sun's disc than at the centre, and the ultra-violet limit would be seen to advance on passing from the centre to the circumference. We have made the comparison by means of our double spectrograph, and we have found no appreciable difference of this kind. The spectrum for the circumference is weaker than that for the centre, but the limit is the same.

There is reason, therefore, if we extrapolate from these results, to believe that the energy curve for the radiation emitted by the sun resembles, in the ultra-violet region no less than in other parts of the spectrum, that of a black body in the neighbourhood of 6000 *K*. If so the intensity must retain a very considerable value right up to the Schumann region, and this is in fact what is assumed in the theory of the formation of ozone which has been explained above.

Duclaux and Jeantet have pointed out* that the ozone layer absorbs strongly the radiation comprised between 2900 and 2200, but that for wavelengths less than this limit it becomes transparent once more, so that we ought to receive the radiations comprised between 2200 and 2000 if ozone were the only absorbent gas. They have tried to find out whether any other gas could absorb this range of radiation, and have ascertained that ammonia possesses a very intense absorption band in the region in question. The quantity of ammonia existing in the lower atmosphere in consequence of the decomposition of organic matter is supposed to be sufficient to cut off this radiation completely. If this theory were correct, we should be screened from radiation in the range 2900-2200 by the ozone of the upper atmosphere, and from the range 2200-2000 by the ammonia of the lower atmosphere. At the top of a high mountain it would be possible to detect the latter range.

This question has been investigated by MM. Chalonge, Déjardin and Lambert during an ascent of Mont Blanc, by means of a double spectrograph specially designed to ensure the complete elimination of diffused light.† They employed oiled plates, in accordance with the technique described by Duclaux and Jeantet,* and also Schumann plates. No trace of radiation in the region 2200-2000 could be found. The question, therefore, remains unsettled. Possibly oxygen in great depth is sufficiently absorbent to explain the absence of such radiation.

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* Duclaux et Jeantet, Journal de Physique, série 6, Vol. 4, p. 115 (1923).

† Revue d'Optique, Vol. 3, p. 277 (1924).

VARIATIONS IN THE QUANTITY OF ATMOSPHERIC OZONE.

Cornu noticed that the limit of the solar spectrum, for a given zenithal distance of the sun, varies from day to day in an irregular manner. M. Buisson and I have observed the same fact, and have come to the conclusion that there is a variation in the quantity of atmospheric ozone. Our method gives for each day the thickness of the ozone layer reduced to atmospheric pressure. On a series of 14 days during 1920 we found values ranging from 2·85 to 3·35 mm. These values are much in excess of the probable error of the observations, and we confirmed them by means of a series of spectra taken on the same plate at noon on several consecutive days.

This question has recently aroused keen interest. An extensive research has been undertaken by Messrs. Dobson and Harrison,† who believe that they have found a connection between the variations in the ozone and the meteorological conditions prevailing in Western Europe, and particularly in Great Britain. On the other hand, Abbot has expressed the view that these variations might be related to the supposed variation of the solar constant. A series of measurements was made during the summer of 1925 by Buisson and Jausseran,‡ at a station where the sky is very clear, in the southern part of the French Alps, near Digne. They found variations in the ozone extending to 15 per cent. but unrelated to local meteorological conditions. That, however, is not necessarily in conflict with the results of Dobson and Harrison, since the Digne station is in the Mediterranean zone, where the meteorological conditions are very different from those of England.

ENERGY ABSORPTION AND TEMPERATURE IN THE UPPER ATMOSPHERE.

The amount of energy absorbed by atmospheric ozone from the radiation of the sun is far from being negligible. It would be possible to calculate it exactly if we knew the whole of the spectral energy curve for that radiation, but the extreme ultra-violet region is only known by extrapolation. If we identify the solar spectrum with that of a black body we find that the ozone absorbs 4 per cent. of the incident radiation when the sun is at the zenith.

The effect of this absorption must be to raise perceptibly the temperature of the upper strata of the atmosphere. From meteorological considerations, Lindemann and Dobson have found good reason to suppose that the temperature of the extreme upper atmosphere is much higher than that of the stratosphere, and perhaps comparable with that of the lower strata of temperate regions.

It will be seen that several of the questions raised by the study of atmospheric absorption still await solution, but already the results have considerably exceeded what might have been expected from a subject at first sight very simple. It is probable that further important results will be yielded by it.

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* Revue d'Optique, Vol. 2, p. 384 (1923).

† Dobson and Harrison, loc.cit.

‡ Comptes-Rendus de l'Académie des Sciences, Vol. 182, p. 232 (1926).

II.—THE CORROSION PRODUCTS AND MECHANICAL PROPERTIES OF CERTAIN LIGHT ALUMINIUM ALLOYS AS AFFECTED BY ATMOSPHERIC EXPOSURE.

By Professor ERNEST WILSON.

Received May 26, 1926.

(*Communicated by* Prof. E. F. HERROUN.)

ABSTRACT.

The Paper gives an account of experiments which have been made upon the electrical conductivities, the corrosion products and tensile properties of high purity aluminium and certain light aluminium alloys, which have been exposed to London atmosphere for a period of 24 years. The elements concerned are copper, nickel, manganese and zinc in varying amounts up to a few per cent. The variation of electrical conductivity, the growth of the corrosion product and its composition have been studied, and the tensile properties before and after exposure have been compared. Finally, there is a note on the corrosion products of high conductivity copper.

THIS Paper deals with further experiments which have been made on the corrosion products and mechanical properties of certain light aluminium alloys.[*] All the specimens are in the form of wire 0·126 inch (0·32 cm.) diameter, and have been exposed on the roof of King's College, London. Certain data relating to electrical measurements and loss of metal by corrosion have been taken from a recent Paper in the "Journal" of the Institution of Electrical Engineers.[†] It is there shown that if $R_0 R_t$ are the electrical resistances at time O, the commencement of exposure, and time t during exposure, $\rho_0\ \rho_t$ the corresponding specific resistances, a_0 the initial volume of unit length of conductor, and b_t the ratio of time t of the weight of metal lost per cm. length of conductor to the specific gravity of the metal, then

$$\frac{R_t}{R_0}=\frac{\rho_t}{\rho_0}\times\frac{a_0}{a_0-b_t}$$

where the ratio R_t/R_0 is separated into the two terms dealing with the variation of specific resistance and loss of metal respectively. In Fig. 1 curves are given to illustrate the variation of the ratio R_0/R_t with time of exposure t. In Table I are given the chemical analysis, the electrical measurements, and the magnetic susceptibility of the corrosion products. In Table II corrosion products are dealt with from the standpoint of the amount of their variation with time during exposure. Table III

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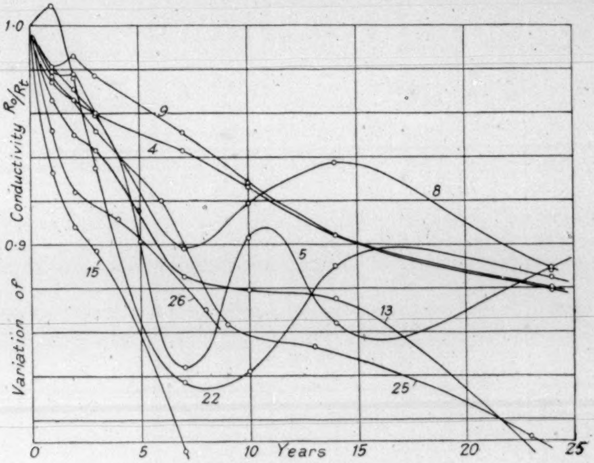
[*] Report of the British Association for the Advancement of Science, 1901, p. 771; 1902, p. 734; 1903, p. 777; 1904, p. 686; 1908, p. 827; 1911, p. 480; 1912, p. 570; 1913, p. 606; 1915, p. 647; also Engineering, 1901, Vol. 72, p. 464; Electrician, 1902, Vol. 49, p. 868; 1903, Vol. 51, p. 898; 1904, Vol. 53, p. 752; 1908, Vol. 61, p. 837; 1911, Vol. 67, p. 907; 1912, Vol. 69, p. 945; 1915, Vol. 75, p. 886; also Journal Inst. Elec. Eng., 1902, Vol. 31, p. 321; and Journal of the Royal Society of Arts, 1901-2, Vol. 50, p. 54.

[†] Journal Inst. of Elec. Eng., Vol. 63, p. 1108 (1925).

TABLE I.

Group.	Number of Specimen.	Time in Years.	Analysis								Initial Values, 1901.		Unexposed Specimens.		Exposed Specimens.						Gms. per cm. of corrosion remaining on wire.	Gms. of metal lost per cm. length of conductor	Magnetic Susceptibility in 10^{-6} C.G.S. Units				
											Sp. res. microhms per cm.^{3} at 15°C.	Resistance per cm. microhms at 15°C.	Sp. res. microhms per cm.^{3} at 15°C.		Sp. res. microhms per cm.^{3} at 15°C.	Resistance per cm. microhms at 15°C.			R_t/R_0				Un exposed		Exposed	Corrosion Product	
		t	Si	Fe	Cu	Ni	Mn	Zn	Mg	Al	ρ_0	R_0	ρ_t	ρ_t/ρ_0	ρ_t	R_t	ρ_t/ρ_0	$\frac{a_0}{a_0-b_t}$	Calculated	Observed			1917 K_v	1926 K_v	1926 K_v	K_m	Sp. gravity
I	25	0	0·14	0·31	...	...	...	...	...	99·55	2·773	34·39	...	...	...	...	...	...	...	...	...	...	4·17	...	...	...	...
	25	8	0·14	0·31	...	...	...	...	...	99·55	2·773	34·39	...	...	2·943	41·12	1·061	1·127	1·193	1·150	0·01383	0·0247	...	...	3·59	105	1·11
	25	23	0·14	0·31	...	...	...	...	...	99·55	2·773	34·39	2·773	1·00	3·032	43·67	1·093	1·160	1·269	1·240	0·0112	0·0362	...	3·73	3·07	161	...
II	4	24	0·38	0·25	0·16	...	...	...	...	99·21	2·88	36·38	2·893	1·005	3·187	42·20	1·107	1·048	1·160	1·140	0·0035	0·0100	...	...	...	...	...
	13	24	0·38	0·25	1·58	...	...	...	...	97·79	3·30	41·56	...	...	4·113	54·09	1·246	1·044	1·301	1·280	0·0030	0·0092	...	...	1·93	...	...
	15	24	0·40	0·40	2·61	...	...	...	...	96·59	3·34	...	...	...	...	...	...	...	...	...	...	...	...	2·32	...	...	...
III	7	24	0·37	0·25	0·05	0·75	...	...	...	98·58	3·05	38·30	...	...	3·327	42·83	1·091	1·025	1·119	1·110	0·0030	0·0054	...	...	...	...	...
	8	24	0·35	0·29	0·09	1·19	...	...	...	98·08	3·24	40·76	3·161	·9757	3·634	45·68	1·121	1·017	1·140	1·130	0·0036	0·0036	...	...	...	...	...
	21	24	0·39	2·57	0·10	1·39	...	...	...	95·55	3·195	40·00	...	...	3·454	44·51	1·081	1·030	1·113	1·106	0·0028	0·0063	7·90	...	5·95	...	...
	22	24	0·37	0·43	1·08	1·29	...	...	...	96·83	3·28	41·44	3·386	·9694	3·797	48·97	1·144	1·030	1·183	1·130	0·0020	0·0064	5·46	...	...	...	...
IV	9	24	0·31	0·35	0·03	...	0·35	...	...	98·96	3·30	41·64	...	...	3·642	47·25	1·104	1·019	1·124	1·136	0·0028	0·0060	...	...	...	...	...
	23	24	0·44	0·56	0·09	...	1·78	...	...	97·13	3·35	42·06	...	...	3·683	48·60	1·100	1·051	1·156	1·160	0·0037	0·0106	8·26	...	2·28	...	...
	26	8	...	...	3·5 to 5·5	...	0·5 to 0·8	...	0·5	...	5·35 (1911)	64·95 (1911)	5·106	·9543	5·929	74·71	1·109	1·038	1·150	1·150	0·01109	0·0083	...	...	0·86	161	...
V	5	24	0·43	0·39	0·09	...	...	2·04	...	97·05	3·07	38·47	...	...	3·368	43·87	1·097	1·040	1·141	1·127	0·0022	0·0084	...	...	...	...	...
VI	Hard-drawn Copper	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	12·8	150	...
	Annealed Copper	4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	4·27	86	1·85

gives the data derived from the tests made of the mechanical properties of representative alloys. Owing to the fact that about half the alloys, numbering 26, were annealed before exposure, the choice of specimens was restricted. It was, however, possible to test unannealed specimens from each of the groups dealing respectively with alloys of copper, nickel, manganese and zinc, in addition to the high purity specimen No. 25.



Variation of the ratio R_0/R_t with time of exposure.

FIG. 1.

TABLE II.

Group.	No.	Exposure. in years.	Specific Resistance. Microhms/cm.^{3} 15°C.	Radius of Metal.	Radial Thickness of Corrosion Product.	Volume of Corrosion per cm. of Conductor.	Gms/cm. of Corrosion remaining on Wire.	Apparent Density of Corrosion Product	Density measured.	Gms. of Metal Lost per cm. of Conductor.	Superficial Area per cm. of Conductor.	Gms. of Metal Lost per sq. cm. of Superficial Area.
				cm.	cm.	cm.^{3}					cm.^{2}	
I	25	0	2·773	0·1603	...	...	...	...	...	...	1·007	...
		8	2·943	0·1510	0·0137	0·01358	0·01383	1·018	1·11	0·0247	0·978	0·0253
		23	3·032	0·1488	0·0137	0·01339	0·0112	0·836	...	0·0302	0·974	0·0310
IV	26	0	5·350	0·162	...	...	...	...	...	...	1·018	...
		2	5·650	0·161	0·0063	0·006496	0·00800	1·23	...	0·00546	1·012	0·00553
		8	5·930	0·159	0·0058	0·005898	0·01109	1·88	...	0·00830	0·999	0·00829

TABLE III.

Group	No.	Time of Ex-posure.	Di-ameter	Area	Elastic Limit Tons per sq. in.		Breaking Load. Tons per sq. in.		Modulus. Tons per sq. in.		Ratio Elastic Limit / Breaking Load		Elongation.				
													Per cent.				Local Extension at Fracture per cent. of Diameter
											Per cent.		Including Fracture on 35 cms.		Excluding Fracture		
					1901	1926	1901	1926	1901	1926	1901	1926	1901	1926	1901	1926	1926
		Years	Ins.	Ins.^{2}													
		0	0·126	0·0125	8·65	5·35	12·6	11·4	3,800	3,750	67·5	47	1·86	1·6	...	1·0	70
						5·30		11·45									
I	25	8	0·119	0·0111	...	4·75	...	11·0	...	3,900	...	43	...	0·95	...	0·55	45
		23	0·115	0·0104	...	5·00	...	11·4	...	3,950	...	44	...	0·9	...	0·5	45
	4	24	0·120	0·0113	7·88	5·6	11·31	10·3	3,670	3,900	69	54	3·4	2·0	...	1·50	60
II	13	24	0·1207	0·0114	14·3	3·8	17·9	7·8	3,690	4,200	83	47	3·7	...	...	3·0	...
III	8	24	0·1238	0·0120	10·4	5·7	14·9	12·8	3,020	4,050	70	44	4·0	0·75	2·5	0·5	27·5
	9	24	0·116	0·0106	9·85	5·9	13·4	13·0	3,920	4,200	73	45	2·0	1·6	1·5	1·0	70
IV	26	8	0·1243	0·0122	...	7 to 13	30(1911)	21·0	...	3,800	...	33 to 62	12 to 15 (1911)	0	...	0	0
V	5	24	0·119	0·0111	9·13	5·2	11·5	10·4	4,180	4,250	80	50	3·2	1·0	2·4	Less than 1·0	40

I. HIGH PURITY ALUMINIUM.

Aluminium for electrical purposes has a purity of 99·5 per cent. or over, and from this standpoint specimen 25 is of special interest. Curve 25, Fig. 1, shows that conductivity diminishes at a greater rate during the first 10 years of exposure than subsequently, and the diminution of conductivity after 23 years' exposure is more largely due to loss of metal than to increase in specific resistance (Table I). The loss of metal in gms. per sq. cm. of superficial area, and also the variation in specific resistance are given in Table II, and illustrated by curves in Fig. 2. Neither of these curves is parabolic, but the latter is more nearly so than the former. Examination shows that the radial thickness of corrosion product remaining on the conductor is substantially the same at eight and twenty-three years' exposure, and this is possibly due to the diminished rate of production of corrosion as time goes on, accompanied by loss of corrosion due to exposure. The values of the apparent density in Table II, as determined by the method of measuring diameters, are necessarily approximate, and are somewhat lower than the separately determined value at eight years' exposure. Examination of the corroded portion, kindly made by Prof. E. F. Herroun, F.I.C., and given in the previous Paper, showed that it consisted mainly of alumina and metallic aluminium. It also contained sulphate in appreciable quantity, and a trace of chloride, also traces of iron, probably derived in part from the emery paper used for its removal, but no zinc or nickel. There was, therefore, a question as to the origin of the iron. A sufficient quantity for further test was removed with the sharp edge of a piece of glass, and further examination shows that there is much more than the 0·31 per cent. of iron contained in the aluminium itself. To test this further, measurements of magnetic susceptibility have been made, and these are given in Table I and illustrated by curves in Fig. 1. The mass-susceptibility k_m was found to be greater after 23 years' exposure than after 8 years. Assuming the density to have an approximate value equal to unity,

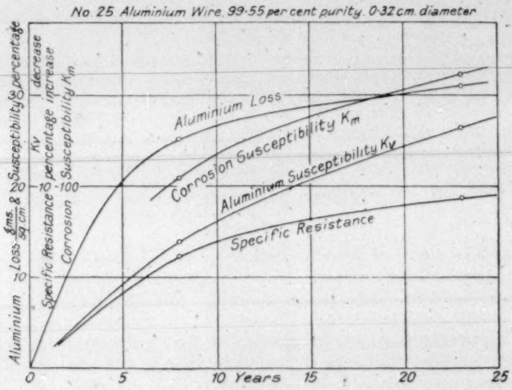


FIG. 2.

the volume and mass-susceptibilities k_v k_m have about equal values, and this enables a direct comparison to be made with the susceptibility of the aluminium itself. The first magnetic tests of these alloys were made on the unexposed specimens in 1917.* The values given in Table I show that as time progresses the magnetic susceptibility of the aluminium itself diminishes in value. Also the susceptibility of the corrosion product is very large in comparison with that of the aluminium itself. The conclusion is that the iron present in the corrosion product must have come from an outside source.

In the mechanical tests, the results of which are given in Table III, the values are those obtained on a first loading. The distance between the grips was 9 inches, and the extension was measured between marked points 3 inches apart on the central portion of the wire. The accuracy obtainable with the extensometer used was 0·0001 inch on 3 inches. The tests bring out one important feature, in that the 1926 values of the breaking load and the elastic limit are substantially the same in the unexposed and exposed specimens. The reduction on the 1901 value of the breaking load amounts in 23 years to about 9 per cent., and that of the elastic limit is 60 per cent. As regards the 1926 elongation values, it will be noted that a very considerable reduction occurs as between the unexposed and exposed specimens. On the other hand, the elongation including fracture obtained in 1901 has only diminished 14 per cent. The break was normal in character, with the usual drawing down.

A comparison of the electrical and mechanical properties of specimen No. 25 with those of present-day wires is given in the recent Institution of Electrical Engineers' Paper.†

II. COPPER.

It is interesting to compare the curves 4, 13 and 15 in Fig. 1 as regards the variation in electrical conductivity. Alloy 4 (curve 4), containing 0·16 per cent. of copper, diminishes in conductivity at a fairly steady rate. Table I shows that its increase in electrical resistance is laregly due to increase in specific resistance, and the loss of metal is small. The presence of the copper has very largely reduced corrosion, and the final resistance R_t is smaller than in the case of alloy 25. Both the breaking load and the elastic limit are smaller than in No. 25, and the elongation is considerably larger.

Alloy 13 was annealed before exposure, so that the 1926 figures cannot properly be compared with those of No. 4. The tests made in 1901 before annealing showed that the tensile strength was considerably increased by the larger proportion of copper, and this was accompanied by an increase in specific resistance. Comparing curves 13 and 25 in Fig. 1, there is a similarity, but the increase in specific resistance in No. 13 almost wholly accounts for the fall in conductivity. It was not possible in No. 13 to estimate the elongation including fracture. The break shows a flaky and brittle material at the surface of the wire, thereby preventing reduction of section at the fracture, which invariably takes place in the grips. The core appears to be very ductile and possibly strong, but the outer material cracks long before the breaking point of the centre portion is reached. When the percentage of copper reaches 2·61, as in alloy No. 15, a most unsatisfactory material is produced. Curve 15, Fig. 1, shows that the conductivity falls rapidly during exposure, and, as was pointed out in 1911,* almost complete disintegration had then taken place.

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* Proc. Roy. Soc., A, Vol. 96, p. 429 (1920).

† Ibid.

III. NICKEL.

One remarkable feature of the nickel alloys is the way in which electrical conductivity varies during exposure. Reference should be made to the curves 8 and 22, Fig. 1, in this connexion. From the information given in Table I it will be gathered that the final increase in resistance is almost wholly due to increase in specific resistance; whereas specific resistance is reduced in value in the case of the unexposed specimens tested.

Alloy No. 8 was not annealed before exposure, and has been tested for its mechanical properties. The breaking load before exposure in 1901 was fairly high, and it has been well maintained, and the total loss of metal by corrosion has the relatively small value of 0·0036 gms. per cm. length of conductor. The elastic limit and percentage elongation have suffered considerable reduction.

Specimens 21, 22 were annealed before exposure, but they have lost very little by corrosion, and are in excellent condition. They had the highest tensile strength of any of the series before annealing. No. 22 is specially interesting, in that its copper-nickel content, treated as a group, has proportions (Cu 45·6, Ni 54·4) more nearly approaching those of the well-known alloy constantan. No. 22 has a relatively low temperature coefficient (0·00173), which is about half that of the other alloys. Its specific resistance is relatively high, as also its coefficient of linear expansion (0·0000252), thereby pointing to a possible direct influence of the Cu-Ni proportions.

Having regard to the great variation of the ratio R_0/R_t in the nickel alloys of aluminium, it was thought that if specimens of No. 25 and No. 8 were submitted to alternate heating and cooling they might vary resistance after the manner of the curves 25 and 8 in Fig. 1. For this purpose a hydraulic motor, controlled by a clock, was used to raise and lower the two specimens from a bath of oil maintained at a temperature of 80°C. The time intervals were 7½ minutes in the oil bath and 7½ minutes in air. The conductors were tested at intervals for electrical resistance. In the case of the high purity specimen 25, the resistance continuously fell from its initial value, but the nickel alloy behaved after the manner of curve 8, Fig. 1. The number of cycles required to produce a fairly constant resistance was 5,122. In Fig. 3, curve 1 shows the variation of specific resistance in alloy 8 during exposure, and curve 2 that of the same alloy during the above heating and cooling experiment.

IV. MANGANESE.

Alloy 26 is a specimen of duralumin first exposed in 1911. Before exposure the breaking load was stated to be 30 tons per sq. inch and the elongation 12 to 15 per cent. It was pointed out in the Reports of the British Association (1913 and 1915) that it becomes brittle on exposure. The specimen was exposed for 8 years, and since then has been loosely coiled up in the laboratory. The breaking load is now 21 tons per sq. inch, and the material is now extremely brittle, and no elongation was measurable. At fracture there is an outside annulus of darker material than at the centre, and there is no contraction at all.

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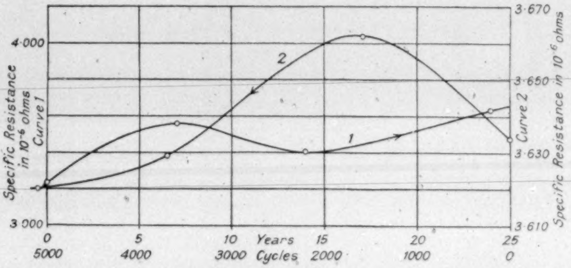
* The Electrician, Vol. 67, p 907 (1911).

It is shown in Table I that an unexposed specimen of duralumin (26) has diminished in specific resistance; whereas the exposed specimen has largely increased, the ratio ρ_t / ρ_0 being 1·109. The loss of metal per sq. cm. of superficial area is about one-third that of alloy 25, the period of exposure being 8 years in each case. The radial thickness of corrosion remaining is practically the same at 2 and 8 years (Table II), and this was also the case with specimen 25. Its temperature coefficient (0·00173) of electrical resistance is identical with that of specimen 22. An interesting comparison is that between curves 15 and 26 in Fig. 1. The former is a copper alloy containing 2·61 per cent. of copper, and it is possible that the high percentage of copper in the duralumin is responsible for the rapid fall in conductivity.

The magnetic susceptibility of the duralumin in the exposed condition is very much smaller than that of the high purity aluminium. This is remarkable, as manganese in the presence of copper and aluminium might be expected to enhance susceptibility. Copper alone (alloy 15) gives a high result relatively to pure aluminium,* and manganese in the presence of small copper content in alloy 23 also gives a high result. It would seem that the low susceptibility in No. 26 is either associated with magnesium present or duration of exposure (*see*, No. 23). The susceptibility of the corrosion product of duralumin is identical with that of the 23 year exposed high purity aluminium 25.



Variation of electrical resistance with cyclical changes of temperature.

FIG. 3.

No. 9 is another manganese alloy, not containing magnesium, which was not annealed before exposure, and which has been tested for mechanical properties. It will be seen in Table III that the breaking load during 24 years' exposure has been well maintained, and the elongation, though diminished, is equal to that in the unexposed No. 25. Curve 9, Fig. 1, shows that the conductivity gradually falls, and this is due almost wholly to increase in specific resistance. Alloy 23 is very similar in its behaviour. The magnetic susceptibility of the exposed metal, though not so small as in duralumin, is very much smaller than in the unexposed specimen.

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* See Landolt-Börnstein, and Kaye and Laby Tables.

V. ZINC.

Specimen 5, containing about 2 per cent. of zinc, shows a variable conductivity, as in the case of the nickel alloys. The loss of metal is small, and about equal to that in the duralumin. As the specimen was unannealed, it was tested for its mechanical properties. The ultimate strength has dropped from 11·5 to 10·4 tons per sq. inch in 24 years, and the elongation has fallen considerably.

VI. HIGH CONDUCTIVITY COPPER.

The corrosion products on annealed and hard-drawn copper have been kindly examined by Prof. E. F. Herroun, F.I.C. In the annealed specimen the aqueous solution from the powder (cold) contained quite an appreciable amount of $Cu\ SO_4$, but no soluble iron. The residue was partly soluble in HCl, giving both $Cu_2\ Cl_2$ from cuprous oxide and $Cu\ Cl_2$ from cupric oxide. It also contained a quite definite amount of iron and traces of alumina, but rather less of the latter than in the case of the hard-drawn specimen. There was a fair amount of copper sulphide insoluble in HCl, and also carbon. After 4 years' exposure the volume-susceptibility had the value $4{\cdot}27 \times 10^{-6}$, and the mass-susceptibility of the corroded portion was 86×10^{-6}. The specific gravity of the corrosion product was found to be 1·85.

The deposit from the hard-drawn wire contained copper sulphide, oxide and basic sulphate. It also contained carbon and iron (probably as an oxide) in considerable amount; also alumina, probably from neighbouring wires. After 4 years' exposure the volume-susceptibility of the copper was $12{\cdot}8 \times 10^{-6}$, and the mass-susceptibility of the corrosion 150×10^{-6}. The mechanical properties of these conductors have not been examined, and the results of the electrical tests were given in the recent Institution of Electrical Engineers' Paper.*

In conclusion, the author wishes to cordially acknowledge the help he has received from Mr. C. M. White, B.Sc., who carried out the mechanical tests.

DISCUSSION.

Mr. J. C. HUDSON: I should like to express appreciation, for Dr. W. H. J. Vernon and myself, of your kind invitation to contribute to the discussion. Dr. Vernon, who greatly regrets his unavoidable absence this evening, has now been engaged in an investigation on atmospheric corrosion for the British Non-Ferrous Metals Research Association for several years. The study of the more fundamental aspects of the problem has left him very little time for field tests, and I was therefore appointed some eighteen months ago to extend the investigation to outdoor conditions of exposure. Unfortunately, the sensitive physical methods (increase in weight and loss in reflectivity) used by Dr. Vernon in his experiments on tarnishing in indoor atmospheres† are not applicable outdoors, beyond the early stages of attack, and ultimately we decided to use the change in resistance of a wire specimen as a quantitative index of corrosion. Our work in this field is thus very similar to that of Professor Wilson, and we are extremely interested in his results.

We have, however, developed our research along slightly different lines. Professor Wilson's work is perhaps more intimately concerned with the mechanical and electrical properties of the materials than with corrosion proper; in consequence he has been led to use wires of dimensions that would be far too unwieldy for our purpose, as we wish to arrange for exposure in several localities and to obtain results in a much shorter time.

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* Ibid.

† W. H. J. Vernon, Trans. Far. Soc., 19, 839 (1924); J. C. S., 2,273 (1926).

The relative increase $\frac{dR}{R}$ in the resistance of a circular wire of radius r, due to the removal of a thin film of metal of thickness dt is given by

$$\frac{dR}{R}=2\frac{dr}{r}$$

It follows that if dr, the depth of the metal corroded, is independent of the dimensions of the wire, the observed change in resistance is inversely proportional to the radius, and the time necessary to bring about a given change can be considerably reduced by the use of thin wire. We are therefore using much smaller specimens than Professor Wilson, and the gain in portability, ease of manipulation and time of exposure will probably compensate for the loss of the mechanical tests.

The equation given above is based on the assumption that the specific resistivity of the material remains constant, and we therefore view with some concern the fact that Professor Wilson reports variations in this property. It is probable, however, that these irregularities, particularly those shown by alloys 5, 8 and 22, might have been prevented by suitable treatment of the materials before exposure, since it is known, for instance, that changes in internal structure may be very greatly reduced by annealing. Thus Matthiessen,* and later Fitzpatrick,† found that the resistance of annealed wires did not vary over many years. The unsatisfactory behaviour of nickel silver has been overcome by modern metallurgical methods,‡ and Professor Wilson's own observation that alloys 8 and 25 reached a constant specific resistance after 5,000 cycles of heating to 80°C. and cooling is further evidence of the efficacy of annealing. Finally, I have found that six hours' annealing in hydrogen at 200°C. is sufficient to bring specimens of copper and brass to a constant resistance; that is, no further resistance change is observed if the annealing is repeated.

Dr. Vernon has asked me to mention that his own work on very pure materials confirms Professor Wilson's view that the iron in the corrosion product is derived from outside sources, There is also no doubt of the high sulphate content of the corrosion product; in some cases the sulphate crystals become visible after a few months' exposure. The fact that the monthly SO_3 deposit varies from 6 tons/Km^2 in London to 0·2 tons/Km^2 at Malvern illustrates the force of Professor Wilson's remark that corrosion may vary from one district to another.

In conclusion, Dr. Vernon joins me in expressing our great appreciation of Professor Wilson's experiments, which constitute pioneer work in this field.

Mr. EDGAR T. PAINTON (The British Aluminium Co., Ltd.): The problems which Prof. Wilson has been studying are of the utmost practical importance, for aluminium conductors are now employed very extensively for overhead transmission lines, and indeed, at the present time are employed for the majority of the longest and highest voltage lines of the world. During the time Prof. Wilson's specimens have been exposed, we, who are engaged in the industry, have been studying from the practical point of view the same phenomena which he had set out to investigate, and it is interesting to compare the deductions to be drawn from his investigations with the facts we have proved from our practical experience.

Prof. Wilson's tests are made on both pure aluminium specimens and on aluminium alloy specimens, and it will be seen from Fig. 1 that as regards resistance to corrosion, the pure specimens are not markedly superior to the alloys and in some instances decidedly inferior. This is not in accordance with our experience and we have found that beyond all shadow of doubt, pure aluminium is capable of better service than alloys—at least than such alloys as Prof. Wilson has investigated. Confirmation is afforded by the fact that while, 25 years ago, aluminium alloy wires were sometimes advocated, at the present time overhead lines are practically invariably of pure aluminium. (In passing it may be remarked that recent discoveries of special non-corrosive alloys may possibly lead to an increased use of aluminium alloy conductors in the future, but this has no bearing on the present point.)

An explanation for this discrepancy can be sought only in the particular specimen of pure aluminium which Prof. Wilson employed, and without going into the subject very deeply it can be stated at once that Specimen 25 though probably typical of good quality aluminium 25 years ago, is not typical of the metal now used for electrical purposes.

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* Matthiessen and Bose, Phil. Trans. A, 152, 1 (1862).

† Fitzpatrick, British Association Electrical Reports, p. 502 (1894).

‡ W. A. Price, "The Measurement of Electrical Resistance" (Clarendon Press).

The point is particularly interesting in that the electrical conductivity of Prof. Wilson's sample was especially good, and in 1901 the wire would have been regarded as eminently satisfactory for overhead line purposes. Our present-day knowledge leads us to surmise that the particular factors which give specimen 25 its low specific resistance are the very factors which render it less capable of resisting corrosive influences.

AUTHOR'S reply : I wish to express my thanks to Dr. W. H. J. Vernon and Mr. J. C. Hudson for their very kind remarks. I appreciate the valuable additions which Dr. Vernon has made to our knowledge of the difficult subject of corrosion, and I look forward with interest to the results of Mr. Hudson's work. His wires are smaller in diameter than mine, and it becomes a question as to how far the depth of metal corroded is dependent upon diameter. Moisture condenses less readily on a fine wire, and this conceivably may play an important part. As regards the variation of specific resistance exhibited by curve 2, Fig. 3, it is only stated that the number of cyclical changes in temperature produced a fairly constant resistance, not that the latter is equivalent to the value obtained before exposure. Wires which have been carefully annealed, and used, for example, in resistance boxes, retain a wonderfully constant resistance ; but in exposed conditions the result may be different. For example, alloy 22 was carefully annealed before exposure at a temperature of 436°C. for half an hour and cooling in air, and it exhibited great fluctuations in specific resistance. The ratio of the specific resistances ρ_t/ρ_0 had the value 1·144 after 24 years' exposure. Alloy 21 is another example, and there are others. As copper melts at about 1,084°C., an annealing temperature of 200°C. appears low, and there is the possible influence of occluded hydrogen. It would be interesting to raise the temperature of Mr. Hudson's copper conductors to, say, 800°C., in an evacuated quartz furnace, as a check upon the value and constancy of specific resistance. I am very pleased to learn that Dr. Vernon, working with very pure specimens, has found sulphate and iron in considerable quantities.

I desire to thank Mr. Painton for his remarks which I feel sure will be read with interest by Fellows of the Society.

III.—THE DISTRIBUTION OF INTENSITY IN A POSITIVE RAY SPECTRAL LINE.—PART II.

By M. C. Johnson, *M.A.*, *M.Sc.*

Received July 29, 1926.

Abstract.

The study of optical spectra of positive rays is continued with the type of tube and Merton wedge described previously. The direction of observation is here longitudinal, in order to isolate the Doppler effect in the " moving " spectrum : accordingly a six-prism spectrograph is substituted for the échelon used with a narrower line in Part I. The results are added to those of Stark, Paschen, Wilsar and Krefft on the same spectrum, and the uncertainties in identifying atomic and molecular carriers are analysed. The distribution of velocity among the particles contributing to the " moving " spectrum is compared with the distribution of velocity in positive rays measured by the electromagnetic method and with some investigations of Retschinsky and Döpel. The appearance of the many-lined spectrum of hydrogen in the positive rays, and the ratio of intensity of the " moving " and " resting " spectra are also considered. The inverse square law of probability of electron capture, and some consequences of the work of Wien and Rüchardt, are the most likely controlling factors in the several phenomena.

I. Introduction.

When a beam of positive rays streams through a perforated cathode, it gives rise to two spectral lines for each radiation of the gas in the tube. One line is emitted by the positive rays, and is known as the " moving spectrum " ; the other is excited by the rays when they strike the stationary gas already behind the cathode. This is known (following Stark) as the " resting spectrum." In a previous Paper* it was found possible, by the high resolving power of an échelon and the use of a Merton wedge, to distinguish these two lines, even when superposed by the axis of the optical train being set perpendicular to the direction of the positive rays. In the present Paper the distribution of intensity in the moving line itself is studied by the Merton wedge ; accordingly the axis of the optical train was placed parallel to the direction of the positive rays. The full Doppler displacement of the moving line in this direction is so large that a six-prism spectrograph was substituted for the échelon used in Part I. Various analyses of the moving line have been made by other workers, chiefly Wien and his pupils, and Stark and Paschen. It is believed that new aspects of the problem of the carriers of the moving spectrum and its conditions of excitation can be investigated by the present method, and the possibility of understanding the conflicting results of previous investigators increased.

II. Experimental Detail.

The discharge tube used was of the type illustrated in Part I, consisting of two glass cylinders waxed on to opposite sides of a central plate which contained the cathodes. In place of the metal end to the positive ray chamber, which in the previous experiments contained the third electrode for the Stark effect, a glass plate was waxed to the end of the chamber and the back of the cathode viewed longitudinally. The gas pressure was uniform in both chambers, and controlled by pumps. and hydrogen reservoir as before. Two cathodes used are seen in Fig. 1.

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* Proc. Phys. Soc., Vol. 38, Part 4, p. 324 (1926). Referred to as Part I.

For details of technique reference must be made to Part I, including discussion of the relation of Balmer to many-lined spectrum intensity in hydrogen, the thermostatic and mechanical stability of the apparatus, and the method and limits of the Merton wedge measurements. In the present case the ammonia-stained panchromatic plates used for H_α were discarded, as the greater dispersion of the prisms at H_β was utilised. Hence, Imperial Eclipse plates were used without any hypersensitising. The chief technical requirement beyond those of Part I was constancy of gas pressure and voltage and current relations during exposures up to ten hours. It was found that after the tube had been running for about 40 hours, pressure increases due to gassing of surfaces were negligible. After several weeks the opposite effect of hardening during discharge began to appear. The best photographs were taken in the intermediate periods, in which the length of the cathode dark space could be guaranteed within one millimetre throughout several hours. This was important, since it was found in Part I that fresh introduction of gas vitiates temporarily the Balmer spectrum. The potential between the ends of the tube was measured by an electrostatic voltmeter. According to Aston,* at low voltages this potential measures the fall in the cathode dark space, and is, therefore, the maximum through which the positive ions can fall. In all cases the 16-in. spark coil was used, as before, driven on half voltage.

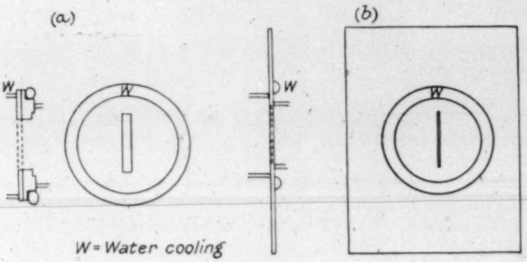

FIG. 1.

III. CALIBRATION OF SPECTROGRAMS.

The spectroscope used was of the Evershed pattern designed for solar prominence work. It consists of two groups of prisms, each comprising a direct vision combination of three; between the two groups is an adjustable mirror, which allows any spectral range of about 300 A.U. wide to be brought on to the plate of an improvised camera. For calibration purposes, advantage was taken of the map of the many-lined hydrogen spectrum published by Merton,† and the list of lines in that spectrum published by Merton and Barratt.* By photographing an old hydrogen tube in which the primary spectrum had decayed, every one of the 17 lines between $\lambda 4860$ and $\lambda 4800$ in Merton's photograph could be recognised. The following stronger lines were used to plot a dispersion curve :—

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* "Isotopes," p. 25.

† Proc. Roy. Soc., 96, 382 (1920).

4873·05
4869·42
4867·05
4849·34
4843·35
4838·25
4832·79
4822·96
4807·33
4797·76

From this curve the dispersion at H_β was found to be 0·039 A.U. per division of the Watson micrometer used.

IV. MEASUREMENTS OF DISPLACED H_β.

Fig. 2 shows the distribution of intensity in three examples with cathode (*a*), the gas pressure, and consequently the fall of potential, being varied. The "moving" line is in each case quite separated from the "resting" line. The steepness and sharpness of the latter as measured by a micrometer on the wedge images indicates the advantages of this method. Except for the measurements of Koch by photoelectric microphotometry, and the thermoelectric microphotometer of Moll, the alternative to Merton's wedge is a visual estimation by compensation of degree of blackening in an image. The difficulty of distinguishing between very sharp rises of intensity by those visual estimations in a Hartmann microphotometer has probably been responsible for obscuring in most previous work the tendency to asymmetry of the moving line. This asymmetry is evident in each example of Figs. 2 and 3, though in order to take a negative of such a faint source, capable of showing detail of the fainter parts if printed and enlarged, the centre of the image is in danger of over-exposure to the point of blurring this feature.

The sharpest and steepest limit to visibility of the moving line is, in each case, on the side nearest to the resting line ; the rise to maximum is hardly distinguishable in steepness from that of the latter. On the contrary, the fall from maximum on the side furthest away from the resting line is more gradual and diffuse. On account of this diffuseness on the violet side, all efforts to do more than assert an asymmetry cannot be relied on ; its existence is beyond doubt, but its shape cannot be identified as a curve, and differences in the falling part of the intensity distribution have not been isolated from the error which, on this edge of the image, is several times as great as on the inner edge. A better elimination of error in wedge measurements has been effected by Merton,† by enlarging the image through a ruled screen and counting the dots in the grain. Unfortunately, the part of the positive ray line which is as intense as Merton's lines is the red side, which is that least susceptible to error, while the diffuse edge ends too faintly to be enlarged on a scale suited to the remainder

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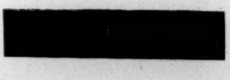
* Phil. Trans., 222, 369 (1922).

† Phil. Trans., 216, 465 (1916).



1.—H_β in Positive Rays. "Moving" and "Resting" Lines. Wedge Image.

2.—H_β in Positive Rays. "Moving" and "Resting" Lines.

3.—Comparison Spectrum of H_β region in ordinary Tube.

To face page 28]

of the line. In the graphs the regions on which it is most difficult to set a micrometer wire are dotted; this indicates the relative error expected, and also shows up the asymmetry by contrast of sharp and diffuse edges.

Fig. 3 shows spectrograms in which attempts were made to alter the intensity distribution by varying the velocity distribution in the positive rays. Plate 12 (*see* Fig. 3) was made with cathode (*b*). According to Aston,* and Gehrcke and Reichenheim,† the distribution of the rays is sensitive to changes in the shape of the cathode. Plate 10 (*see* Fig. 3) was made with a condensed discharge. The significance of this will be discussed later in connexion with the actual nature of the velocity distribution. In both of these cases the asymmetry of the moving line still appears. In view of the maximum error being on the violet side, it is not thought advisable to attach any meaning at this stage to the apparent slight lessening of the asymmetry in Plate 12 (*see* Fig. 3).

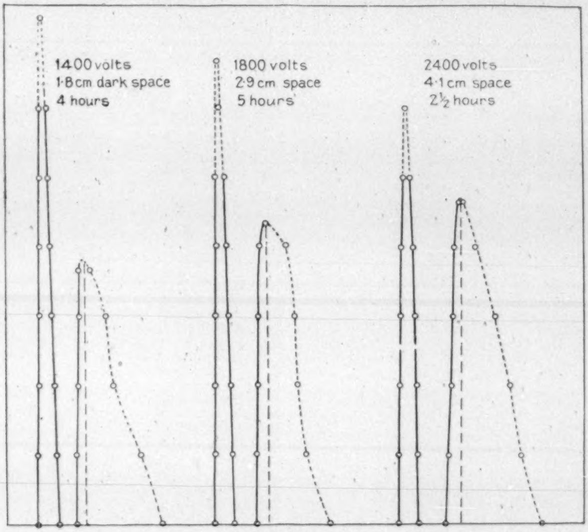


FIG. 2.

V. THE MANY-LINED SPECTRUM.

The many-lined or secondary spectrum of hydrogen is always fainter than the Balmer lines in the positive rays, as discussed in Part I. Nevertheless, measurement of it is important here for two reasons: Firstly, it is necessary to make sure that the observed structure of the moving Balmer line is not complicated by the superposed presence of one of the secondary lines—a precaution I cannot find dealt with in the previous work on this problem. Secondly, Rau* is the only investigator out of many who has notified a Doppler effect in the secondary spectrum of hydrogen positive rays. G. P. Thomson† considers its usual absence‡ is due to the comparatively small concentration of molecules in most tubes which have been used.

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* Proc. Camb. Phil. Soc., 19, 321 (1919).

† Wien: Kanalstrahlen, p. 237 (1923).

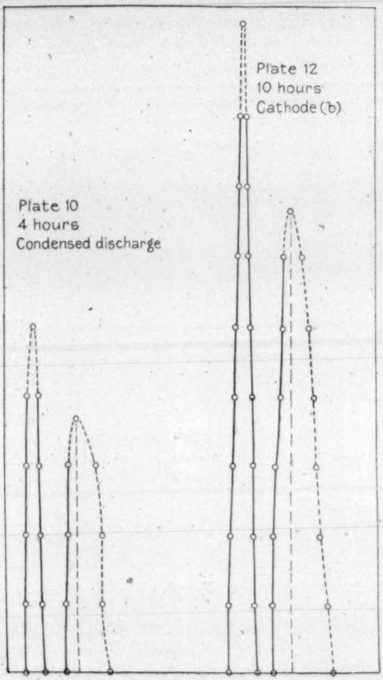

FIG. 3.

In the present experiments the extreme faintness of this spectrum made its examination by the wedge impossible, but measurement of a plain positive-ray spectrogram resulted in the identification of certain lines. In the immediate neighbourhood of H_β the lines found were the ten lines used for calibration in paragraph 3, with the sole absence of λ4807·33. Since these were chosen for the calibration because of their intensity in the ordinary discharge tube, their appearance in the positive rays indicates a like intensity distribution under those conditions of excitation.

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* Wien : Kanalstrahlen, p. 226 (1923).

† Phil. Mag., 40, 245 (1920).

‡ Vegard : Phil. Mag., 41, 558 (1921).

In addition to these known lines, there is a faint radiation at λ4853 showing through the displaced H_β. This is not in Merton's spectrogram and not in the Merton and Barratt list. That it is not a separated intensity component of the moving H_β is shown by its appearance also in comparison spectrograms taken of the same region with an ordinary tube.

A second addition is at λ4864, and though not in Merton and Barratt, it is seen in the comparison spectrograms and in Merton's photograph, as well as in the positive rays.

The other possible error in measuring the moving H_β would occur if the sharpness of its edge were increased falsely by a third additional line, which appears near λ4858 in the comparison spectograms and in Merton's plate, but not in Merton and Barratt. This, however, is exceedingly faint and far below the intensity of the faintest positive-ray line seen.

None of these 11 lines appear as anything but fine and narrow, as in the comparison spectra: there is no doubt of the absence of Doppler effect in this part of the secondary spectrum even under conditions fulfilling G. P. Thomson's requirement.

The positions of the lines also show that the secondary spectrum cannot contribute to the apparent shape of the moving line H_β.

VI. Comparison with Previous Work.

Several previous investigators have found that the moving line itself in the Balmer series has more than one intensity maximum. Others have found—as in the present experiments—only a single maximum. The appearance of a double maximum has been attributed to the fact that both atoms (H_1) and undissociated molecules (H_2) can be positively charged and accelerated in the positive ray stream. Their velocities will be in the ratio $\sqrt{2}/1$ since the equations governing motion in the dark space are

$$\tfrac{1}{2}m(H_1)v^2=eV$$
$$\tfrac{1}{2}m(H_2)v^2=eV$$

The Balmer lines are emitted by the neutral H_1,* but if this atom has until just before emission been undissociated as H_2, the Doppler effect in the line will be that appropriate to the mass of the molecule. A recent research of Blackett and Franck† has made far more plausible the underlying assumption that a hydrogen molecule can be both dissociated and excited to Balmer atomic emission in one simultaneous act.

Unfortunately, the measured ratio of the Doppler displacements at maxima, instead of being equal to $\sqrt{2}$, vary between 1·3 and 2·5. The discrepancies among the results of previous workers in this field make it impossible to ascribe with certainty the maxima in the published curves to H_1 and H_2, and prevent the identification of the hypothetical H_3 of J. J. Thomson* with a third maximum seen in some cases. They also make it necessary to find other criteria to decide whether atomic or molecular carriers are responsible for the results of the present Paper.

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* Wien: Annalen der Ph., 69, 328 (1922).

† Zeit. f. Ph., 34, 389 (1925).

Without, therefore, attempting at the beginning to specify H_1 or H_2 as the carriers, comparison with previous work will be made from the ratio of velocity calculated from the Doppler effect to velocity calculated from the potential across the tube. For the former,

$$v_D = c\frac{d\lambda}{\lambda};$$

for the latter,

$$v_C = \sqrt{\frac{2e}{m}V}.$$

Let v_{D_2} be the velocity at the maximum intensity of the moving line. Let v_{C_1} be calculated for H_1.

Collecting the measurements of those investigators from whose Papers numerical values can be obtained, we can construct the following table :—

		Paschen.†	Wilsar.‡	Krefft.§	Johnson.
v_{D_2}/v_{C_1}	Slow max.... ...	0·3 to 0·4	0·2 to 0·4	0·25 to 0·6	0·37 to 0·42
	Intermed. max. ...	...	...	0·4 to 0·6	...
	Fast max.... ...	0·7 to 0·85	...	0·5 to 0·7	...

From this it appears that the single maximum of the moving line in the present experiments agrees with the slower rather than the faster maxima of previous writers. The identification with slower rather than faster streams is supported by the fact that Krefft finds only the slow stream at the lower voltages, and the faster streams not visible until the voltage is above the highest in the present Paper.

But according to the simple view so far taken to explain these phenomena, namely, that the mass of the singly charged particle is the deciding factor in the speed of the positive ions in the same dark space, the slow stream, with which the ions of the present experiments seem identified, should be that of the greatest mass. Hence the carriers of the moving spectrum should be hydrogen molecules only dissociated just before atomic emission.

This, however, is disproved by the absence of Doppler effect in the molecular spectrum, which possesses no moving lines. Further, the fact that three maxima are seen in some curves would, in terms of that explanation, ascribe the slowest maximum, which is very intense, to the rare and unstable H_3 molecule. A third discrepancy is the fact that Stark and Steubing‖ find the slower stream increases in intensity relative to the faster stream as voltage is increased. Krefft finds the faster maximum increases relative to the slower as voltage is increased. Paschen finds the slower maximum stronger at high voltages.

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* "Positive Rays," p. 202 (1921).

† An. der Ph., 23, 247 (1907).

‡ An. der Ph., 39, 1251 (1912).

§ An. der Ph., 75, 82 (1924).

‖ An. der Ph., 28, 974 (1909).

It becomes necessary, then, to abandon the identification attempted previously, of order of Doppler effect with inverse order of mass, and to seek some other identification of the carriers of the spectrum.

VII. The Carriers of the Spectrum.

Fig. 4 shows the relationship of the velocities measured at different parts of the moving line and the velocities calculated for atomic and molecular hydrogen. The independent variable is conveniently taken as the length of the cathode dark space.

Let v_{D1} be the smallest and v_{D3} the greatest velocity in the Doppler effect, and let v_{D2}, as before, be the velocity at maximum intensity. Let v_{C2} be calculated for H_2 and v_{C1}, as before, for H_1.

The graph shows at a glance that while either H_1 or H_2 could give rise to the Doppler displacement at its maximum intensity or at the smallest separation from the resting line, only H_1 could give rise to the largest displacements. These last, measured on the diffuse edge of the line where the error is greatest, vary so much that the only definite conclusion possible is that they are greater than the calculated value for H_2, which is the highest velocity that could be imparted to a molecule in the tube; on the other hand, all are smaller than the value for H_1, except one whose excess is well within the limit of error of the measurement at that edge of the line.

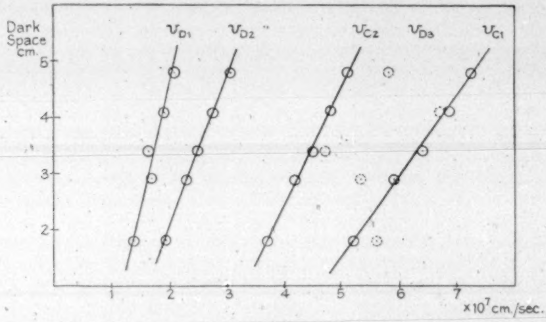

FIG. 4.

Hence, if the spectrograms represent a single stream, the carriers of the Balmer spectrum in the present case cannot have been accelerated as H_2 molecules, as their identification with the slow streams of other investigators would have suggested.

There remains a chance of each spectrogram covering two streams. Paschen's Stream II is diffuse on its violet edge owing to the superposed presence of Stream I at 3,100 volts. But in Paschen's case this asymmetry is transient and only occurs with the smallest dark space, while here it occurs in all photographs. Hence the present spectrograms cannot be ascribed to a double stream, and must represent the single maximum found by Krefft at these voltages and by Wilsar. The carriers must in that case be atomic.

Once the theory of two streams as atomic and molecular is abandoned, their explanation can be begun on the lines opened by J. J. Thomson,[*] and by Döpel,[†] who investigated electromagnetically the velocity distribution in hydrogen and distinguished two streams, both atomic and isolated from the molecular positive rays. This will not be further discussed here as the present experiments only relate indirectly to multiple streams.

VIII. THE DISTRIBUTION OF VELOCITY.

It is necessary to consider next whether the results obtained can provide any clue as to why the simple order of velocities according to mass of the atomic and possible molecular aggregation does not control completely the Doppler effect.

There are two means of studying the velocities of particles in the positive rays. The Doppler effect shows the velocities of those particles which are emitting light, and the electromagnetic method of J. J. Thomson shows the velocities of the particles which are electrically charged, by analysis of their deflections in magnetic and electric fields.

Now, Retschinsky[‡] has compared the two methods, with respect to the ratios of velocities found thereby to velocities calculated from the fall of potential in the tube. In the parabola measurements he finds velocities which are 15 per cent. lower than the theoretically possible velocities calculated from the dark space. This discrepancy is not as great as those found from the present Doppler velocities at maximum intensity, though it would fit the velocity of Paschen's fastest stream, and would agree with the case of the fastest velocities found in the fainter part of Figs. 2 and 3. Retschinsky and Krefft find no relation between the "beading" of parabolas, i.e., the appearance of several intensity maxima, and the multiple maxima in the Doppler curves of Paschen, Krefft, and Stark.

But a further aspect of the comparison between parabola and Doppler measurements is instructive. The slight asymmetry in the wedge analysis of the latter was shown in Paragraphs 4 and 5 to be unmistakably separable from any contamination by the secondary spectrum: this asymmetry means a preponderance of low-velocity ions in the light emission, since the intensity maximum is near the undisplaced side. Now, hydrogen parabolas can be examined in many examples published by J. J. Thomson and Aston, and most recently in the Paper of Döpel referred to. In the majority of cases there is an asymmetry in the parabola velocity distribution, but in the opposite direction to that in the Doppler effect, i.e., there is a preponderance of the higher velocities, the "head" of the parabola being stronger than the end furthest from the undeflected spot.

J. J. Thomson[§] and Aston[||] explain this asymmetry in the normal parabola distribution by the intense ionisation at the boundary of the dark space furthest from the cathode. A greater proportion of the ions is formed at this boundary than anywhere else in the dark space, and hence there is a concentration of the positive ions about that velocity which is derived from falling through the whole of the cathode fall of potential.

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[*] "Positive Rays," p. 85 (1921).
[†] An. der Ph., 76, 1 (1925).
[‡] An. der Ph., 50, 369 (1916).
[§] "Positive Rays," p. 85 (1921).
[||] "Isotopes," p. 29.

If there were an equal chance of any of these positive ions being neutralised and excited to Balmer emission, the spectroscopic observations should show a similar distribution of intensity—i.e., the moving line should be more sharply bounded and more intense on the violet side. The present experiments show that any asymmetry there is is a sharpness and intensity maximum on the redder side nearest to the undisplaced line.

That this is not due to an individual characteristic of these experiments, appears from its persistence through change of cathode shape, involving change of distribution of the actual positive ion velocity; especially is this significant in the case of the condensed discharge of Fig. 3. Now the main visible effect of condensing the discharge was to blur the boundary of dark space and negative glow, the former becoming much brighter. This relaxes considerably the condition which J. J. Thomson and Aston laid down for the parabola asymmetry. In spite of this change from the usual distribution of positive ray velocity the opposite asymmetry of the Doppler distribution persists. The conclusion is strengthened that while in the positive ray stream the velocity distribution concentrates a majority round the higher speeds, the greatest number which are neutralised and excited to Balmer emission are concentrated around the lower velocity limit.

We here approach evidently some velocity criterion which decides which of a stream of positive ions can capture an electron.

Rüchardt* has investigated this problem from the point of view of his experiments on Wien's Free Path, the mean distance L_1 traversed by H_1^+ before becoming H_1. He finds L_1 increases with increase of velocity; this is the physical basis of the distribution we have found in the moving line. Rüchardt states, however, the velocity criterion of neutralisation as follows: The condition for capture of an electron is

$$\tfrac{1}{2}mv^2 \leqq \frac{e^2}{r},$$

$$r \geqq r_1,$$

where e and m are charge and mass of electron, v is the relative velocity of ion and electron, and r_1 is the radius of the first Bohr orbit in the hydrogen atom.

This criterion would account for the fact that a Doppler velocity is never more than a certain fraction of the calculated positive ray velocity, but it gives no reason for there not being a sharp violet edge to the moving line in the present experiments. The gradual fading of the violet edge is, however, more definitely accounted for if we adopt as a criterion for capture the law put forward by Milne,† which was suggested by thermodynamics of radiation and fits a large number of astronomical and experimental facts. According to this law, if P is the probability of an electron being captured when it approaches a positive ion with relative velocity v,

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* Zeit. f. Ph., 15, 164 (1923).

† Phil. Mag., 47, 209 (1924).

$$P=K_1/v^2,$$

where K_1 is a constant for the atom.

Stated in terms of Wien's Free Path for neutralisation,

$$L_1=K_2v^2.$$

IX. THE RELATIVE INTENSITY OF MOVING AND RESTING SPECTRA.

Fig. 5 shows the relation between the greatest intensity of resting and moving lines of H_β.

Vegard* stated the relative intensity of resting lines increased with potential. Dempster† published a series of drawings of intensity curves for H_β, in which the ratio of resting to moving intensity increased as the dark space increased. McPetrie‡ remarks that as the positive ray velocity increases, the moving line intensity diminishes.

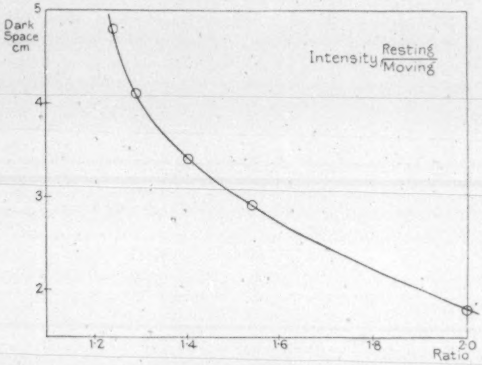

FIG. 5.

These three results are contrary to the phenomenon shown in Fig. 5, where increase of potential increases the moving line intensity towards a limit.

Stark§ found conditions under which increase or decrease of moving intensity are both possible, but he keeps constant quantities which cannot be isolated in the present arrangement.

Vegard, Dempster and McPetrie all worked at higher voltages than those obtained in the experiments from which Fig. 5 is plotted, and so it seems probable that the ratio has a minimum above the highest potential of the present experiments.

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* Phil. Mag., **41**, 558 (1921); An. der Ph., **39**, 111 (1912).

† Astroph. J., **57**, 193 (1923).

‡ Phil. Mag., **51**, 1093 (1926).

§ An. der Ph., **39**, 1185 (1912).

and below the lowest of the previous writers. This would be accounted for if two opposing factors underlie this ratio of intensities. In the present state of knowledge the following seems adequate as a hypothesis for both the present and earlier results.

The emission of the moving Balmer line requires the neutralisation of a positively charged atom by capture of an electron. But in the positive ray chamber there is not the free electron concentration there is in the discharge tube, where cathode rays are streaming towards the anode. Wien* and Rüchardt† consider the required electrons are those just previously liberated from stationary atoms or molecules by the impact of ions which immediately capture them.

The emission of the resting line is independent of electron capture, as particles already neutral in the stationary gas can be stimulated to Balmer emission, by inelastic collisions from particles in the positive ray stream, with exchange of less than ionising energy.

If we consider both processes as the result of collision, we are in this case concerned with positive ions as projectiles in place of the electrons of ordinary and thermionic tube spectra; the energy exchanges required can be classified as follows:

1. For moving line: (*a*) Ionisation of atom, or (*b*) ionisation of molecule.

2. For resting line: (*a*) Excitation of atom, or (*b*) dissociation of molecule and excitation.

The great strength of the Balmer relative to the many lined spectrum in this type of tube indicates a predominance of the (*a*) type of process in each case, as in the "red" stage of the ordinary hydrogen tube.

The increase of potential fall in the lengthening dark space of Fig. 5, will correspond to a tendency for 1 (*a*) to become more common, and a decrease in the ratio of resting intensity over moving intensity will result. A quantitative relation between the two processes, based on the value of the respective critical potentials, cannot be derived, as we are not dealing with a gas in thermodynamic equilibrium.

But if the inverse square law of probability of capture is adopted, the increased number of ionisations by impact will not continue indefinitely to increase the intensity of the moving line. For although more electrons become liberated, the increasing relative velocity of positive ion to electron will decrease the chance of capture by the fast ion, the electrons will tend instead to be captured again by their own or other stationary atoms. At voltages higher than those employed here the ratio of resting to moving intensity will be expected to cease to diminish and begin to rise again, as found by Dempster and others working at very high potentials.

It is a pleasant duty to acknowledge again gratefully the generosity of Prof. S. W. J. Smith, F.R.S., in encouraging and equipping these experiments, and the skill of Mr. G. O. Harrison in instrument construction.

DISCUSSION.

Prof. O. W. RICHARDSON said that, though the interpretation of the author's results might be debatable, there was no doubt as to their importance. The new lines mentioned by the author were in the list published by Tanaka, and there was an additional line within about 1 Å.U. of H_β; but in view of the author's assurance that the distance between the sharp edge of the moving spectrum and the edge of the resting spectrum, which depended on the voltage employed, was in the neighbourhood of 8 or 10 Å.U., it was clear that this particular line could not be a factor. In fact, he agreed with the author's conclusion that the effects were not vitiated by secondary spectrum lines shining through the moving H_β, all the relevant lines being too weak to affect the observations. He was not convinced that the results afforded very definite support for the law of electron capture suggested by Eddington and Milne. The difficulty he felt was not the diffuse broadening of the line on the side towards the violet, which might occur with many different laws of capture, but the existence on the side towards the red of a sharp limit which did not appear to bear any simple relation to the electric field.

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* Proc. Phys. Soc., 37, 324 (1925).

† Loc. cit.

Dr. B. DASANNACHARYA : The pronounced intensity maxima occurring in the displaced line nearer the slower velocities is, I believe, due to the atoms from the dissociation of the accelerated molecules. If, as Mr. Johnson objects and points out, that the maximum velocity which such molecules may attain is lower than the maximum velocity represented in the displaced line, the conclusion is that the atomic stream is to be supposed as existing side by side. The latter is, under the conditions of Mr. Johnson's investigation, weakly represented. Mr. Johnson's second objection to the molecular stream hypothesis is that no Doppler displacement has been observed in the many lined spectrum of hydrogen. Ran actually observed such a displacement in the positive ray behind the cathode by employing a very ingenious method (Wien's "Kanalstrahlen," p. 227). But the effect was very faint, which would justify the assumption that, *behind the cathode*, only a few molecules in the ray are present—in agreement with the results of the electromagnetic analysis—and that the molecules are easily dissociated. In front of the cathode Ran was not able to detect any Doppler displacement of the molecular lines, an interesting fact, from which it has been concluded that the many lined spectrum is due to the neutral molecule, and that in front of the cathode the molecules move as charged particles only, or they must be dissociated.

As regards the double stream theory, Mr. Johnson's objection is that the ratio of the velocities corresponding to the two intensity maximas is not equal to $\sqrt{2}$, but varies between 1·3 to 2·5. The reasons for the variation are the following : The molecules and the atoms in the discharge tube do not behave alike. It is well known that, even in front of the cathode, the canal ray could be seen, which shows that some of the moving atoms of the positive ray, there, are neutral. That the atoms change their state from the charged to the uncharged, and vice versa, is well established. It is obvious, therefore, to expect that the atoms will not get the full acceleration that would correspond to the cathode fall and that the velocity may vary within certain wide limits. The molecules, as I have remarked above, are easily dissociated—the more so the higher the voltage. So it is reasonable to expect the variation of the ratio of the average velocities of the atomic and the molecular streams from the simple value of $\sqrt{2}$.

Secondly, the amount of light given out by an atom in a second is a function of its velocity and the pressure of the gas in the chamber in which the excitation to the emission of light takes place. This is dealt with in a Paper of mine (Ann. d. Phys., 77, 597, 1925), and treated further in a Paper, now in print, in the Journal of Science (Maharajah's College, Vijayanagaram, India). An abstract appears in the reports of the British Association meeting in Southampton in 1925. There is no exact equivalence to be expected between the distribution of the velocity in the Doppler effect and that deduced for the neutral atoms from that for the charged atoms in the electromagnetic analysis. There is also another reason for this difference, because the positive ray traverses a high vacuum in the electromagnetic analysis, whereas for the spectroscopic investigations the pressure is usually of the order of a hundredth of a millimetre of mercury. The curves given in my Paper in Annalen to illustrate these facts are interesting.

AUTHOR'S reply : In reply to Prof. Richardson's question as to why there is a space with no radiation between the resting and moving lines, I believe J. J. Thomson's remark about the minimum energy needed for a positive ray to be effective is the only suggested explanation so far. The rise to maximum on the red side of the moving line appears to represent the same type of probability distribution as the rise on each side of the resting line itself.

In reply to Dr. Dasannacharya, I do not agree that the double maxima have a satisfactory explanation as due to atomic and molecular carriers. The discrepancies between published results for their velocity ratio up to 2·5 and the theoretical 1·4 are what led me to use a new identification method for the carriers. The results in the Paper show that these are all atomic, in spite of agreeing in velocity with the slower stream of the Continental writers, previously regarded as molecular. The mass is not the main controlling factor. In addition to this method for atomic identification, the absence of a molecular stream is shown by the constancy of the smooth intensity curves under all conditions. At one or other of the voltages and pressures used the molecular and atomic maxima would have separated or changed their relative intensity if both were, as Dr. Dasannacharya suggests, present in the single curves.

IV.—A RAPID BOLOMETER MADE BY SPUTTERING ON THIN FILMS.

By H. Dewhurst, *A.R.C.S.*, *D.I.C.*, Imperial College of Science and Technology.

Received June 11, 1926.

SUMMARY.

Thin Films.—The method of making thin films of collodion by drying weak solutions in ether and alcohol on the surface of clean mercury, and their subsequent removal, is described. Flexible films are also made in the same way, using a modified formula, producing very robust films which withstand great distortion, and will stand being punctured without fracture. Both types can be produced rapidly and cheaply down to a thickness of one wavelength of light.

Sputtering.—Two novel types of sputtering apparatus are described, together with working data. A table is given containing sputtering data for 25 metals, nine of which appear for the first time. Notes relating to the effects of Shape and Age of Kathode, Shape of Anode, Form of Dark-Space and Kathode Temperature appear, together with novel modifications in the design of the Wehnelt break. Approximate temperature coefficients of resistance are assigned to eight new metals and more accurate values to three others, that relating to gold being new. The apparatus used for these determinations is described. Notes and curves on ageing appear regarding six metals and resistances for films of standard size for ten metals. A curve is given throwing, perhaps, some light on the formation of a sputtered film.

Bolometer.—Apparatus is described by which rings are moulded for supporting the thin collodion films and providing a reliable contact for overlying sputtered metallic films. The method of making the bolometers, together with holders of various types, and apparatus for blackening, are detailed. The comparative sensitivity of these new bolometers is discussed, and curves and an empirical formula given from which an estimation of the speed of the instruments can be determined.

The new type is roughly 400 per cent. faster than a representative bolometer of the Lummer and Kurlbaum type. The thickness of the component metal film is estimated.

I. On the Making of Thin Collodion Films.

Object.

FOR the construction of the proposed bolometer it was necessary to devise a method of stretching a very thin film of electrically-insulating material across the aperture of an annular disc. The dimensions of the disc, or ring, were arbitrarily fixed at $1\frac{3}{8}$ in. exterior diameter, with an aperture of $\frac{3}{4}$ in., the thickness of the annulus being $\frac{1}{8}$ in.

Question of Suitable Material.

The most promising material of which the thin film could be made appeared to be one of the following: Mica, glass, celluloid, gelatine or collodion. No Papers dealing with the production of very thin films in a permanent form seemed to have been published, with the exception of one dealing with films of gelatine and soap,* which would be much too thick for the purpose in view.

It may be interesting to describe very briefly how thin films of mica and glass were obtained; which, although thin enough to show interference colours, were otherwise unsuitable. The method adopted in the case of mica was the successive splitting of a thin sheet. The mica was "sandwiched" between two plane brass plates, with a film of adhesive between each plate and the mica face adjacent to it. When the adhesive had set sufficiently the plates were pulled apart; the process to be repeated as often as necessary with the successively thinner sheets of mica.

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* "Durable Films," R. Malagoli, N. Cimento, 11, pp. 351-53.

It is clear that the adhesive used must effect a greater cohesion between metal and mica than that one can imagine to exist between adjacent mica laminæ. Of the adhesives tried—shellac, shellac and beeswax, "Chatterton's compound" and sealing wax—only the latter was successful. Contrary to expectations, the mica usually split the first time, leaving a film on one metal plate sufficiently thin to show interference colours. Unfortunately, the film thus obtained was not of uniform thickness. The whole area was split up into small patches, each of uniform colour, bearing a resemblance to an aerial photograph of country divided up into a number of fields. The average thickness of the mica, however, was of the required order; but on attaching the annular ring and dissolving ring and film away from the metal plate in alcohol, the various small areas of uniform thickness broke away from their neighbours, the whole film thus disintegrating. Mica, therefore, was unsuitable for the purpose in view, unless specimens could be obtained containing fairly large areas of uniform thickness in some particular well-defined lamina. As, however, such laminæ would be distributed in depth in the thickness of the mica, the chances of finding one in the right location for splitting were very remote.

In the case of glass the ordinary method of blowing a sphere would be inadmissible, as it would be necessary at a later stage to attach flat electrical contacts to the finished film, the curvature of the glass making this impossible. The method tried, therefore, was that of blowing two bubbles at the same time, both pieces of the molten glass used for blowing being connected to the mouth through rubber tubes of the same diameter by means of a Y-piece. If the expanding spheres of glass touch each other when of the same size, thickness and temperature one would expect a plane film of double thickness to result. It is, however, practically impossible to get all the conditions for success fulfilled at one and the same time. In the one case in which a reasonably flat film was obtained it was found that the glass was corrugated in circular waves, these being due to the imprisonment of air between the two component glass films.

There remained celluloid and similar substances which could be dissolved in some solvent, the solution spread over a plane area, leaving a thin film on evaporation of the solvent. It was known that thin celluloid films were being made* by a process, no details of which appear to have been published, using rock-salt. A solution of celluloid is poured on to an optically flat face of a rock-salt disc, allowed to dry, the rock-salt being finally dissolved away in water, leaving the film attached to (say) a ring, which had previously been stuck on the film-covered face of the rock-salt. This process, whilst not giving very uniform films, is a lengthy and very costly one; both factors rendering it prohibitive in the present case, where a large number of films would be required for tests to destruction.

A modification was therefore tried in which the celluloid or collodion solutions were poured on to the surface of water and allowed to dry. In both cases the solvents used were soluble to a greater or less extent in the water itself, thus tending to lack of uniformity in the films produced.

* Adam Hilger, Ltd.

Outline of Method Adopted.

At this stage mercury was substituted for water, this proving to be the basis of the successful process to be described. Collodion was finally adopted in preference to celluloid, owing to the greater standard purity in which it could be obtained. Gelatine was ruled out, owing to the time required for water solutions to dry. Briefly, then, the method finally used was the pouring of a few drops of a weak collodion solution in ether and alcohol on to a clean surface of mercury. The solution spreads, forming a disc which dries very rapidly, usually from the periphery to the centre. Just before the film is quite dry the annular ring which is to hold the film is placed on top of the film-covered mercury, the surplus film outside the ring removed, and finally the ring and film taken off the mercury as described later.

Factors Affecting the Final Product.

The collodion used was that known commercially as " Mawson's." This is a somewhat viscous solution, and requires considerable dilution before use. Dilution by ether alone, after a certain point, gives an imperfect solution, and consequent cloudiness. The addition of a small quantity of alcohol, however, effects a perfect solution. The amount of alcohol permissible is rather critical. Too high a proportion of alcohol gives films which contract and corrugate on drying; too high a proportion of ether, on the other hand, giving whitish films instead of the usual clear product. A solution sufficiently dilute to give a film showing interference colours on drying was found by rough trial. Such a solution, however, will only give consistent results provided the mercury surface is absolutely clean and the collodion solution reasonably pure. The slightest trace of contamination on the mercury surface, whether due to grease, moisture or amalgams, is fatal. Contamination prevents the solution " wetting " the mercury surface. Unless the solution wets the mercury properly, gravitational action will be insufficient to spread the solution; one being left with a small, oily, distorted result bunched together by its surface-tension, whose drying will be slow and restricted, with consequent variation of the mutual proportion of the two solvents. It is not surprising that under these circumstances a sticky white film will result, and of a thickness greater than that normally obtained.

The following proportion of the constituents was found to give good results:

Mawson's Collodion	1·0 ccs.
Ether (anæsthetic)	9·5 ccs.
Alcohol (absolute)...	0·5 ccs.

the two solvents being first mixed together and the collodion then added. Standardising the solvent proportions, a decrease in the collodion content tends to make messy and fragile films.

The rate at which the solution is allowed to dry is very important. Rapid drying gives clear films of uniform colour, whereas slow drying produces oily, contracted and unevenly coloured results.

As contamination of the mercury used has such a deleterious effect on the final product, the mercury has to undergo several cleaning processes before it is suitable for film making. It is twice passed, in a finely divided condition, through dilute nitric acid, and twice in a similar manner through distilled water; being finally distilled in a partial vacuum whilst a slow current of air passes through the boiling liquid to oxidise base metals.*

Solid contamination is removed by "filtration" through a small hole in the apex of a folded filter-paper in the usual manner. This gives better results than squeezing through chamois leather. The presence of grains of dust, fibre, etc., on the mercury, however, is by no means so serious as other types of contamination, and, even if present, can always be removed by forming a film over the foreign matter and stripping both from the surface. In fact, this method would appear, from the experience gained, to be the most certain method of obtaining a perfectly clean surface. It should be noticed that filtration must always immediately precede film making, whereas the washing and distillation as described need only be carried out once for any given batch of mercury.

Detailed Method of Film Production.

The mercury is contained in a glass funnel sealed at the apex of the cone (Fig. 1). A containing vessel of this shape gives sufficient depth for the operation of removal to be described, whilst at the same time necessitating the use of considerably less mercury than would be required to fill a cylindrical vessel to the same depth. More important still, the funnel shape decreases very considerably the liability to circular waves being set up on the surface, due to adventitious shaking; such waves tending to puncture the film in the centre.

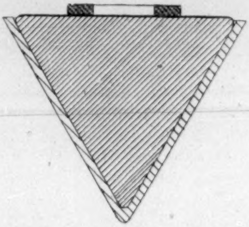

FIG. 1.—SIMPLE APPARATUS FOR MAKING THIN COLLODION FILMS.

The collodion solution is poured on to the mercury by means of a 1 cc. pipette. Roughly, $\frac{1}{3}$ cc. is allowed to fall in a continuous stream. The solution first spreads rapidly, and afterwards more slowly, until the increase in size is held up by drying or the walls of the funnel. Drying proceeds in waves, beginning at the periphery and ending at the centre, and becomes visible owing to the change in tone. The last wave leaves a clear iridescent film quite distinct from the dull appearance of an imperfectly dried film. It is just before the final disappearance of this dullness at the centre that the ring should be placed on the film. Too long a time elapsing between the drying of the film and the placing of the ring will involve a lack of adhesion between the two, rendering it impossible to remove the film from the mercury surface. The superfluous film round the outside of the film can now be removed by means of a needle. The process by which the ring and the film are removed from the surface is illustrated in Fig. 2. A fairly large globule of mercury is poured gently into the aperture of the ring, so that the film at the bottom is sandwiched between mercury (Fig. 2A). The ring is now pushed down at one side below the surface until the mercury above and below the film has joined together (Fig. 2B). The ring is tilted further, whilst keeping the film wholly below the surface, until the ring is vertical, when it can be lifted straight up out of the mercury, with the film stretched across it (Fig. 2C). By this means the pressures and surface-tensions on both sides of the film are maintained practically the same throughout. Collodion films are so tightly stretched when dry, and are so slightly extensible, that a very small distortion is sufficient to break them. Occasionally one can manipulate them with sufficient skill to slide them horizontally off the convex boundary of the mercury surface. The method is a very uncertain one, however.

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* "Practical Physics," W. Watson, pp. 487-9 (1917).

It will be found that in many cases tiny pellets of mercury are left adhering to the ring, and in some cases (when the mercury is insufficiently clean, or the film not properly dry before removal) to the actual film itself. In both cases they can be removed by wiping carefully with a small pledget of very wet filter paper.

Once all the necessary conditions for success, as outlined, have been obtained, the process is a quick one, the time necessary to produce a film being only two or three minutes. It will often be found, however, that, although all the precautions have been taken to ensure clean mercury and a properly working solution, several films will prove unsuccessful, and must be removed one after the other until the surface has become sufficiently clean in the process. In all cases the first film is abortive, thus lending additional weight to the assertion that the removal of a film from the surface leaves it in a much more perfect state of cleanliness than is obtained by the more usual methods. It is important to notice, also, that succeeding films, to be successful, must be made more or less immediately.

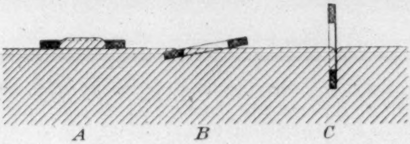

FIG. 2.—METHOD OF REMOVING THIN FILM FROM MERCURY SURFACE.

Flexible Thin Films.

It has been mentioned that films made by the above process were tightly stretched and inextensible. Owing to this reason they were found quite unsuitable as a base upon which a thin metallic film could be formed by the process of "sputtering." A large number of films were tried under sputtering varied in every way possible, but in all cases the films were shattered by the particles hurled from the kathode. It was necessary, therefore, to devise some modification of the film formula, or make use of some new material, which would result in films that were flexible and which would not shatter when punctured. The solution of such a problem would lie, in the first place, in the trial of various promising substances which, when added to the collodion, would confer the property of flexibility.

Success was attained with the addition of critical proportions of Canada balsam and castor oil. Neither of these alone will confer the requisite property of flexibility, for, as will be seen immediately, the addition of the balsam results in white brittle films, whereas the oil gives oily and sticky products. In retrospect, it is rather remarkable that a formula should exist at all which will give films sufficiently tough to withstand the kathodic bombardment of sputtering, but yet, at the same time, prove amenable to the very delicate process of thin film making. The data will now be quoted relating to the effect of variation of the constituents from considerations of which the final formula was eliminated.

For convenience, arbitrary standard solutions of the constituents were made up, these being given short pseudonyms to obviate unnecessary repetitions. It will be convenient to retain these names for the purpose of this Paper. The solutions were :—

40	per cent. solution of	Mawson's Collodion in ether		"Collol."
2	,, ,,	Canada balsam	... ...	"Balsol."
1	,, ,,	castor oil	... ...	"Castol."
5	,, ,,	ethyl alcohol	... ...	"Ethol."

Although, of course, a large number of experiments were carried out to obtain the best proportions, only a few of the more conclusive ones will be outlined here.

Preliminary trials had shown that the probable composition would lie between the values :—

Collol.	*Balsol.*	*Castol.*	*Ethol.*
0·5 to 2.	0 to 2.	0 to 2.	2·5 ccs.

Experiments were therefore made using 0·8 to 1·0 ccs. of collol. Additional alcohol was found necessary to the extent of about 0·3 ccs. in order, possibly, to allow for the complete solution of the additional balsam and oil. Tabulated results follow of a series of films in which the proportions of the balsam and oil to each other were varied, whilst other factors were kept constant.

TABLE I

Constituents.						Spread.	Drying.	Removal.		Film Characteristics.	
Solution.	Collol.	Balsol.	Castol.	Ethol.	Alcohol.	—	—	Condition.	Flexibility.	Colour.	Clarity.
1	0·8	0	0	2·5	0·3	Normal	V. good & fast	V. clean corrugated	Tears easily	V. bright	V. good
2	...	0	2	...	...	...	V.uneven & spotty	V. oily ...	Sticky ...	...	Poor
3	...	2	0	...	...	...	Contracts	V. white	V. brittle	None ...	Bad
4	...	2	2	...	...	...	Fair ...	V. oily ...	Sticky ...	Poor ...	Poor
5	1·0	2	2	...	...	...	Good ...	Clean ...	Tough ...	Fair ...	Good
6	...	1	2	...	...	...	...	...	Tears easily	...	...
7	...	2	1	...	...	...	...	V. white	V.brittle	None ...	Bad
8	...	1	1	...	...	...	...	White ...	...	Poor ...	Poor

In the first four, 0·8 ccs. of collol were used, whilst in the last four this was increased to 1·0 in order to reduce the proportion of the balsam and oil combined to that of the collodion. It will be noticed that while none of the films 1 to 4 is at all satisfactory, 2 indicates that an excessive quantity of castor oil makes the resultant film oily and sticky, while excess of balsam confers a white brittleness quite fatal to flexibility. Film 5, owing to the increase of the collodion to balsam-and-oil ratio, is very nearly up to requirements, the lack of brilliance in the interference colours indicating the possibility of too heavy a proportion of balsam and oil combined to the remaining constituents. Films 6 and 7 merely tend to confirm the conclusions arrived at from 2 and 3.

Using film 5 as a basis, experiments were made varying the proportion of ethol by 50 per cent. each way, retaining the 1 cc. collol, 2 ccs. balsol and 2 ccs. castol ratio. The results were somewhat inconclusive, and point to the proportion of ethol not being a critical one. Retaining, therefore, the proportion of ethol at 2·5 ccs., the ratio of balsam and oil were varied within a narrower range. In the first series, retaining the balsol at 2 ccs. throughout, the castol was tried between 1·6 and 2·4 ccs., by steps of 0·2 ccs. at a time. Secondly, keeping the castol at 2 ccs., similar variations in the balsol were experimented with. In both cases the best result was obtained when there was 0·2 ccs. less balsol than castol. Thus the best films in each series were obtained from :—

	Collol.	*Balsol.*	*Castol.*	*Ethol.*	*Alcohol.*	
(1)	1·0	2·0	2·2	2·5	0·3	ccs.
(2)	1·0	1·8	2·0	2·5	0·3	ccs.

As a last preliminary, the amount of collol was varied, retaining the other proportions according to either of the above, between the limits 0·6 and 1·4 ccs. From the results it appeared that 1·2 ccs. met the case to a better extent than the others.

Having by a method of successive approximations arrived at a formula presumably approaching the best possible, a final trial, tabulated below, was undertaken to obtain the best mutual proportion of the balsam to oil—the most critical factor.

TABLE II.

Constituents.						Spread.	Drying.	Removal.		Film characteristics.	
Solution.	Collol.	Balsol.	Castol.	Ethol.	Alcohol.	—	—	Condition.	Flexibility.	Colour.	Clarity.
1	1·2	1·8	2·0	2·5	0·3	Normal	Uneven, slow	Clean ...	Tough ...	Good ...	Good
2	...	1·6	2·0	...	...	...	Fair ...	...	...	...	...
3	...	1·4	2·0	...	...	...	Good ...	Inclined oily	...	...	...
4	...	1·4	1·8	...	...	...	...	Clean ...	...	...	...
5	...	1·4	1·6	...	...	...	...	Inclined oily	...	...	...
6	...	1·2	1·8	...	...	...	Fair ...	Clean ...	Inclined brittle	Fair ...	...
7	...	1·2	1·6	...	...	...	...	...	Inclined to tear	...	...

Of the seven films above it will be seen that only one gives a really satisfactory film all round, whereas all the others have some factor or factors precluding their use. Reducing the values in film 4 to terms involving only the original constituents, the composition of the solution giving satisfactory flexible films will be :—

Collodion	0·48	ccs.
Canada balsam	0·028	,,
Castor oil	0·018	,,
Ether	6·25	,,
Alcohol	0·43	,,

It is not asserted that other combinations are not so good, nor even better, perhaps, for the purpose required; but the above formula appears to work well under most conditions (all the usual precautions being observed), whereas other solutions, which on occasion give equally good results, may be found useless, as, for example, in a humid atmosphere.

Later experiments were performed using 10 per cent. ethol, thus obviating the addition of 0·3 ccs. of alcohol on each occasion.

It is to be noticed that in the case of these flexible films removal can be effected in all cases by sliding horizontally off the mercury surface. They are, in fact, so extensible that they may be distorted into hemispheres by blowing without fracture; in addition, they may be punctured by needles, for example, and prove self-sealing (in the case of small holes).

Thickness of Collodion Thin Films.

Interferometer tests have shown that the collodion films made as above are roughly two wavelengths of sodium light in thickness—i.e., approximately 0·00011 to 0·00012 cm.

A later type of film was made of roughly one-half the above thickness, a modification of the film-formula being the addition of 7 ccs. of 10 per cent. ethol, instead of the previous 2·5 ccs. in the standard case. It was clear that the " black film " stage was very near, as the films were a deep yellow by reflected light, and in one or two cases actual blackness was attained. Interferometer tests confirmed this, the thickness being of the order of one wavelength only, although any accurate measurement for displacements of only one wavelength are, of course, precluded. The thickness of the later type of film was therefore half the previous value, or roughly 0·00005 cm. It was noticed in the case of the black films that they were extremely brittle and had very little cohesion. It therefore seems probable that the limit has been reached in the thinness of films made by the method outlined. The standard thickness type of film and the latter half-thickness type were both used in making of bolometers.

II. ON SPUTTERING.

Outline.

The thin collodion films obtained by the process already dealt with, and mounted on an electrically non-conducting flat annulus containing two metal inlaid contacts, had now to undergo the process of " sputtering," or kathodic deposition, in order to lay a very thin strip of metal from contact to contact across the collodion film. The changes of resistance of this metal strip or film, resultant upon changes of temperature due to the absorption of radiant heat, constitute the principle of the bolometer.

Sputtering, as a method of depositing metals in mirror-like films, has been known for about fifty years, although, even now, we are not too well acquainted with the laws governing the process. A comprehensive treatment of the subject, together with an exhaustive bibliography up to 1912, is contained in a Paper by Kohlschutter.* A general summary, including the above, appears in an article by Kaye,† which forms a useful basis of information for work on this subject.‡

The present Paper does not treat of sputtering, or any of its phases, in particular; but merely records observations made whilst using the process as a means to an end—viz., the production of a new-type bolometer. Consequently, observations relative to sputtering alone will be mentioned only when they supplement, confirm or refute earlier work.

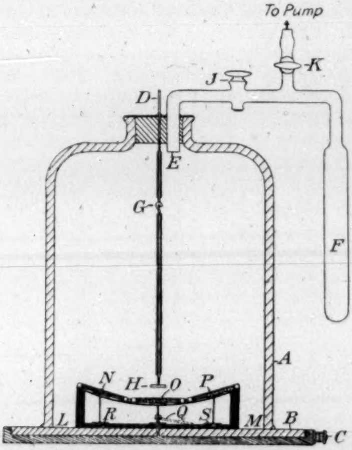

FIG. 3.—APPARATUS USED FOR SPUTTERING ON THE THIN FILMS.

Apparatus and Process Adopted.

The apparatus used in the first instance is represented in section (Fig. 3§). This consists of a bell-jar (*A*) resting upon a plane ground glass (*B*), the latter provided with a metal base and terminal (*C*). The aperture of the bell-jar is closed with a rubber cork, whose top is covered with a smooth layer of sealing-wax, forming an airtight joint. Through the cork pass an aluminium kathode (*D*) and a glass tube (*E*), leading through glass taps to the pump and charcoal-tube (*F*). Inside the bell-jar the kathode is broken at the swivel-joint (G), the lower part consisting of an aluminium rod attached at the end to a rectangular slab (H) of the metal to be sputtered. The aluminium wire, except at the swivel, is clothed with glass tubing to heighten the kathode-fall of potential at the end of the electrode. Underneath the kathode is the "sputtering-table," the top of which is a rectangular strip of mica, whose length is bent into the arc of a circle, centre G, and is supported at each end by upright pieces of ebonite fixed to an ebonite base-plate. Along the top of the mica are three strips of aluminium foil, about $\frac{1}{2}$ in. wide, running parallel to each other across the width of the mica. Similar aluminium strips are fixed to the ebonite base-plate, vertically below those on the mica, each pair being connected by a vertical aluminium wire at one edge of the table. Fixed to the ground-glass base, and connected electrically to the metal base-plate, is a selector contact (Q), which swings about a vertical axis, and can be brought to rest by tilting the apparatus upon any one of the three strips on the ebonite base-plate. The metal base-plate being the anode, it is clear that any of the three strips on the mica table-top can be connected to the anode, to the exclusion of the other two. Resting upon each strip of the table-top is an aluminium mask (N, O, P), sketched in Fig. 4 (shown upside down), covering the rings and films to be sputtered, the aperture of each mask being in the shape of a long slit ($\frac{1}{4}$ in. wide), overlapping at each end the two contacts inlaid in the ring below.

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* Jahrbuch d. Radioact, 9, pp. 355-418 (1912).

† Chemical World, 11, pp. 149-52, 183-5, 211-4 (1913).

‡ *See* also F. Simeon, Phys. Soc. and Opt. Soc. joint discussion, p. 26 (Nov., 1920).

§ All the diagrams of apparatus in this Paper are to scale, and therefore actual dimensions of components can be ascertained from the drawings if it be remembered that the diameter of the sealing-wax rings, which occur explicitly or implicitly in each, is 1⅜ in.

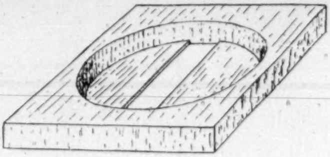

FIG. 4.—SPUTTERING MASK.

Briefly the process of sputtering is as follows: The primary of an induction coil (4 in. spark) is connected in series with a Wehnelt interrupter to a lighting circuit (110-220 volts), the secondary being connected, through a thermionic rectifier,[*] to the electrodes of the sputtering apparatus. The vacuum pump (Gaede oil) is put in connection with the charcoal tube by closing tap J (Fig. 3) and opening tap K, the charcoal being raised to a dull red heat by means of an electric heater. The latter is now removed, and, the charcoal being cool again, tap J is opened and the bell-jar pumped out. Liquid air is applied to the charcoal tube until the dark space round the kathode has a radius of between 1 and 2 cms. In general, under these conditions, sputtering takes place and a metallic film of the same shape as the mask aperture will be deposited on the collodion film.

Follow the experimental data which governed the design of the apparatus used and the process adopted, where such are of interest or show novelty.

It will facilitate matters if one divides into five arbitrary groups the factors affecting the process of sputtering. These factors are: Nature of kathode, kathode fall of potential and gas pressure, current density, nature of the gas and temperature of the kathode.

[*] Mullard, "U.30."

Nature of Kathode.

Due to the fact that most metals appear to have a very much smaller temperature-coefficient of resistance in the sputtered condition than that which they have normally, a large number were tried to find one with a coefficient sufficiently large for the purpose in view. No quantitative results[*] of any great practical value have appeared relating to the amount of any particular metal that will be sputtered under given conditions, owing, doubtless, to a very real difficulty in specifying the conditions. Qualitative lists have, however, been quoted by several writers, giving the order of facility in which sputtering takes place. The first is that given by Wright[†] in 1877, he apparently being the first to investigate the subject. His list in order of facility of sputtering is :—

(1) Bi, Au, Ag ... ... (readily)
(2) Pt, Pd, Pb, Sn, Zn, Cd ... ... (less readily)
(3) Cu, Fe, Ni, Co ... ... (with difficulty)
(4) Al, Mg ... ... (very difficult)

From this table he draws the conclusion that the heavy metals are more readily sputtered, whilst fusibility, tenacity and electrical conductivity all appear to exert some influence. Crookes,[‡] in 1891, gives his list :—

Pd, Au, Ag, Pb, Sn, Pt, Cu, Cd, Ni, Ir, Fe, Al, Mg,

which differs somewhat from that of Wright, and which he notes as being roughly in order of the voltaic series. Holborn and Austin,[§] and also Kohlschutter,[‖] state that, other things being equal, the amount of sputtering is roughly proportional to the chemical equivalent. These divergent opinions are not, of course, intended to be taken too seriously ; and one would say that the actual physical phenomena involved in sputtering are, as yet, little understood.

Comparative results of a large number of the present experiments, conducted under fairly constant conditions, are tabulated below :—

TABLE A.

Vapourizes.	Melts.	Thick deposits possible.		Thin deposits.	Very faint deposits only.	No appreciable deposits.
		Readily.	Not so readily.			
Hg*	Wood's* metal	Bi	Cd	Ir	Mn*	W
		Bi* (alloys)	Ni	Sn	Cr*	Al
		Au	Tl*	Pd	Zn	Se*
		Pt	Te*		Fe	
		Ag	Co		As*	
		Pb				
		Cu				

The metals marked with an asterisk (*) have not, apparently, been previously investigated.[*]

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[*] Since writing this Paper some valuable quantitative results for 25 metals have appeared, together with a very possible law relating these. *See* A. Güntherschulze, Zeit. f. Physik. 36, p. 563, Nov. 8, and 38, p. 575, Nov. 8 (1926).

[†] Am. Jour. Sci. and Arts, 13, pp. 49-55 (1877).

[‡] Proc. Roy. Soc., 50, 88 (1891).

[§] Wissensch. Abhandl. d. Phys. Techn. Reichsanstalt., Bd. 4, p. 101 (1904).

[‖] Loc. cit.

Apart from mercury and "Wood's metal," which will be dealt with later, it will be seen that bismuth heads a list of seven metals, all of which sputter readily to give thick, opaque films, if and when required. (The bismuth alloys were with tin and silver.) Five others follow which will give similar films, but require a longer "exposure" in which to do so. The first group require a period ranging roughly from ten minutes to half-an-hour, under the conditions to be specified later, whereas the second require from one-half to three hours. Up to a period of three hours the remaining metals give the results indicated. A few peculiarities should be noted. Tellurium deposits in a widespread manner, covering the whole of the bell-jar interior, whereas the deposition of other metals is more or less confined to the nearest object whose surface is normal to the direction of sputtering. Zinc and arsenic both scintillate vigorously, the former, however, only during the first few minutes. Selenium dilates, as if on the point of melting.

In the case of mercury the electrodes were reversed, the liquid being contained in a watch-glass supported on the mica table-top (kathode), with the collodion film (anode) suspended above it. Under the action of the discharge, the mercury was apparently in a state of ebullition, volatilization being widespread over the bell-jar. Something of the order of grams of mercury was sputtered in a few minutes; in all cases the greyish deposit coagulating into beads of all sizes. This experiment, which does not appear to have been carried out previously, offers, it is thought, striking confirmation of the theory that sputtering is a form of volatilization.*

Eliminating the metals not common to all, the table just quoted agrees with that of Wright in placing bismuth at the head of the list and palladium lower down, whereas agreement is closer with that of Crookes in placing copper and nickel higher. In the case of tin a lower value than in either of the others is assigned. Even if one constructs a table embodying the mean results of the three lists, it is difficult to find confirmation for any of the suggestions advanced as to the order being roughly that of the voltaic series or proportional to the chemical equivalents.

From considerations of the present experiments, it would appear that metals in Groups I, II and VIII of the Periodic System, together with the metals of high atomic number—Hg, Tl, Pb, Bi (11th row)—sputter with greater freedom than the majority.

Nature of Kathode: Minor Factors.

There are other factors of experimental importance which effect the deposition of metals. These are age of kathode, shape of kathode, shape of anode and form of the dark space.

It has been stated that the previous history of the kathode is of no importance;† in other words, that old and new kathodes would sputter to the same extent under otherwise similar conditions. The present experiments do not tend to confirm this, for it is found in the majority of cases that the deposit obtained from a second sputtering immediately following a previous one is superior in quality and obtained with greater freedom. One may instance the case of bismuth. An electrode of this metal, unused for a few weeks, gave with difficulty a black film, whereas an immediate repetition yielded the usual mirror-like deposit characteristic of sputtered metals. One can only credit the phenomena to the formation of a film of oxidation, to the adsorption or condensation of gases, or to all of these.

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* A. C. Longden, Phys. Rev., 11, p. 40 (1900).

† A. M. Tyndall and H. G. Hughes, Phil. Mag., 27, pp. 415-27 (1914).

The shape of the kathode is of importance, especially as regards the uniformity of the deposit obtained. That this is so follows from the conception, usually adopted, of sputtered particles leaving the kathode surface normally and travelling in straight lines. One expects, therefore, the shape of the sputtered film to be similar to that of the kathode. A rectangular electrode will, in general, give a rectangular image (somewhat blurred), and so on. Since, however, the sputtered particles are negatively charged, they will only deposit freely on surfaces which are positive to the kathode. The shape of the anode is therefore a factor in the modification of the shape of the resultant sputtered film. Again, the shape of both electrodes will affect the kathode-fall of potential, which is a vital factor in the rate of sputtering obtained. As an example, in these experiments all kathodes were rectangular plates (with a few exceptions), of dimensions slightly greater than those of the mask through which sputtering was to take place. In most cases uniformity of deposit was thus obtained. In one or two instances, however, where the anode distribution of potential had been modified by some means or other, it was found that greater uniformity was attained with the length of the kathode at right angles to the length of the mask.

The shape of the dark space is dependent, of course, on that of the electrodes and on the kathode fall of potential. It has been stated that no deposition will occur inside the dark space.[*] This statement would appear ambiguous, for the dark space will always be delimited in the proximity of the anode. For unrestrained deposition a surface positive to the kathode is required. A kathode then, although surrounded normally by a dark space of an inch, say, will yet deposit on an anodic surface at a distance of less than an inch.

It is important to notice, however, that in the present case, where sputtering is to take place on non-conducting collodion films, it is always advisable to use a kathode distance in excess of the dark-space depth, as it will be very difficult to maintain the film within the dark space sufficiently anodic to the kathode. In such a case it is only too true that sputtering will not occur within the dark space, although matters are changed once a preliminary thin deposit of metal, connected to the anode, has been sputtered on the collodion film.

Other Factors Affecting Deposition.

In the case of kathode fall of potential and gas pressure,[†][‡][§] no quantitative experiments were made, although qualitative remarks concerning them will appear later on. The relation between current density[†] and the rate of sputtering was not determined; whilst all the present data were obtained from metals deposited in air (nature of the gas).[†][‡][§]

Kathode Temperature.

Recent work has shown that, in the case of tungsten, the temperature of the kathode has no effect on the rate of disintegration.[‡]

On the other hand, Crookes found that, in the region of the melting point, the rate of sputtering is markedly increased in the case of the less refractory metals—tin, cadmium and lead. In the present experiments the effect of the discharge on Wood's metal is striking. This alloy is rapidly melted within a minute or so of the discharge being started. It is very doubtful whether the temperature of the metal is actually raised to the melting point, especially when one remembers the case of mercury, already quoted, in which the liquid was apparently in a state of ebullition, although the temperature could not conceivably be in the region of the boiling point. One can only conclude that sputtering is a form of electrical evaporation without considerable rise in temperature; the temperature of the kathode being of little importance, except where it approaches that of the boiling or melting points of the metal concerned, in which case the amount of sputtering is increased out of all proportion.

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[*] Kaye, loc. cit.

[†] Tyndall and Hughes, loc. cit.

[‡] Research Staff, G.E.C., Phil. Mag., 45. pp. 98-112 (1923).

[§] Kingdon and Langmuir, Phys. Rev., 20, p. 107 (1922).

Apparatus Detail.

Referring back to Fig. 3, the apparatus and method adopted can be examined in greater detail in the light of data obtained. Here, as in other later instruments, temporary vacuum-tight joints were made by using soft red wax. The wax is prepared for use by being squeezed through a small circular aperture into the form of a long thread. A piece of this thread would be attached all round the ground glass opening of the bell-jar (*L*, *M*), the latter being then placed on the ground-glass plate *B*. On applying the pump, the thread is squeezed into a wax tape, which effectively seals the apparatus. The function of the swivel joint at *G* is two-fold. By tilting the whole instrument one is enabled to present the kathode *H* opposite any of the three masks (*N*, *O*, *P*) containing films to be sputtered. Usually the centre mask is left vacant, so that a preliminary sputter, in the position shown, will ensure the kathode being rendered clean enough to give a normal deposit with discharges immediately following. By tilting the instrument the kathode will swing into position opposite to either *N* or *P*, thus enabling one to sputter two films in turn with only one evacuation of the jar. As it is found that sputtering deposits most freely on surfaces positive to the kathode, it is expedient to make the film to be coated as positive as possible, and to reduce otherwise the area of anode surfaces within the jar. This is achieved by the selector attachment previously described. When the jar is tilted to the right, the moving selector-contact *Q* will rotate on to the aluminium strip *S*, which is connected by a wire to the aluminium strip *P*, immediately under the film to be sputtered. At the same time the other four aluminium strips on the sputtering table are disconnected from the anode. (Adjacent masks are insulated from each other by wax threads.) This device tends to more rapid deposition than when no precautions are taken to reduce useless anode area; and considerably more than when no steps are taken to make the film positive.

In general, the conditions under which sputtering was conducted were as follows: Discharge current, 7 millamperes; dark space, 1 to 2 cms.; kathode distance, 2 to 3 cms.

It is found that currents in excess of 10 milliamperes, or gas pressures of less than that associated with 2 cms. dark space, appear to give black non-conducting deposits.

Wehnelt Interrupter.

A modified form of this break was designed for use with the sputtering apparatus. With the usual type, trouble is experienced in burning out the platinum anode. In addition, variations in the current strength are not permissible to any extent; the sulphuric acid electrolyte becomes vitiated after a time, making long runs of an hour or more impossible; and the temperature of the electrolyte rapidly increases. The design finally evolved, shown in Fig. 5, consists of a glass jar (*H*) containing an electrolyte (*A*) composed of a solution of one part phosphoric acid in seven parts of water. This constitution remains unchanged, excepting the evaporation of water, there apparently being no chemical changes taking place in this instance. More important still, the break works indefinitely.

The electrolyte is cooled by the lead helical piping (*B*), through which flows cold water, the piping also acting as the kathode. A novel feature is the form of anode. Instead of the usual platinum point projecting from an insulating tube within the liquid, there is here merely a platinum wire (*C*) dipping into the surface of the electrolyte. The platinum wire is fused at its upper end into a stouter aluminium wire (*D*), sealed into a glass tube (*E*). The latter protects the joint from acid spray, and also slides within the rubber cork (*F*), thus permitting variation in the depth of immersion of the platinum wire. The length of wire immersed provides an admirable means of regulating the strength of the current, acting, presumably, as a variable resistance. It was, in fact, found possible to dispense with any additional resistance in the primary circuit. Phosphoric acid has apparently a higher specific resistance than sulphuric acid.* The apertures in the wooden lid (*G*) permit the escape of evolved gas.

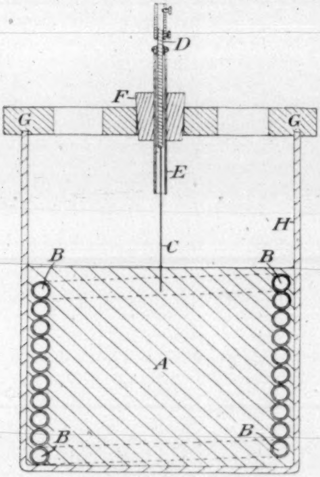

FIG. 5.—IMPROVED WEHNELT BREAK.

At a later stage, an alternating current of 5 amperes at 70 volts and 50 cycles was passed through the primary of the induction coil, instead of using the Wehnelt break. It was found that for large currents the Wehnelt was apt to " kick " on occasion, the resulting surge in the secondary giving rise to minute sparks along the edges of the metal contacts in the sealing-wax rings, thus destroying the conductivity of any metal film sputtered across them.

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* F. H. Newman, Proc. Roy. Soc., 99, pp. 324-30 (1921).

General Notes.

A few observations made should be noted before passing on to the more tangible results attained.

The inclusion of a thermionic rectifier in the secondary circuit is desirable. Without one, it was found impossible to exceed a unidirectional current of 4 milliamperes in the secondary. The inclusion of a rectifier, however, results in a secondary current proportional to the immersion of the platinum wire in the electrolyte of the Wehnelt break. Again, without the rectifier, the anode of the bell-jar will be acting during part of the time of discharge as a kathode, not only lengthening the total time of sputtering required (for two reasons), but also tending to contaminate the kathode with deposits from the component metals of which the anode may be composed. A contaminated kathode can only result in a film of poor quality. Similar effects have been noticed elsewhere.*

The effect of the discharge on the collodion films to be sputtered is interesting. It is noticed that after the deposit has been made the films become somewhat flaccid, the part actually coated often developing corrugations. If this same film be again subjected to the discharge, the slackness and corrugations are taken up, the film becoming quite taut. It appears, therefore, that the discharge has the effect of tightening up collodion films, and it is not, therefore, surprising that non-flexible films shatter under the process.

With high orders of vacua, corresponding to a dark space of 2 in. and more, these films are thrown into violent vibration by the discharge. Continued exposure will, in fact, result in the fracture of the film.

Temperature Coefficients of Resistance.

As stated previously, it was necessary to deposit on the collodion films a metal which, in the sputtered condition, would have a maximum temperature coefficient of resistance.

Of the various metals sputtered, a good number were abandoned for purely negative reasons. In some cases the metal films were too thin or non-existant. In others sputtered films, otherwise satisfactory, failed to make contact with the metal insets provided for the purpose in the insulating ring supporting the collodion film. In the case of silver, for example, contact is apparently impossible on brass, but readily obtained on nickel.

Of the metals which gave satisfactory conducting films, two methods were used to determine the temperature coefficients, the first adopted being fairly accurate, but slow; a later method, a rapid one, giving approximate results only.

Dealing with the latter first, it will be sufficient to state that it consisted in exposing the metal film to radiation from a focus lamp at a fixed distance. Given the numerical value of the temperature coefficient of resistance of one of the metals so tested, approximate values can be assigned to the others. Metals tested in this way, and their respective temperature coefficients, follow; the values given depending on that found for bismuth, which was determined by the alternative method described later.

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* L. F. Curtiss, Phys. Rev., 15, No. 6, III (1920).

TABLE B.

Metal.	Change of resistance with standard illumination.	Calculated temperature coefficients of resistance.	Notes.
Bi	0·273 per cent.	−0·0011	(B)
Pb	0·43 ,,	+0·0018	(B)
Tl	0·3 ,,	0·0012	(A) through glass plate
14 Bi 1 Sn ...	0·24 ,,	−0·0010	(B)
Cu	0·07 ,,	0·0003	(A)
Ag	0·05 ,,	−0·0002	(A)
Pd	zero	zero	(B)
Ir	...	...	(A)
Ni	...	...	(B)

NOTES: (A) signifies film unblackened; (B) blackened.

With the exception of bismuth, values of the above temperature coefficients do not appear in previous work. Those given above, of course, are very approximate only, and should be regarded as giving merely the order of the true coefficients in each case.

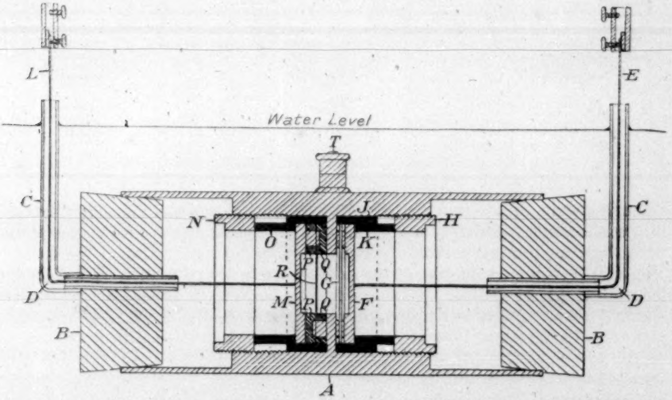
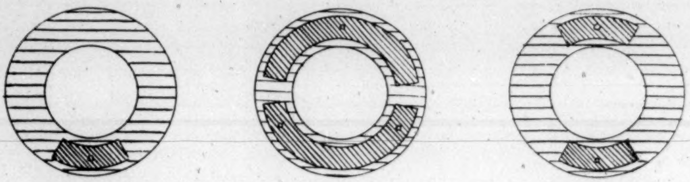


FIG. 6.—APPARATUS FOR DETERMINING TEMPERATURE COEFFICIENTS OF RESISTANCE.

Apparatus.

The apparatus used for a closer estimation of temperature coefficients is shown, in section, Fig. 6. This consists of a brass tube (*A*), closed at each end by large rubber corks (*B*) (*B*), through which pass glass tubes. Attached to the outside end of each glass tube are rubber tubes (*C*) (*C*). Passing through these tubes are stout copper wires, which can be bent at the points *D D* to the shape shown. To insert the various components, the copper wires are bent straight at *D D*, the corks and tubes being slipped off. The whole apparatus being immersed in water to the level indicated, it is clear that the liquid is excluded from the interior, although the latter is in communication with the atmosphere. One of the copper wires (*E*) is soldered to a brass disc (*F*), which is pressed into contact with one side of a platinum resistance thermometer (*G*) by the screwed brass ring (*H*). This "thermometer" is merely the business part of a hot-wire microphone,[*] and consists of an annular disc of mica across whose centre is stretched a grid of very fine platinum wire, roughly 0·0015 cms. in diameter. Contacts are made by two annular discs of tin-foil, one on each side of the mica. Thus, the wire *E* is in electrical connection with one end of the thermometer grid *G*, whilst the other end is connected to the brass casing (*A*). Insulation, where required, is provided by ebonite tubes (*J*) (*K*), the latter taking the pressure from the brass ring *H*. On the other side, the copper wire *L* is soldered to a brass disc (*M*), which is pressed, by means of the screwed brass ring *N* and the insulating ebonite tube *O*, into contact with a sealing-wax contact disc (*P*), constructed as described later and shown in Fig. 7A. This contact disc is identical in construction with those used for supporting the finished sputtered films. One of the latter (*Q*, Fig. 6) is in contact with disc *P*, the sides shown in Fig. 7 (top) being face to face. The other side of *Q* (Fig. 6), containing one contact, is pressed against the brass casing. By this means one end of the sputtered bolometer strip (*R*) is in electrical contact with the wire *L*, whilst the other end is in connection with the casing (*A*). Again, the bolometer (*R*) and the thermometer grid (*G*) are near each other, and enclosed by blackened metallic masses in close proximity. If the water surrounding the apparatus be raised in temperature, heat will be conducted through the walls of the brass tube *A* and radiated to the bolometer strip and the thermometer grid. Owing to the tenuity of both, they will each respond almost instantaneously to changes in temperature of the enclosure walls. The experimental procedure is now clear. The bolometer strip is included in one arm of a Callendar bridge by means of wire *L* and the common terminal T, whilst the thermometer grid is connected up in a second bridge of this type by the wire *E* and terminal *T*. The apparatus being



Bottom
(A)

Top
(A and B)

Bottom
(B)

FIG. 7.—THE CONTACT RINGS.

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[*] W. S. Tucker and E. T. Paris, Phil. Trans, A, Vol. 221, pp. 389-430.

immersed in melting ice, the resistance of both the bolometer and thermometer are measured. The ice-and-water bath is fitted with two mechanical stirrers. The stirring must be really efficient, and in these experiments the two stirrers were so arranged that water was continuously rotating round the brass tube at a fast rate. The bath is now heated at a uniform speed up to about 30°C., and parallel resistance readings taken on each bridge at intervals of one-half to one minute. Although the temperature of the blackened enclosure may lag behind that of the water bath by several degrees, yet the temperature of the bolometer and thermometer will always be that of the enclosure, and therefore the same at any given instant. If any assumptions are made here, their justification appears in the results obtained.[*] Three metals were tested in this manner (platinum, gold and bismuth), the resistance temperature curves being given in Figs. 8, 9 and 10. They are all straight lines within the limits of experimental errors. That for bismuth is especially interesting in that two curves are given relating to two different specimens. The curves are parallel, and the coefficients therefore the same, within the limits of experimental error, although their resistances at 0°C. are different. It would appear probable, then, that the resistance and age of a sputtered film have little or no effect on the temperature coefficient.[†]

The coefficients obtained were :—

TABLE C.

Metal.	Resistance at 0°C.	Temperature coefficient.	Temperature coefficient of metal in bulk.	Ratio of coefficients.
Pt	477·8 ω	+0·000040	0·0038	1/95
Au	28·54 ω	+0·00072	0·0040	1/5·5
Bi	1430·0 ω	−0·00110	0·0042	1/3·8
Bi	1399·0 ω	−0·00112	...	...

that for gold not having been previously determined elsewhere. Those for platinum and bismuth compare with values obtained by other workers as follows :—

TABLE D.

Metal.	Temperature coefficients found.	Becker and Curtiss.[‡]	Richtmyer & Curtiss.	Koller.[§]	Reynolds.[¶]
Pt ...	+0·000040	...	...	From small negative up to +0·002	From small negative up to +0·003
Bi ...	−0·0011	−0·001 to −0·003	−0·0016 to −0·0029	...	...

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[*] The lag in all experiments was 9·4°C., and was constant within one-half of 1 per cen

[†] Cf. F. K. Richtmyer and L. F. Curtiss, Phys. Rev., **15**, No. 6, II (1920).

[‡] J. A. Becker and L. F. Curtiss, Phys. Rev., 15, No. **6** (1920).

[§] L. R. Koller, ibid.

[||] F. K. Richtmyer and L. F. Curtiss, loc. cit., ibid.

[¶] F. W. Reynolds, Phys. Rev., 24, pp. 523-31 (1924).

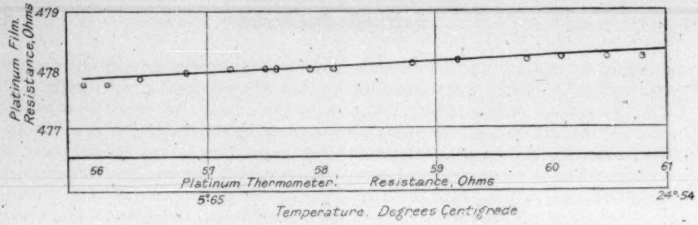

FIG. 8.—TEMPERATURE-RESISTANCE CURVE FOR PLATINUM.

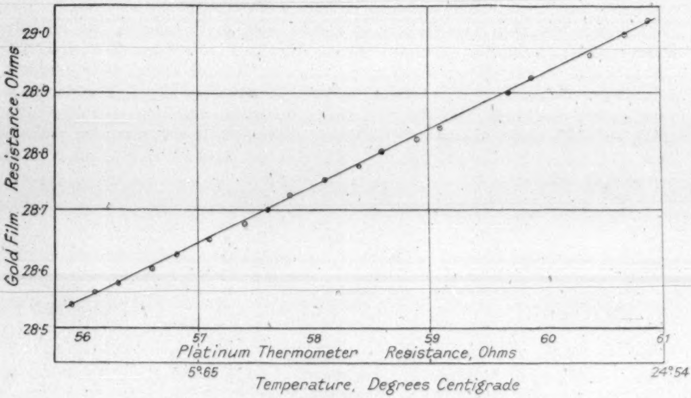

FIG. 9.—TEMPERATURE-RESISTANCE CURVE FOR GOLD.

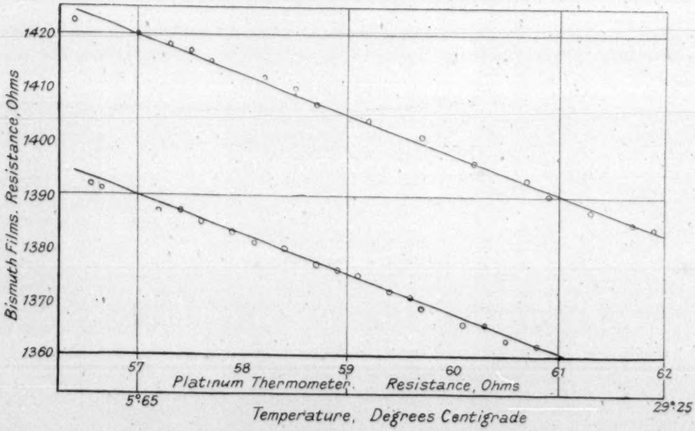

FIG. 10.—TEMPERATURE-RESISTANCE CURVES FOR BISMUTH.

In the case of bismuth the coefficient determined comes within the ranges quoted by others. For platinum, although only one result is quoted, a number of films were tested roughly, and in all cases a very small positive coefficient was found. Koller obtains his high positive coefficients by ageing at 200°C., whereas his unaged very thin films have negative coefficients. Reynolds observes the same, and attributes the reversal of the coefficient to absorption of oxygen whilst hot, as heat ageing alone is insufficient.

The new coefficient quoted for gold is probably reliable in cases where heat or chemical ageing has not taken place, a number of films roughly tested giving the same result.

It appears, therefore, that the four metals with the greatest temperature coefficients are :—

Lead	+0·0018	(approx.)
Thallium...	0·0012	(estimated)
Bismuth	−0·0011	
Gold	+0·00072	

Bismuth has, so far, been chosen as the metal to be used in the construction of these bolometers, as both lead and thallium oxidise in air, the latter with remarkable rapidity. Even when enclosed in a vacuum of a fraction of a millimetre of mercury of air, a film of thallium is completely oxidised in four days. Experiments are being made, however, with a view to constructing bolometers of lead and thallium enclosed in inert gases at low pressures.

Ageing.

The temperature coefficients of resistance should be considered in connection with the specific resistance of the film and its previous history. With regard to the latter, any process of ageing through which the film may have passed is important. Of the causes of ageing, possibly that of heat and that of time (atmospheric and inter-molecular action) show the greatest effects. No definite experiments were undertaken to determine these effects, but some data were incidentally obtained which may prove of interest.

Before being used as bolometers, the thin metal films obtained were blackened by smoking over a flame, a rise in temperature being thus involved. In every case a change of resistance was noted. These are tabulated below.

Metal.	Resistance.	Change.
Ni	579ω	+ 0·5ω
Pb	240ω	+ 55·0ω
Pd	3,280ω	+ 474·0ω
Bi	2,692ω	+ 186·0ω
Bi	4,500ω	+ 735·0ω
Bi	6,190ω	+2,980·0ω

The rise in resistance on heating in the case of nickel is remarkably small, amounting almost to constancy in resistance on heating. The three cases quoted for bismuth tend to show that a greater rise in proportion to the original resistance takes place the higher this initial resistance may be. Of the above, bismuth only figures in previous work, Richtmyer and Curtiss having dealt with this increase in the resistance of bismuth on heating in air.

A table follows giving the increases in resistance of various films acquired on being left in air for several days.

Metal.	Resistance.	Change.	Time.
Au	28·75 ω	+ 0·025 ω	1 day.
Pb	295·2 ω	+ 716·0 ω	4 days.
Ag	1,260·0 ω	− 13·0 ω	10 ,,
Bi	1,058·0 ω	+ 229·0 ω	6 ,,
Bi	1,370·0 ω	+ 377·0 ω	6 ,,
Bi	5,235·0 ω	+2,365·0 ω	14 ,,

Here gold displays a very small increase, but silver* is unique in showing an actual decrease. The three cases of bismuth, as in the case of heat ageing, seem to show an increase in resistance out of proportion to the original resistances. In this connection, the curves shown in Figs. 11 and 12 are not without interest. They relate to three other specimens of bismuth and show that the resistance does not increase linearly with time, but tends to a practical limit.† It also would appear that the curves are steeper, the greater the original resistance, confirming the deduction from the three cases of bismuth in the table just given. The break in the second curve, Fig. 11, is due to heat ageing—the film having been blackened at this point. The increased steepness in the resistance-time curve with the increase in "original" resistance is very striking.

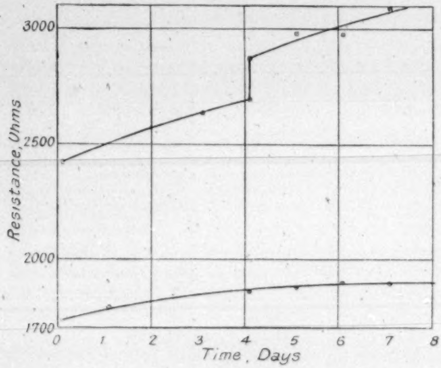

FIG. 11.

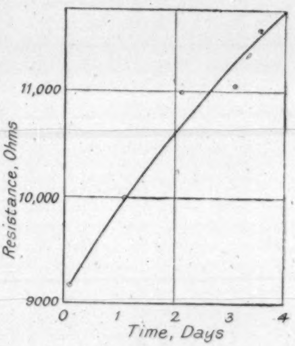

FIG. 12.

RESISTANCE : TIME CURVES FOR BISMUTH.

Ageing with time in air does not appear to have received a great deal of attention. The practical limit for ageing in air, in the case of platinum, found by Koller, is attributed by him to adsorption of air, hydrogen or oxygen.

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* Cf. decrease with heat ageing. J. Patterson, Phil. Mag., 6, 4, p. 652 (1902).

† Cf. case of platinum. L. R. Koller, loc. cit.

Resistances.

The resistance of a bolometer film of the size used ($\frac{1}{4} \times \frac{3}{4}$ inch) varies, of course, with the metal of which it is made. It is interesting to compare these, although, owing to variations in thickness, it may occasion surprise that one should associate any definite resistance with a particular metal. It is found, however, that there appears to be, when considered from the view-point of its ultimate purpose, an optimum range of resistance (possibly associated with a particular range of thicknesses) for each metal. Below are given examples of resistances possessed by various metals, sputtered under similar conditions, which may give some idea of the order of resistance that can be expected for any particular metal.

Metal.	Resistance.
Au	60 ohms.
Pb	270 ,,
Cu	470 ,,
Tl	310 ,,
Pd	3,200 ,,
Ir	3,600 ,,
Ni	580 ,,
Ag	1,300 ,,
Pt	270 ,,
Bi	2,010 ,,

These resistances are representative only of films immediately after sputtering.

Improved Sputtering Apparatus.

At this stage, the need for a sputtering apparatus of greater versatility became apparent. Improvements and additional facilities were needed in the following: (1) A bell-jar is needlessly large in view of the smallness of the area required to be sputtered, and involves longer periods of evacuation. (2) The separation of the bell-jar from the base-plate after sputtering, in order to remove the films, is difficult owing to the large area of contact. The wax joint has invariably to be warmed prior to breaking. (3) Provision should be made to fill the apparatus with gases other than air. (4) In the apparatus already described it is necessary on completion of sputtering to remove the film from the bell-jar (after admitting air), and to mount it in some type of holder providing electrical contacts, before its resistance and temperature coefficient of resistance can be measured. Apart from possible oxidisation of the film, this procedure is a delicate one fraught with numerous possibilities of accident to the fragile films. It would be an advantage, therefore, if the ring supporting the collodion film were already mounted in a holder providing electrical contacts to the outside of the apparatus before subjection to sputtering. In this way, the resistance of the film may be measured without breaking the vacuum. Not only this, but continuous readings of the resistance could be made during the whole process of sputtering. (5) If the apparatus can be so arranged that a beam of radiant heat be directed on to the film, an estimation of the temperature coefficient of resistance is possible at any stage of manufacture. (6) In the case of metals oxidisable in air, or more freely sputtered in other gases, it would be necessary to remove the sputtered film in its holder rendered gas-tight, by some device or other, before admitting air to the apparatus. In other words, the bolometer should be capable of being "bottled" in vacuo. (7) Owing to the fact that the supporting rings are made of sealing-wax and there is a definite heating effect associated with the discharge, some form of cooling is necessary to prevent distortion of these rings.

All these improvements are effected in the apparatus shown in section, Fig. 13. This consists essentially of a cylinder of glass (*A*), ground flat at each end, on which fit two aluminium shoulder-pieces (*B*) (*C*) of such a shape that distortion due to atmospheric pressure (and consequent fracture of the joints) is reduced to a minimum. The top piece (*B*) carries at its centre a circular disc of rock-salt (*D*) with optically worked faces. There are, in addition, two glass tubes fitted with taps (one only shown, *E*), and a ground-glass joint carrying the aluminium kathode lead (*F*), the inner member of which is capable of rotation about a vertical axis. Screwed round the rock-salt disc is a brass tube (*G*), on which can be fitted the standard source of radiant heat (a Nernst lamp—not shown) when required. The kathode lead (*F*) ends in a rectangular slab of the metal to be sputtered (*H*), and, together with the modification (*J*), the purpose of which is described later, is clothed in glass tubing. It will be seen that a partial rotation of the ground-glass joint at *F* will remove the kathode from over the central orifice in (*C*) to the side, where a preliminary sputter can be made to clean the kathode. The whole of the aluminium base (*C*) can be made the anode by fitting the conducting-plug (*L*). Beneath the base-plate is attached the film-holder (*M*) (also shown separately in Fig. 21). This holder, when removed, constitutes the complete bolometer itself, as will be shown. There is only one aperture to this holder which, when in the position drawn, is totally within

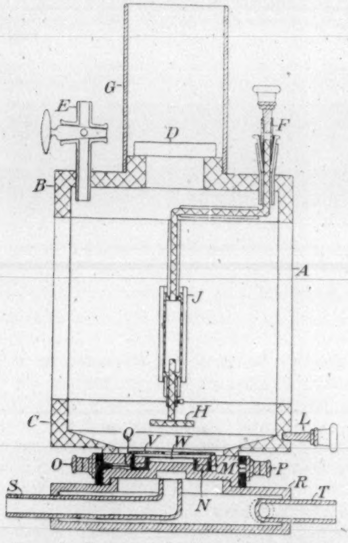


FIG. 13.—IMPROVED SPUTTERING APPARATUS.

the apparatus, and is therefore gas-tight. Inside the holder at *N* (Fig. 13) is a sealing-wax contact-ring (Fig. 7A), with two contacts on top and one beneath, the latter making contact with the body of the holder. The other contact, which only occupies a part of the depth of the ring, is connected by means of an insulated wire to the terminal *O* (Fig. 13), the aperture in the holder through which the wire passes being rendered air-tight by a conical plug of "Chatterton's" compound. The terminal (*P*) is in contact with the body of the holder. The sealing-wax ring, with a collodion film stretched across its aperture, is placed on top of the contact-ring (*N*), and screwed down by the brass ring (*Q*). This film-supporting ring is modified, as shown in Fig. 7B, in that it has two contacts on each face, the top pair providing contacts for the sputtered metal film, whilst the lower pair make contact with those of the ring below. When the metal film-strip has been sputtered on to the collodion film and its supporting ring, it is obvious that one end will be in connection with terminal *O* (Fig. 13), and the other end to *P*, both terminals being outside the apparatus. Screwed into the base of the film-holder (*M*) is the water-cooling jacket (*R*) with its supply pipes arranged so that the jacket is always full of water and the exit pipe (*S*) so placed that adequate cooling is ensured to the base of the film-holder. The entry pipe (*T*) is bent at right-angles inside to give the water a rotary motion within the jacket. By this means the sealing-wax rings above are kept efficiently cool by conduction.

All joints in the apparatus described are rendered air-tight by means of soft-wax tapes. The liquid-air apparatus, similar to that shown fixed to the bell-jar in Fig. 3 is here kept separate, connection being made by means of rubber pressure-tubing to one of the glass taps fitted to the top. In this way the sputtering apparatus can be turned upside down when required without reversing the liquid-air tubes.

With this instrument the resistance of the sputtered film can be measured continuously during its manufacture by connecting the terminals *O* and *P* (Fig. 13) in series with a cell of known E.M.F. and a micro-ammeter with shunts (calibrated, if desired, direct in ohms). At any period during its formation, the temperature coefficient of resistance of the sputtered metal can be measured by rotating the kathode (*FJH*) clear of the film, and attaching the Nernst lamp to the tube *G* on top of the instrument.

The volume of air to be evacuated in this apparatus is much smaller than in the previous bell-jar. The joint to be broken (that between the base-plate (*C*) and the film-holder (*M*)) is so much smaller that preliminary heating of the wax is unnecessary, and makes for economy in wax. Being fitted with two taps, it is possible to fill the apparatus with any gas desired.

In the case of a metal film which is adversely affected by air, it is possible to bottle it in vacuo by an auxiliary apparatus (not shown), containing a rock-salt disc at one side, interposed between the base-plate and the film-holder. On the completion of sputtering, the whole apparatus is inverted and a subsequent tilting slides the disc into position immediately beneath the film-holder. By again placing the apparatus in an upright position, the rock-salt disc will fall into position over the aperture in the film-holder (surrounded, before evacuation, by a thread of soft wax). Admission of atmospheric pressure to the apparatus forces the disc into contact, effectively sealing the film-holder from the atmosphere. The holder can then be removed and forms a complete air-tight bolometer. The reason for the modification in the design of the kathode at *J* is now clear. Owing to the thickness (about ½ inch) of the subsidiary apparatus inserted between the base and the film-holder the kathode will normally project somewhat into the centre of this auxiliary apparatus. On reversing, however, the modification in the kathode is automatically shortened by gravity, and clears the central space sufficiently to allow the rock-salt disc to be slid into position. Normally the mask which controlls the shape of the sputtered film is located, as shown, at *V*. When, however, the subsidiary apparatus is in use the mask is placed inside the latter, and is swung clear when the rock-salt disc slides into position.

In some cases the film-holder itself was made the anode instead of the base-plate by interposing a thin ebonite washer between *C* and *M*, and making terminal *P* the anode. This always accentuated a peculiar phenomenon noted in all cases—viz., a subsidiary current, due to the discharge, set up in the circuit connected to the terminals *O* and *P* for measuring the resistance of the sputtered film. This current is due, of course, to an E.M.F. set up on the film-contacts, owing to an asymmetric kathode-fall of potential between the electrodes. This is very marked when the rectangular kathode is so placed that it is not truly parallel to the mask aperture. It is usually made visible by the anode glow on the contacts, the glow disappearing when true parallelity is attained. As an example, a current, due to the discharge, of 150 micro-amperes was reduced to 7 micro-amperes with correct adjustment. In all cases, too, this discharge-current is increased when the water-cooling is used, the increase being of the order of 10 times. This is possibly due to inefficient insulation from the earth of the coil and other apparatus used in producing the discharge. In any case, however, the anode glow disappears, together with the "discharge-current," on increasing the vacuum (say, of that associated with a dark-space of 1·5 cm.).

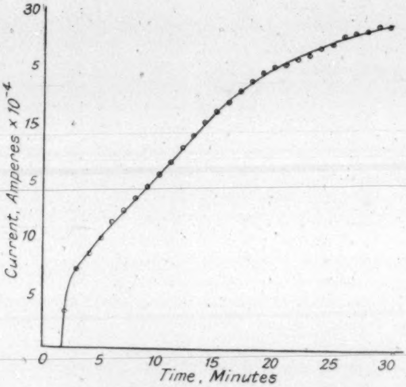


FIG. 14.—GROWTH OF A SPUTTERED FILM.

With this new sputtering apparatus an interesting result has been obtained, throwing some light, perhaps, on the process of sputtering. The curve in Fig. 14 shows the increase of current through the film (a criterion of its decreasing resistance) against time of sputtering. The curve (for bismuth) seems to be made up of three parts, of which the middle part is approximately a straight line. The first part, curved, can be explained if we imagine that the film has been sputtered in part before contact begins to be made. Contact will occur suddenly some time after commencement of sputtering and will become more and more widespread as an ever increasing number of linkages are made between the various aggregations of sputtered particles. After a time contact is " perfect " throughout, and the straight second part of the curve shows conductivity as strictly proportional to thickness. The third curved portion is more difficult of explanation. When it is known that frequently at the end of such a curve conductivity often ceases quite suddenly, a reasonable supposition is that the aggregation of molecules, after a certain thickness is attained, is accelerated ; the aggregations eventually becoming large enough to split the film across, although such splitting may be quite invisible. This " ageing " may be due, of course, to the higher temperature, derived from the discharge, which would be attained with a greater thickness. Herein lies the basis for the previous statement that there is a range of optimum resistance for each metal ; this range coinciding, one would say, with the straight portion of its characteristic curve.

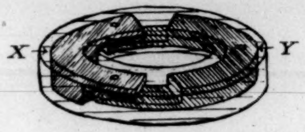


FIG. 15.—SKELETON VIEW OF A CONTACT RING.

It has been repeatedly noticed that if attempts are made to obtain much thicker deposits of low resistance by protracted sputtering, conductivity ceases suddenly soon after the optimum resistance has been attained, but will build up again and again to the same value. In the case of bismuth, for example, it seems impossible to obtain a permanent film of less than 1,500-2,000 ohms.

III. ON THE CONSTRUCTION AND PERFORMANCE OF THE NEW-TYPE BOLOMETER.

General Construction.

A description of the component parts of the bolometer follows. The process by which the necessary thin collodion films were made, and the method of coating these with thin films of metal by sputtering, have already been described.

To support the thin collodion film, a flat annulus of sealing-wax is used, inset with metallic contacts. These are shown in plan, top and bottom, in Fig. 7 ; also in skeleton, Fig. 15. The collodion film adheres to the top of the ring ; thus there are, beneath the film, two wide contacts, one for each end of the metallic overlying strip subsequently sputtered. In one type of ring there is one contact only going through to the bottom (Fig. 7A) ; in the other, both go through (Fig. 7B).

Moulding Apparatus for Contact-rings.

The apparatus used to make these rings is shown in Fig. 16. This consists of a flat, circular vessel, composed of a base-plate (*A*) and a rim (*B*), split into two semi-circular halves, all of brass. This vessel rests in a circular depression in an iron plate (*C*) fitted with two upright iron guide-rods. Sliding on these rods is a bridge (*D*), carrying the mould (*E*). The brass contact-pieces, shaped as shown in skeleton below, are screwed on to the base-plate (*A*) by means of four screws. The first (*F*) secures the bottom part of contact *X* flush against the base-plate, whilst the second (*G*) has the top end of its thread turned off to form a collar, the latter holding contact *Y* 1-32nd of an inch from the base-plate. (In the case of rings of type *B*, Fig. 7, contact *Y* and its associated screw are modified so that *Y* is in contact with the base-plate in order to give the second bottom contact.) There are two further screws (*H*), one of which is incorrectly shown, for the sake of clearness, in the plane of the section, both being turned off with the two collars of different diameters, which hold the laminated ends of the contacts *X* and *Y* 1-32nd of an inch apart. Interposed between the base-plate and the metal contacts is a very thin disc of asbestos paper (*J*), the purpose of which will appear immediately.

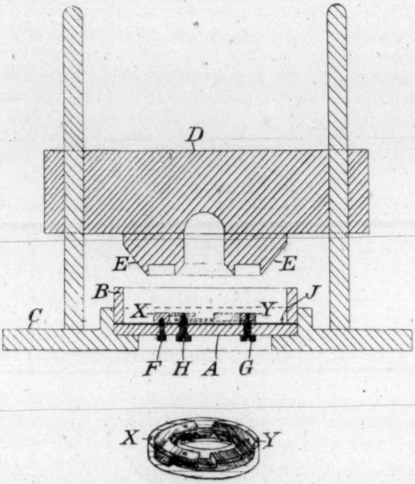


FIG. 16.—MOULDING APPARATUS FOR CONTACT RINGS.

Process of Moulding.

The process of ring manufacture is very simple. The bridge and mould are removed, the remainder of the apparatus being heated to the temperature of melting sealing-wax. A small quantity of the molten wax is poured into the cavity, and when no further air bubbles appear and the wax smooth, the bridge and mould are replaced and forced on to the wax in a press. In this way the wax is made to adhere to the metal contacts by reason of their high temperature, but not to the mould owing to its large mass of cold metal. Neither will the wax stick to the hot base-plate, owing to the interposition of the asbestos paper. When cold the mould is pulled away, the screws removed and the brass box (*A*, *B*) split apart, leaving a complete ring adhering to its asbestos paper. Both sides of the ring are rubbed down on fine glass paper until the paper is removed from the bottom and the contact faces, top and bottom, are flush with the surrounding wax. The contacts being quite level with the wax surface, the superimposed collodion film will overlie the junction without the formation of a ridge or gap. Consequently when the thin metal strip is sputtered across the whole from contact to contact there is no chance of continuity being broken at a " precipitous " junction of metal and wax.

The Contact Rings.

The appearance of the finished ring is shown in plan in Fig. 17. Here *A* is the wax ring, *B* and *C* the metal contacts. A collodion film (*D*) is superimposed and its periphery removed by wiping with alcohol so that it just covers the aperature of the ring and the junction between the metal and wax, leaving the majority of the metal contact surface exposed, ready to receive the ends of the sputtered bolometer strip (*E*). It is clear that the latter will be in electrical contact at its ends with *B* and *C* and yet will be practically flat throughout. The only possible deviation from the horizontal will be at the boundary of the collodion film. As, however, the film is " shelved " off owing to the action of the alcohol, and in any case its thickness is of the same order as that of the bolometer strip, it is found that the method provides a certain and perfect contact. In the diagram, portions covered by others are shown unshaded.

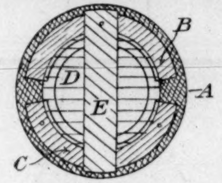


FIG. 17.—DIAGRAMMATIC SKETCH OF A FILM BOLOMETER.

It should be noted in passing that the overlapping of the laminated ends of the metal contacts within the enclosing wax augments the mechanical strength of the rings. So much so, that they may be dropped on the floor without fracture, whereas without the overlap the rings are almost too brittle to be handled.

The metal inset contacts were made of brass, nickel and steel. The steel contacts are most satisfactory from the point of view of oxidisation, provided that they are kept reasonably free from moisture.

Blackening.

For use in air, these bolometers have now to be blackened—the sputtered strip possessing normally a mirror-like surface. Blackening is carried out by means of the apparatus shown in Fig. 18, the purpose of which is to provide finely-divided smoke at a temperature sufficiently low to obviate destruction of the collodion film. Smoke is obtained by means of the paraffin lamp (*A*) providing a small, smoky flame. This rises up through the bell-mouthed metal chimney (*B*), taking with it a large quantity of air. Over the aperture of the chimney is a metal disc pierced at its centre with a small hole. The disc is held clear of the chimney top by two pieces of wood (*F*) and supports, in its turn, spaced in the same way by two further pieces (*E*, *E*), the wax ring carrying the metal film to be sputtered facing downwards. Smoke will be rble to escape from under the disc (*C*), but a small quantity of cooler smoke will pass through the hole, playing on the film surface ; part being deposited, the remainder escaping from underneath.

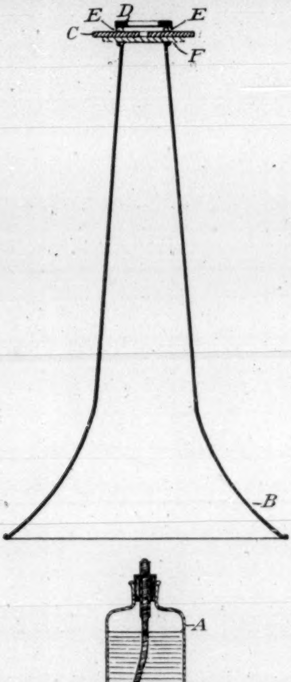


Fig. 18.—Apparatus used for Blackening Film Bolometers.

Types of Bolometer Holders.

The finished bolometer, when smoked, is mounted in the holder shown in Fig. 19. This consists of a brass cylinder (*A*), covered with an ebonite sleeve (*B*). Inside, at the centre, is an ebonite cylinder (*C*), fitted in the middle with an ebonite disc (*D*). Adjacent to this disc is another of brass (*E*), against which presses the bottom face of the ring carrying the bolometer film (*H*), its one contact on this side (*F*) connecting electrically to the disc (*E*). Pressing against the top surface of this ring is a second ring of the same type (Fig. 7, *A*), the two top faces being in contact. The contact coming through to the bottom (*G*, Fig. 19) connects to the brass washer *J*, the whole being pressed into intimate contact by the screwed tube *K*. From the two brass discs *E* and *J* wires pass to the terminals *O* and *P* through insulated slots (not shown) in the body of the tube *A*. One end of the bolometer strip will thus be connected to the terminal *O* and the other end to terminal *P*. Air draughts are excluded by the tube *L*, sliding within *K*, which carries two baffles at *M* and *N*, the shape of the apertures being rectangular, similar to that of the bolometer strip.

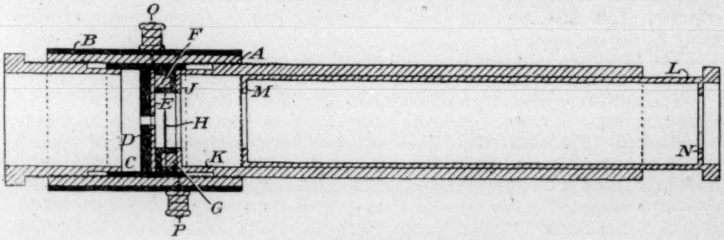


Fig. 19.—Holder for Film Bolometer.

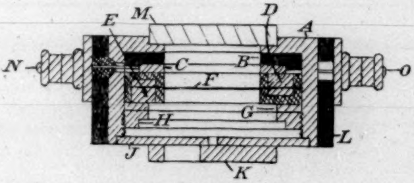


Fig. 20.—Gas-tight Holder.

The discs *D* and *E* are pierced with apertures at their centres in order that the bolometer may be alined on to any source of heat by sight.

In cases where the bolometer films are to be kept from contact with the air, the gastight holder shown in Fig. 20 is used. Here exactly the same method of mounting is observed as in the air type above, with the exception that the casing is used as one terminal (*O*). The other terminal (*N*) is connected to the contact washer (*C*) by means of a wire passing through the casing, the aperture being sealed with "Chatterton's compound." The top of the holder is sealed with a rock-salt disc (*M*), the joint being made with wax tape. The bottom is sealed by a brass disc (*J*), waxed at the rim, pierced with a small hole at the centre. Sliding over the disc, on a film of soft wax, is another thicker one (*K*), with a much larger hole placed eccentrically as shown. The bolometer having been made and mounted up inside, the disc *J* is waxed on and the second disc (*K*) placed in position with its aperture over that in *J*. The end of a glass tube, connected to an air pump, is inserted in the aperture of *K* and waxed in, the holder then being evacuated. Sealing under vacuo is effected by sliding the disc *K*, slightly warmed, until the apertures no longer coincide. The glass tube can then be removed, leaving the bolometer mounted ready for use in vacuo.

The two types of holder described above are for use in the case of those films made in the bell-jar apparatus (Fig. 3).

The third type (Fig. 21) is a component part of the improved sputtering apparatus (Fig. 13), and forms a complete holder to the interior of which air need never be admitted. The connections have already been described. The one aperture is sealed by the rock-salt disc *J* (Fig. 21). It should be noted that the film surface is within 1/32nd of an inch of the top of the holder, this form of construction being expedient owing to the fact that the mask (placed of necessity on top of the holder) must be very close to the film to ensure a clean image of its aperture being thrown in sputtering.

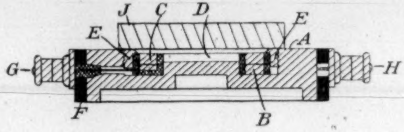


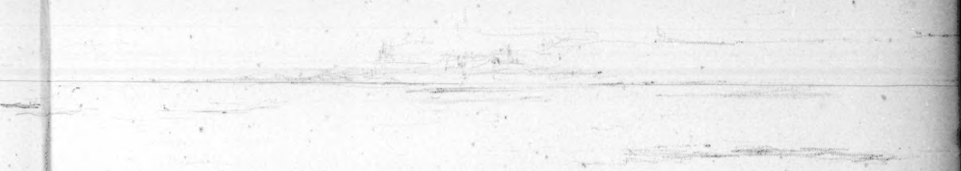
FIG. 21.—VACUUM HOLDER.

The Bolometer in Use.

The intention in designing the new type was that of reducing, still further than in existing types, the thickness of the bolometer grid. It should be remembered that one may sputter an almost impalpable metal film, the only restriction being that it must conduct an electric current. Practically transparent metal films could therefore be used. One is handicapped, however, by the need of a supporting film of some insulating substance; and, speaking generally, it is the combined thickness of the metal and supporting films together which will determine the lag of the instrument. Unless this combined thickness is less than that of the metal grids in other types, obtained by rolling or chemical means, the new type will show no advantage in the matter of speed of response.

Sensitivity.

Tests have shown that a definite advance in "speed" has been obtained, but with the sacrifice of a certain amount of sensitivity. In the case of the bismuth bolometer, for example, it will be remembered that in the sputtered condition bismuth has a temperature coefficient of resistance a little less than a third of that of platinum in bulk. It is possible that the lead type, with a coefficient of roughly a half, may



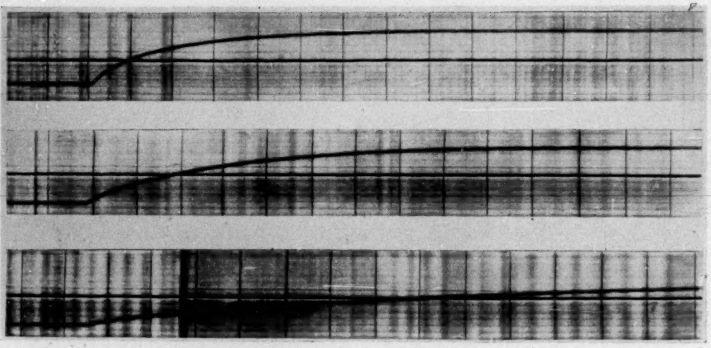


FIG. 22.—PHOTOGRAPHIC RECORD OF FASTER RESPONSE OF FILM BOLOMETERS (TOP PAIR).

prove better eventually, whilst thallium may show still better results. At the moment, however, we must accept the sensitivity of the new type as roughly a third of that of previous types (using platinum grids), *ceteris paribus.*

Sensitivity Data.

To determine the response of a film bolometer to radiation of known intensity, a focus lamp radiating 0·95 gm. calories/cm.²/min. was placed at a distance of 500 mms. The change in resistance having been measured, the Lummer and Kurlbaum[*] was substituted at the same distance.

Intensity of radiation.	Change of Resistance.	
	Film type.	L. & K.
10·56 milliwatts/cm.²	0·316 p.c.	1·07 p.c.

The ratio of sensitivity is therefore $\frac{L\ \&\ K}{\text{Film}}=\mathbf{3{\cdot}39}$.

Speed.

To determine what advance had been gained in speed, comparative photographs were taken, using the Einthoven string galvanometer. Comparison was made with a Lummer and Kurlbaum bolometer,[*] which would appear to embody the greatest advances in speeding up these instruments by reduction of the grid thickness to the lowest possible mechanical limits.

A preliminary photograph was taken with the Einthoven to ensure that no extraneous mechanical or electrical vibrations were being recorded. A second film depicted the effect of a commutated current in a non-inductive circuit. Inspection of this photograph enabled one to estimate the natural frequency of vibration of the string itself, and to ensure that its period was of a different order to that expected in the response of the bolometers to heat impulses.

Time markings, perpendicular to the travel of the photographic film, were made every one-fifteenth of a second by an apparatus whose driving motor was controlled by a tuning-fork of known frequency, a stroboscopic attachment ensuring that the control was visibly effective. The galvanometer string was connected to a Wheatstone bridge containing the bolometer, and on the latter being exposed suddenly, by means of a "Compur" shutter, to a beam of radiant heat, the bridge is thrown out of balance and the change in resistance results in a lateral displacement of the string-shadow in the camera.

The motor drive for the film having been started, this apparatus would be brought into play, thus photographically recording the growth of the bolometer current due to exposure to a heat source from some definite starting moment up to the attainment of the full deflection.

Fig. 22 shows three actual photographs taken in this way, the bottom one due to the Lummer and Kurlbaum, whilst the upper pair refer to two different types of "film" bolometers. The equidistant parallel lines are the time marks, and give us time abscissæ; whilst the distance between the film shadow and the straight line down the middle of the film gives ordinates of deflection. It will be seen that the response of both film bolometers is much quicker than that of our standard of comparison, especially in the case shown in the top photograph. The latter refers to the later type of instrument, previously mentioned, whose supporting collodion film is roughly one-half the thickness of that of the original specimen, of which the middle photograph is representative.

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[*] **Ann. der Physik, Bd. 46, p. 204 (1892).**

These photographs were measured up on a travelling microscope, the results being plotted as in Fig. 23. It was found that measurements of deflection could be repeated to within 1/50th of a millimetre, the full deflections ranging from 10 to 15 millimetres. The same accuracy was probable in the case of the time measurements. These orders of accuracy are in excess of possible experimental errors. As far as the timing was concerned, this was never further out than one beat in 180, so that an accuracy of 1 in 150 is certain. Two sources of error are possible in the experimental recording of deflections. The first, which in view of the results may be neglected, is the possible distortion of the photographic film in the camera. The second, an irregular electrical vibration of the galvanometer string, occurred in some cases, and is due to fields set up by the electric arc used as an illuminant. In trial photographs of a commutated current, previously referred to, these irregularities are distinctly traceable to variations in the strength of the arc currents, as the latter are registered at varying periodic intervals parallel to the time markings, similar to those visible in the three photographs in Fig. 22. The two irregularities occur simultaneously. These displacements of the string shadow, due to extraneous causes, are of the order of 1/10th of a millimetre as a maximum. We shall therefore be justified in regarding our measurements of deflection as accurate only to 1 per cent. This inaccuracy, due to an illuminant unsuited to its purpose, could not be avoided, as no other was available; most of the apparatus, as a matter of

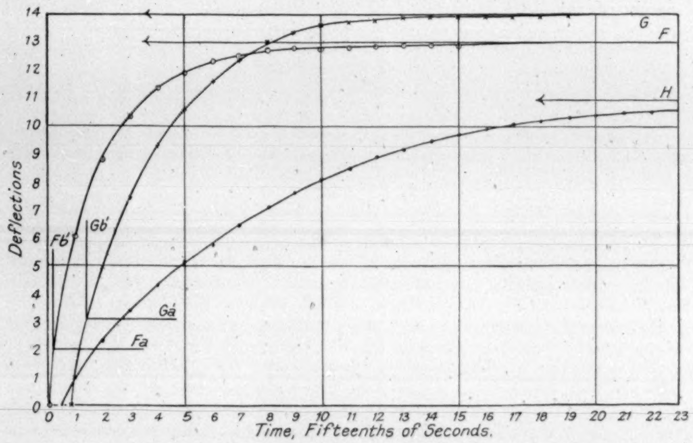

FIG. 23.—RESPONSE OF COMPARATIVE BOLOMETERS TO RADIANT HEAT.

fact, with the exception of the galvanometer itself, was made up in the laboratory. In passing, it should be remarked that the test film referred to showed that the galvanometer was practically dead-beat at the string tension used, the minute oscillations present dying down in less than 1/100ths of a second.

In Fig. 23, the faster type of film bolometer gives the graph F, the slower one G; H being due to the Lummer and Kurlbaum standard. It will be seen that each curve approaches its final deflection asymptotically, and each can be roughly represented by the formula

$$\theta = \theta_\infty(1 - e^{-at}),$$

where θ gives the deflection at any moment, θ_∞ is the final deflection, a is a constant and t the time.

It is a very difficult matter to arrange the apparatus so that each bolometer gives the same final deflection on the photographic film. To compare the performances of the three instruments, however, it is useful to compare them under conditions such that each will give identical final deflections. If these curves can be represented by the formula quoted, then we may shift the origins of the two curves of greatest amplitude, so that they shall all represent performances with the same final deflections. The new axes required for curves F and G are shown at Fa', Fb', Ga' and Gb', whilst the resulting comparative curves are shown together in Fig. 24. From an inspection of this it can be seen immediately what are the relative times for each bolometer to attain any given fraction of its full deflection. Taking the times for nine different deflections (2 to 10), the mean comparative speeds are :

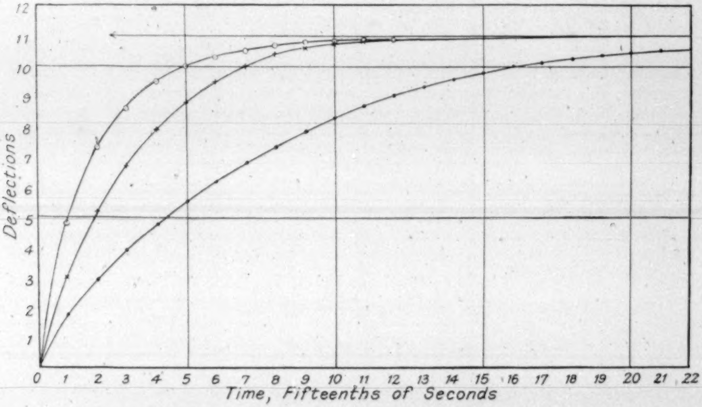

FIG. 24.—COMPARISON, UNDER SAME CONDITIONS, OF BOLOMETERS' RESPONSE.

$$\frac{F}{H} = 3{\cdot}90 \qquad \frac{G}{H} = 2{\cdot}21 \qquad \frac{F}{G} = 1{\cdot}77.$$

Roughly, therefore, the thinner film bolometer is four times, and the thicker one twice, as fast as the Lummer and Kurlbaum.

This determination of relative speeds depends on the accuracy with which the simple exponential formula represents experimental observations. It is found, however, that in all three cases substitution of corresponding values of t and θ in this expression give different values for the constant a. If the values of " a " so obtained are plotted in their turn against " t," it transpires that the variation of the constant in each case decreases smoothly with increasing time, following closely a logarithmic curve of the type

$$a=a_0+K(t+c)^n$$

However, no formula of this type, representing the results with sufficient accuracy, suits our immediate purpose, and one falls back on representing this variation in the constant by means of the more empirical expression

$$a=b+ct+dt^2$$

Our original formula therefore becomes

$$\theta=\theta_\infty\{1-e^{-(bt+ct^2+dt^3)}\}$$

Choosing the range $t=0$ to 5, in order that the greater curvature may result in greater precision, this expression represents the results with sufficient accuracy, although, of course, extrapolation for values of t beyond 5 will be meaningless. The values of the constants obtained are

	b.	*c.*	*d.*
Film Bolometer *F*	+0·765	−0·107	+0·0113
Film Bolometer *G*	+0·418	−0·038	+0·0045
Lummer and Kurlbaum *H*	+0·196	−0·0245	+0·00275

The respective " speeds " will be given in each case by

$$\frac{d\theta}{dt}=\theta_\infty\,(b+2ct+3dt^2)e^{-(bt+ct^2+dt^3)}$$

As a particular case, when $t=0$, the speeds will be

$$\left[\frac{d\theta}{dt}\right]_{t=0}=\theta_\infty\,.\,b,$$

and if the final deflections θ_∞ are taken, the same for each, to give a fair comparison the respective speeds work out as

$$\frac{F}{H}=3{\cdot}90 \qquad \frac{G}{H}=2{\cdot}13 \qquad \frac{F}{G}=1{\cdot}83$$

These figures refer only, of course, to the speeds of the respective " kicks " at the start of each bolometer's response. They will not, owing to the departure of the empirical formula from the simple exponential form, represent the merits of the three instruments at later stages. Further information is required as to what time each bolometer requires to attain some fraction of its full deflection. Examining this question, let the respective times be found for each to attain $1/n$th of its full deflection whatever θ_∞ may be.

The general equation concerned is

$$\theta=\theta\infty\ \{1-e^{-(bt+ct^2+dt^3)}\}$$

or writing θ_D for $(\theta\infty-\theta)$ and taking logs

$$\log_{10}\theta_D=-(bt+ct^2+dt^3)\ .\ \log_{10}e+\log_{10}\theta\infty$$

and

$$\frac{\log\dfrac{\theta\infty}{\theta_D}}{\log e}=bt+ct^2+dt^3$$

Now if θ be put$=\dfrac{\theta\infty}{n}$

then

$$\frac{\log\dfrac{n}{(n-1)}}{\log e}=bt+ct^2+dt^3$$

or putting N equal to the constant expression on the left

$$N=bt+ct^2+dt^3$$

For two instruments, therefore, F and H say, the respective times to attain $1/n$th of the final deflection $\theta\infty$ in each case will be given by

$$b_Ft_F+c_Ft_F{}^2+d_Ft_F{}^3=b_Ht_H+c_Ht_H{}^2+d_Ht_H{}^3$$

as both sides of the equation will equal N.

If, now, the expressions on each side of the equation be plotted against t, and any straight line drawn parallel to the axis of time to intersect the two curves, the values of t at the intersections will be inversely proportional to the respective speeds. A series of such straight lines can thus be drawn, and the mean value of the respective speeds obtained. The mean relative speeds are

$$\frac{F}{H}=4{\cdot}25 \qquad \frac{G}{H}=2{\cdot}39 \qquad \frac{F}{G}=1{\cdot}78$$

Thus the actual mean speeds of the film bolometers relatively to the Lummer and Kurlbaum standard are slightly in excess of those obtained when $t=0$.

It is, however, clear that the graphical comparison shown in Fig. 24 is sufficiently close for the purpose of this Paper, where one is merely endeavouring to ascertain what advance has been obtained in speed. Various models of the same type would, of course, differ slightly amongst themselves in this matter of speed; and it suffices to state that the thinner type is roughly four times, and the thicker twice, as fast as the Lummer and Kurlbaum.*

Thickness of Bolometer Films.

It has been seen that the thickness of the standard supporting collodion film was 0·00012 cm. As far as the sputtered metal film is concerned, interferometer measurements fail, the film being much thinner than a wavelength. One can only estimate the thickness from considerations of the specific resistance of the metal and the superficial dimensions of the film. The thickness would be given for unit resistance by $\rho \times$length$\div$width. The latter are 0·75 and 0·25 in respectively, but the specific resistance is unknown. Patterson* has stated that the specific resistances of sputtered metals are several times greater than the metals in bulk. Can any estimation of the specific resistance of sputtered bismuth be made from the fact that the temperature coefficient of resistance is 0·00111, whereas that of the metal in bulk is 0·0042—roughly four times as great? Supposing one accounts for the discrepancy on the supposition that only a portion of the total resistance is inherent in the metal itself, the remainder being imagined to exist as "bad contacts." In other words, within an aggregation of molecules the specific resistance of the metal applies, but between these aggregations the transference of electrons is much more difficult, requiring a greater potential gradient. If now the temperature be raised, the resistance due to passage within the molecule-aggregations will change, as it would in the metal in bulk; whereas that due to the bad contacts would remain practically constant. Under these circumstances, if the temperature coefficient of resistance be only one-fourth of its true value, the resistance due to the metal itself will be one-fourth of the whole, provided the bad contacts are regarded as all in series with the molecule aggregations. The effective specific resistance of the film will therefore be four times that of the metal in bulk. If this view be correct, the thickness of a typical metal film works out roughly at 0·0000007 cm., or about 170 times as thin as the supporting collodion film.†

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* cf. Speed of sensitive thermopiles prepared by Dr. Moll. Recent Papers in Phil. Mag. and Proc. Phys. Soc.

In that case the thickness of the metal film is negligible in comparison with that of the support, and the latter would be the vital factor in determining the speed. That there is something to be said for this view is the fact that the later type of collodion film, which is only one-half the standard thickness, gives, when sputtered, a bolometer which is twice as fast as the normal, although the thickness of the sputtered metal is the same in both cases.

It appears also that the thickness of the smoke blackening is negligible in comparison with that of the collodion film.

Film Thermo-piles.

Thermo-piles have been constructed on the same principle as the film bolometer described. In the first instance a single thermo-junction was used. The method of construction was exactly that used in the case of the bolometers, with the exception that one-half of the sputtered metallic strip would be of one metal, whilst the other half would be of another—a very slight overlap being allowed. Couples were made up in this way of the following pairs:—

Bi-Te Pt-Au Bi-Au Bi-Ag

These worked quite satisfactorily, but the necessarily low E.M.F., combined with, as a rule, a high resistance rendered them of little use. It is interesting to note that the E.M.F.s observed were not usually those associated with the same metals in bulk. As an example, a Bi-Te couple gave 1 200 microvolts, whilst under the same temperature difference Pt-Au gave 126 microvolts, a ratio of roughly ten, whereas in bulk the ratio would be very considerably greater. Again, the order of sputtering is important. Tellurium must be sputtered before bismuth; if the order be reversed, the deposit of the second metal is greatly retarded, or rendered abortive. The thermo-electric order should therefore be followed.

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* Phil. Mag., 6, 4 (1902).

† cf. Extremely thin translucent metal films prepared by C. Müller. Engineering, p. 199 (Feb. 12, 1926).

Attempts were made to construct instruments of ten or more couples. The procedure was to sputter one metal through a ladder-like mask, the apertures of which were of the same thickness as the rungs. The ladder would then be shifted half a step, so that the rungs would cover the sputtered portions and leave the interstices between open to sputtering by the second metal. Using bismuth and silver as the two metals, on account of their high thermo-E.M.F., low resistance and facility of sputtering, the method broke down owing to the apparent impossibility of sputtering bismuth through the tiny openings. The silver presented no difficulty, although it appears that all metals are retarded when sputtered through small apertures. The experiments were not proceeded with, as those relating to bolometers appeared much more promising. In any case, owing to their high resistance, it would appear that a thermo-pile of this type, if made, would have to be used in such a way as to take advantage of the bolometric effect at the same time.

Acknowledgment.

In conclusion, I must express my gratitude to Prof. Callendar for his invaluable surveillance throughout the research.

Mr. Colebrook has taken a constructive interest in the design of apparatus and greatly facilitated their completion, the apparatus being made with admirable precision by Mr. Hellary. To Mr. Bradfield I am indebted for his excellent photographs of the drawings.

DISCUSSION.

Dr. A. B. **Wood** said that, knowing by experience the difficulty of such work, he would like to congratulate the author on having produced such thin and tenacious films. In describing Fig. 14 of the Paper, the author had stated that during the sputtering process the bismuth film passes repeatedly through a cycle in which its conductance increases up to a point and then suddenly ceases. Might not this be due to oxidation of the bismuth under sputtering conditions? The greater part of the thickness of the author's films was contributed by the collodion. It might be of interest to note that very thin purely metallic films had been described by Carl Müller in a report of the Physikalische Reichsanstalt (February, 1926). These had been produced by the electro-deposition on copper of gold, silver, platinum, nickel or iron, the copper being subsequently dissolved away with the exception of a supporting ring. In this way gold membranes 0·02μ thick and nickel membranes 0·04μ thick and 6 cm. in diameter had been obtained. The speaker had produced, by rolling, metal films whose bolometer-effect reached half its final value in 1/10 sec.

Dr. **Ezer Griffiths**: I should like to congratulate the author on having worked out a difficult piece of technique. I think the bolometer might have applications in the study of radiation from gaseous explosions, etc. I should like further information on two points. (1) Whether the sputtered film is permanent in its physical properties. My experience with the sputtered films on the quartz suspension of an electrometer has not been encouraging. Perhaps this is an exceptionally difficult case as it is necessary to effect electrical contact between the film on an exceedingly thin quartz fibre and the hooks. (2) Whether the bolometer which the author employed as a standard of comparison was one which had the shortest period attainable with that form of construction.

Prof. F. L. **Hopwood** said that the Paper constituted a valuable guide to the technique of thin film production. Were the author's films permanent, or would their volatile constituents evaporate in a high vacuum? He had seen another method successfully applied by Dr. A. Dauvillier in the case of magnesium, which was deposited on a collodion film by evaporation.

As regards the sputtering of aluminium, he had seen this carried out at the Cavendish Laboratory before the war by using an atmosphere of an appropriate gas, probably chlorine, whereas the Paper stated that aluminium could not be sputtered.

The **Author**, in reply to the discussion, said that the resistance of the films, when these had been aged, might be regarded as permanent, since it increases exponentially to a limiting value within a short time after manufacture. As regards the possible volatilization of the collodion in a vacuum, this certainly did not occur in the course of a week. Aluminium could admittedly be sputtered under appropriate conditions, but these were not the conditions necessitated by the present work.

V.—A HYGROMETER EMPLOYING GLYCERINE.

By Ezer Griffiths, *D.Sc.*, *F.R.S.*, and J. H. Awbery, *B.A.*, *B.Sc.*

(From the National Physical Laboratory.)

Received July 24, 1926.

Abstract.

The variation of refractive index of glycerine solutions in equilibrium with air of various humidities has been studied; the time for equilibrium to be reached, when thin films of glycerine are used, has also been investigated.

It is concluded that this property may be used very conveniently in a hygrometer.

Introduction.

In those hygrometers which utilise the change in some property of a hygroscopic body as a measure of humidity, there is frequently not only considerable time lag, but the indications depend to some extent on the past history of the material. Evidently this difficulty would be much lessened, if not eliminated, by using a liquid rather than a solid, such as hair or goldbeater's skin. Among hygroscopic liquids, glycerine is one of the commonest and most reproducible, and suggests itself for this purpose.

This idea had previously occurred to Giraud, who in 1913 published a brief Paper on the subject in the "Journal de Physique."

He employed cigarette paper soaked in glycerine, which he laid on the face of the prism of an Abbé refractometer, and determined the variation of refractive index with humidity. No details of his experiments are given beyond the remark that he employed a wet and dry bulb hygrometer as the standard for the humidity measurements.

We have repeated the experiments of Giraud, determining the relation between refractive index and humidity, but as a practical method of hygrometry found it to be somewhat unsatisfactory; it was difficult to define the exact line of demarcation between the bright and dark portions of the field, an effect due to the diffusing effect of the cigarette paper. Trial showed that this could, in fact, be dispensed with, and the glycerine film applied direct to the glass prism. Fig. 1 gives a diagram of the apparatus, including the optical system.

A careful series of experiments was then made to ascertain the relation between the relative humidity of the surrounding atmosphere and the refractive index of a thin film of glycerine in equilibrium with it. Experiments were also made to find the time taken by a fresh film of glycerine to attain equilibrium with the atmosphere.

Calibration.

To calibrate the instrument and obtain a curve of refractive index against humidity, it is necessary to expose the glycerine film to an atmosphere of which the humidity is known and is under control, whilst when equilibrium has been attained the refractive index must be read, which requires the manipulation of the refractometer.

This difficulty was overcome by the arrangement of apparatus shown in Fig. 2. A vessel with a glass front was provided with means for altering the humidity of the air in it, and with a dew-point tube and thermometer.* In addition, a short length of large diameter brass tube was inserted through one side, and a rubber glove securely attached to this tube.

To carry out an experiment the refractometer with glycerine on the prism, freely exposed, was inserted in the case, and the front closed. The humidity being adjusted to the desired value, a sufficient time was allowed to elapse for the establishment of equilibrium, and then, by means of the rubber glove, the hand could be inserted in the case without opening it or otherwise disturbing the humidity, and the cover provided on the Abbé refractometer closed, thus preventing the access of air to the glycerine. The method will be understood easily by a reference to Fig. 2. The front of the case could now be opened, and the instrument withdrawn, and a reading taken.

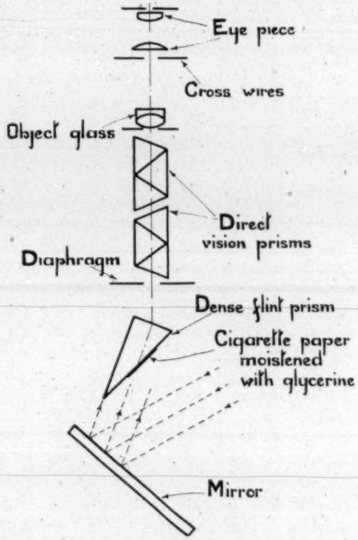

FIG. 1.

In this manner the calibration curve shown in Fig. 3 was obtained. It may be stated here that the results were the same, whether the glycerine was applied directly or whether it was applied to the prism by means of cigarette paper.

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* This vessel has already been described by one of us. (*See* Proc. Phys. Soc., 1922).

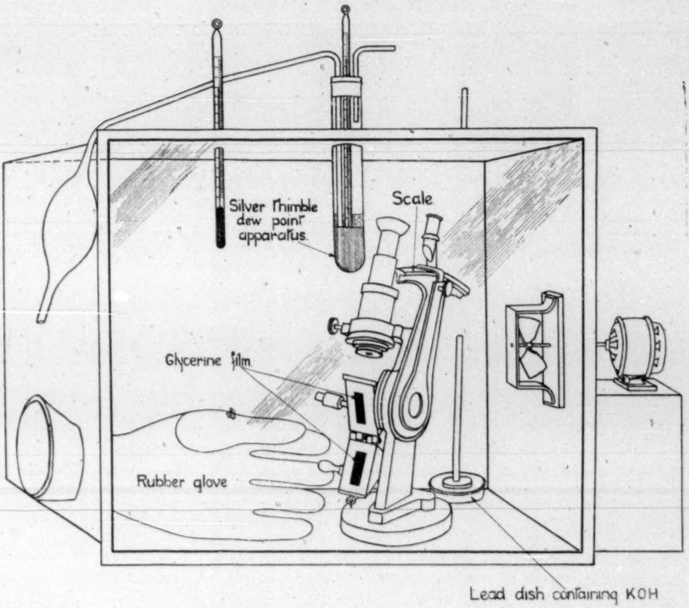
Silver thimble
dew point
apparatus
Scale
Glycerine film
Rubber glove
Lead dish containing KOH

FIG. 2.

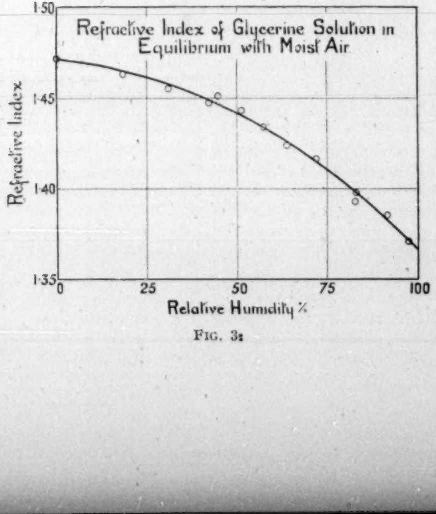
Refractive Index of Glycerine Solution in Equilibrium with Moist Air
Refractive Index
1·50
1·45
1·40
1·35
0
25
50
75
100
Relative Humidity %

FIG. 3.

TEMPERATURE COEFFICIENT.

To investigate the temperature coefficient, a complete calibration was carried out in the same manner in a room of which the temperature was maintained at approximately 30°C. It was noticed that at this temperature, and at high humidities, where the glycerine solution is weak, the viscosity was so low that the liquid was inclined to run off the prism and readings were a little difficult. Nevertheless, the curve (Fig. 4) shows the effect of temperature very clearly. For comparison, the smooth curve of Fig. 3 is there reproduced. An attempt was also made to obtain a calibration curve in a cold-storage chamber at -5°C. Unfortunately, the results were extremely erratic and no curve could be drawn through them. It is probable that freezing of the solution sometimes occurred at those concentrations for which the freezing point was above the temperature of the chamber.

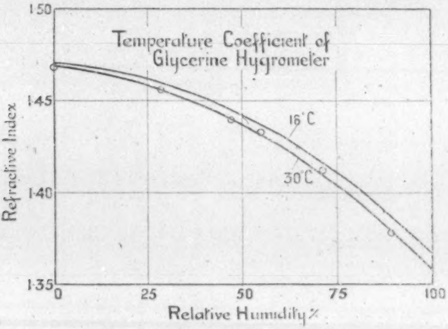

FIG. 4.

Relative Humidity.	Refractive Index.	
	At 16°C.	At 30°C.
25	1·462	1·460
30	1·459	1·456
35	1·455	1·452
40	1·451	1·448
45	1·447	1·443
50	1·442	1·438
55	1·437	1·432
60	1·431	1·426
65	1·425	1·420
70	1·418	1·412
75	1·410	1·404
80	1·402	1·396
85	1·394	1·387
90	1·385	1·378
95	1·376	1·369

TIME LAG OF THE INSTRUMENT.

To obtain data as to the time taken to reach equilibrium, the refractometer was placed in the calibration chamber referred to above, and allowed to become steady at a humidity of about 12 per cent. It was then transferred rapidly to the open room, where the humidity was 54, and readings taken at intervals. The curve is shown in Fig. 5, and it will be observed that half the change is completed in about 1·4 minutes ; however, the curve is not accurately exponential, and the time to cover 99 per cent. of the range is of the order of 15 minutes.

A second experiment was also made in which the instrument was transferred from a higher to a lower humidity in the same way, and the results for this are also shown in the same figure.

It will be noted that the time lag is about the same in the two cases, but it must be anticipated that the lag would vary with the exact thickness of glycerine actually placed on the prism surface.

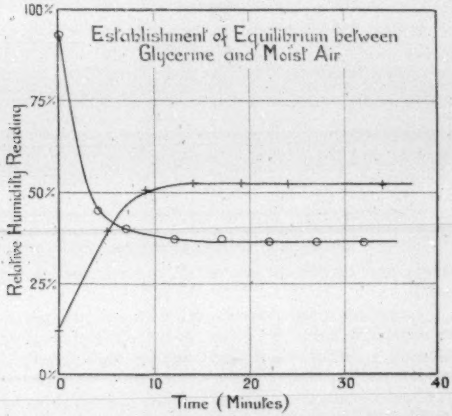

FIG. 5.

It may be of interest to calculate approximately the concentrations of glycerine concerned at the usual humidities. For this purpose two methods are available. The refractive indices of glycerine solutions for the *D* lines have been measured by Strohmer at 17·5°C. from concentrations of 50 to 100 gms. per 100 gms. of solution ; his curve could be extrapolated with some certainty, since the refractive index of water gives the end-point of the curve. Unfortunately, our calibration is for white light and his for sodium, but the difference is comparatively small, and no correction was applied. The concentrations for the refractive indices corresponding to humidities of 10, 20, 30 per cent. etc., were read off from this curve, and are shown in the table below.

A second method of estimating the concentrations makes use of the humidities directly, utilising the vapour pressure measurements of Dieterici (at 0°C.) for concentrations from 20 to 400 gms. glycerine per 100 gms. of water. This again is not directly comparable with our experiments, owing to the temperature, but, assuming his value for the vapour pressure of water at 0°C., the vapour pressures at other humidities were calculated and the corresponding concentrations read off.

Converting both sets of results to the same units, we obtain the following results :

Relative Humidity. %	Concentration (% by weight of glycerine).	
	From refractive index.	From vapour pressure.
0	100	100
10	98	...
20	94	...
30	90	...
40	85	85
50	80	79
60	73	71
70	66	62
80	58	51
90	48	33
100	32	0

The agreement is good, considering the nature of the data, up to 80 per cent. humidity. Above that, it is clear that there must be discrepancies, since by the nature of the instrument there is always glycerine present, even at the highest humidities. Thus, even at saturation, the refractive index does not fall off to the value for pure water, whereas the curve of concentration against refractive index does do so.

PRACTICAL CONSIDERATIONS.

The advantages and disadvantages of the instrument may be briefly summarised as follows :—

(1) Although it requires a definite setting to be made before the humidity can be obtained, the refractive index scale could be engraved directly in humidities, so that the instrument is simpler than a wet-and-dry bulb hygrometer, or than the dew-point instrument, in that there are no calculations necessary.

(2) Its lag is about 15 minutes. In many cases this would not be excessive.

(3) It is portable and reasonably robust.

ACKNOWLEDGMENT.

The above experiments were carried out on behalf of the Engineering Committee of the Food Investigation Board who kindly sanctioned their publication.

We are indebted to Mr. F. W. Morris, Junior Observer, for assistance in taking the observations.

DISCUSSION.

Prof. C. V. Boys (communicated) : I have read Dr. Griffiths's Paper on a glycerine hygrometer with especial interest, as I have long intended to try a glycerine hygrometer consisting of a piece of paper wetted with glycerine and suspended from one arm of a special light balance most easily made with wheat-straw, needles and sealing wax, so as to indicate perhaps 10,000ths of a grain, but with sufficient stability to remain within its arc of swing for such variations of relative humidity as, e.g., 50 per cent. to 80 per cent. or 90 per cent. With such stability the delicacy of weighing might be less but still ample. Such a hygrometer could be made to go in a space not more than 2 in. long and 2 in. high, and a lag for half an hour or so would for many purposes be quite immaterial.

Each instrument would, I fear, need to be calibrated separately, but the information supplied by Dr. Griffiths indicates that it might well give consistent results, and that the time of arrival would not be excessive.

VI.—EFFECT OF WORKING ON THE PHYSICAL PROPERTIES OF TUNGSTEN.

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(Communication from the Staff of the Research Laboratories of the General Electric Company, Ltd., Wembley.)

Received August 22, 1926.

Abstract.

Changes in the density and resistivity of tungsten during swaging and drawing have been investigated.

The densities of pure and thoriated tungsten (0·63 per cent. thoria) from the sintered bar down to wires 0·02 mms. in diameter are given. The experimental advantages of the use of benzene as the displaced liquid are mentioned.

Measurements of the densities of specimens cleaned in successive stages by etching reveal the presence of low density surface layers.

Variations of heat treatment likely to occur in practice are shown to have no appreciable effect upon the density of worked tungsten.

The density rises rapidly during swaging to a maximum value within 0·5 per cent. of the density of the perfect tungsten crystal. Further working produces a steady fall in density, which becomes more marked in the finest sizes.

Measurements have been made of the resistivity of the same specimens.

The resistivity falls rapidly in the early stages of working and reaches a minimum when the density is a maximum. It then increases at a uniform rate, which is approximately an exponential function of the diameter

I. Density and Resistivity.

THE structure developed in a drawn tungsten wire when heated above its recrystallisation temperature depends upon the mechanical treatment to which the metal has previously been subjected. In particular, it has been found that the percentage reduction in cross-sectional area and the temperature at which the metal has been worked have a marked influence on the structure of the recrystallised wire. The changes which take place in the physical properties of the wire during working have been measured in order to throw light on the mechanism of recrystallisation. In the present Paper the results of measurements of density and resistivity are given.

Part I.—Density.

The density of the perfect tungsten crystal can be calculated from the lattice spacing as determined by X-ray measurements and the atomic weight. The most accurate determination is that of Davey,[*] who found a value of $3{\cdot}155 \pm 0{\cdot}001$ Å. for the lattice constant, using a sample containing 99·9995 per cent. tungsten. Taking the atomic weight as 184·0, this gives a value for the density of $19{\cdot}32 \pm 0{\cdot}02$ qms./cm^3.

The changes in density which take place during the swaging and drawing of tungsten rods have been investigated by a number of workers, but their results do not show satisfactory agreement. The values found by Lorentz (Table I) show a progressive fall in density with decreasing diameter, whilst Schriever[†] found an increase in density (Table II), the value for the finest wires being considerably higher than that calculated from X-ray measurements. The density does not, however, appear to change in a regular manner.

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[*] Davey, Phys. Rev., 25, 753 (1925).

[†] Schriever, Iowa Acad. of Science, 24, 235 (1917).

TABLE I.—*Density of Worked Tungsten (Lorentz).*

Material.	Diameter (mms.).	Density.
Swaged rod	1·27	19·1
Drawn wire	0·51	18·8
,, ,, ...	0·254	18·5
,, ,, ...	0·127	17·9

TABLE II.—*Density of Worked Tungsten (Schriever).*

Material.	Diameter (mms.).	Density.
Drawn wire	0·227	19·13
,, ,, ...	0·176	19·03
,, ,, ...	0·126	19·48
,, ,, ...	0·078	19·35
,, ,, ...	0·024	20·26

Geiss and Van Liempt* have measured the density over a wide range of wire sizes, and their results, given in Table III, show a steady increase in density down to the finest sizes, for which the values closely approach the theoretical value of 19·32.

TABLE III.—*Density of Worked Tungsten (Geiss and V. Liempt).*

Material.	Diameter (mms.).	Density.
Sintered	...	10—12
Formed	15·0	16·88
Hammered	14·8	17·62
,, ...	11·8	18·28
,, ...	6·2	19·16
,, ...	5·0	19·19
,, ...	3·85	19·22
Drawn	0·150	19·23
,, ...	0·110	19·22
,, ...	0·020	19·41 ± 0·04
,, ...	0·0125	19·28 ± 0·06

In view of the fundamental differences in the results quoted, the authors have repeated the measurements both for pure tungsten and for tungsten containing 0·63 per cent. thoria, and have studied the effect of some variations in the mechanical and thermal treatment of the wires.

Materials.

Tungsten rods 5·6 × 5·6 × 17·5 mms. were prepared from metal powder, which was pressed and formed in the usual manner, of which numerous accounts have been published.†

The following particulars give the essential difference between the two batches of metal used in the investigation.

Metal A.—Pure Tungsten.

The sintered bars contained more than 99·99 per cent. tungsten, and had a fine grained structure.

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[*] Geiss and Van Liempt, Annalen der Physik., 77, 105 (1925).

[†] Dudding and Smithells, B.E.A.M.A., 13, 221 (1923).

Metal B.—Thoriated Tungsten.

The sintered bars contained 0·63 per cent. ThO_2, other impurities being less than 0·01 per cent. The bars had a very coarse structure.

MECHANICAL WORK.

After forming at about 3100°C., the bars were sufficiently strong to be worked mechanically. The preliminary reduction was carried out in swaging machines in which the bar was hammered to a round section and increased in length. The temperature, which was lowered progressively, as shown in Table IV, was throughout below the temperature at which any visible recrystallisation takes place in the metal.

TABLE IV.

Wire Diameter. Mms.	Swaging Temperature. Degrees C.
6·0	1,300
4·0	1,200
3·0	1,100
1·5	1,000
1·0	900
0·8	750

After the metal had been reduced to a diameter of 0·8 mm., further reduction was carried out by drawing through diamond dies. The reduction in diameter at each step was between 8 and 10 per cent. The wire was heated before entering the die by passing an electric current through the wire, the temperature being lowered progressively from 800° to 500°C. as the wire diameter decreased.

METHOD OF MEASUREMENT.

The density of wires larger than 1 mm. in diameter may be determined by weighing in air, and in distilled water, using a sensitive chemical balance, the samples weighing from 10 to 50 grams.

Considerable difficulty was found in obtaining an equal accuracy with finer wires. A torsion balance, which would have enabled rapid readings to be taken, was not found to give sufficient accuracy, and eventually a chemical balance weighing to ±0·0001 gm. was used. It was hoped to obtain increased accuracy by using as the displaced liquid, methylene iodide, which has a density of 3·34; but surface tension effects, where the supporting wire passed through the surface, more than counterbalanced the advantages of increased upthrust. Water was found to be as bad in this respect, even when a fine and well-cleaned platinum wire suspension was used. It was not the magnitude of the surface tension which caused trouble, but its irregularity of action, by which it seemed to cling to the wire cutting the surface, and to hold it even against several milligrammes excess weight on the opposite side of the balance.

Alcohol and benzene were both found to give freedom from this source of difficulty and error, the suspension wire passing smoothly through the surface and the balance being as sensitive to added weights as if the body were being weighed in air.

In order to obtain a sufficiently accurate value of the density of the finer wires a considerable length of wire had to be used. This necessitated winding it in a loose coil. It was found extremely difficult to free such a coil from air bubbles when immersed in water, even by boiling under reduced pressure. Distilled water also has an appreciable oxidising action on fine tungsten wires. The use of benzene avoids both these difficulties. By boiling the coil of wire for a few seconds in benzene in the vessel in which it is to be weighed all sign of air bubbles is removed ; and the benzene has no chemical action on the tungsten, and cools conveniently rapidly.

Benzene was therefore used in conjunction with a sensitive chemical balance to determine the density of the finer wires.

The benzene was re-distilled from " pure recrystallised " benzene, and a constant boiling fraction taken. Its density was determined by the displacement method, using two hollow glass " sinkers " ballasted with lead, which were weighed in the benzene and in distilled water. After correcting for the expansion of the glass sinkers, this gave the density as 0·8873 at 12°C. in air. The accepted relation* between density and temperature for benzene was used in the subsequent work. The fine wires were wound into coils on a former of such design that the coils could be secured and removed from it intact. This former consisted of a hollow steel cylinder divided into three parts by three parallel axial cuts, fitted on to a steel core. The whole was held together by two brass clips, with nuts and bolts to tighten them. Three to five short pieces of thin tungsten wire were held in place by and between the clips, and the wire under test was wound on top of them. The ends of the short pieces were then pulled out of the loosened clips, and twisted together, as also were the two ends of the coil. On removal of the steel core, one of the three sections of the outer split cylinder was pushed out of place and the coil easily removed intact. Tightening the binding-wires then gave a coil of fine wire about one inch in diameter, which was convenient to handle for density measurements.

This coil was boiled for a few seconds in a separate lot of benzene before being boiled in the benzene in which it was finally weighed.

The finer wires were suspended by a tungsten wire 0·05 mm. in diameter, and the larger ones by a nickel wire 0·13 mm. diameter. Correction for the weight of the suspension wire, and the surface tension effects upon it, was made by weighing it alone partly immersed in the liquid as when in use.

After being weighed suspended in benzene, the coil was dried by heating on a watch-glass over a steam bath, allowed to cool and re-weighed.

EXPERIMENTAL RESULTS.

Tungsten wire must be lubricated with graphite during drawing, and the finished wire has a shiny black surface. Chemical analysis of several samples of wire 0·05 mms. in diameter showed that the weight of carbon on the wire was 0·05 per cent. of the weight of the wire. This corresponds to a layer $0{\cdot}05\mu$ thick, which in the case of wire 0·05 mm. in diameter would only affect the density by 0·35 per cent. The first measurements were, therefore, made on the wires as drawn, and without any preliminary cleaning beyond boiling in benzene.

The results of these measurements for Metals A and B are summarised in Table V and shown graphically in Fig. 1.

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* Landolt Börnstein, Tabellen, 1025 (1912).

The accuracy of the determination varies according to the weight of material available for each specimen, and this has been indicated in the table.

TABLE V.—*Density of Tungsten Wires as Drawn (Black Wire).*

Material.	Diameter. Mms.	Density.*			
		Bar No.	Pure Metal A.	Bar No.	Thoriated Metal B.
Formed bar ...	(6.02) – (6·24)	107	17·29	35	17·33
,, ...	,, ,,	...	...	67	17·21
Swaged rod ...	5·1	112	18·34	31	18·41
,, ...	4·0	...	...	30	18·94
,, ...	2·95	102	19·10	32	19·08
,, ...	2·03	104	19·10	33	19·07
,, ...	1·4	111	$19{\cdot}1_2$	37	$19{\cdot}0_4$
,, ...	1·05	111	$19{\cdot}0_9$	76	$19{\cdot}0_0$
Drawn wire ...	0·61	102	19·00	37	...
,, ...	0·50	...	...	33	$18{\cdot}8_5$
,, ...	0·295	110	19·05	37	$18{\cdot}8_1$
,, ...	0·150	108	$19{\cdot}0_0$	33	18·79
,, ...	0·140	...	...	37	18·75
,, ...	0·117	...	...	33	$18{\cdot}7_1$
,, ...	0·093	...	...	37	$18{\cdot}7_1$
,, ...	0·072	108	$18{\cdot}8_0$	33	$18{\cdot}6_2$
,, ...	0·058	...	...	37	18·7
,, ...	0·047	108	$18{\cdot}7_5$	33	18·4
,, ...	0·030	...	...	37	18·4
,, ...	0·030	...	...	33	18·3
,, ...	0·021	...	...	33	18·2 ± 0·2
,, ...	0·021	...	...	37	17·8 ± 0·2

The general shape of the density-diameter curve is the same for both metals. A rapid increase in density occurs in the early stages, when the porous bar is compacted by hammering, and after reaching a maximum a steady fall in density occurs,

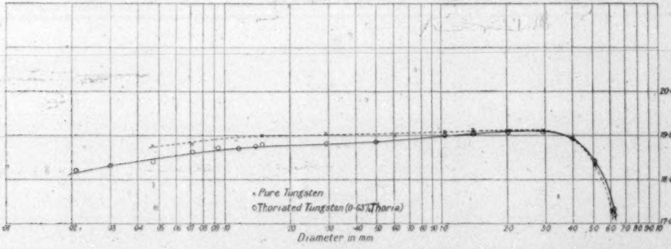

FIG. 1.—DENSITY OF PURE TUNGSTEN AND THORIATED TUNGSTEN WIRE (UNCLEANED).

which in the case of the thoriated metal (Metal B) becomes very marked in the finest sizes. Below 0·03 mm. diameter these wires became difficult to draw, and the very low values may be attributed to internal splits.

Taking the density of the thoria particles as 10·0, the theoretical maximum density for Metal B is 19·22, whilst that for pure tungsten is 19·32. It is, therefore, clear that neither metal attained its full theoretical density. It seemed possible that this, and also the subsequent fall in density, might be due to foreign material on the wire surface, since this would produce a greater effect on the smaller wires. The effect of removing the surface layers from the specimens was therefore investigated.

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* The accuracy is indicated by the significant figure.

EFFECT OF CLEANING.

A "black" tungsten wire can readily be cleaned by raising it to incandescence in dry hydrogen. To determine the total weight of volatile material removed in this way, about 60 metres of fine wire of Metal A (diameter 0·045 mm.), cut into short lengths, were flashed in hydrogen. The total loss in weight represented 0·51 per cent. of the weight of the wire.

Repeating the operation with the same wire produced no further change in weight, showing that the original loss was not due to oxidation. Since the carbon represents only 0·05 per cent. by weight, the bulk of the volatile material most probably consists of oxides of tungsten. Tungsten is readily oxidised at the temperature at which it is swaged, and a certain amount of oxidation occurs before the graphite lubricant, which protects the finer wires, is used.

The black wire can be cleaned by treatment with reagents which dissolve tungsten oxides, and so loosen the graphite. Hydrogen peroxide and ammonia, which reacts slowly, was found to give easy control; but for more vigorous etching a hot solution of sodium peroxide, following by boiling in ammonia, was used. The specimens of Metal A previously used were thoroughly cleaned in this way, and their densities re-determined. The densities of the cleaned wires were found to be higher than before, the effect being most marked with the finer wires. The results are given in Table VI and Fig. 2, where the values for the uncleaned wires are reproduced for comparison. The maximum density, which is now only 0·5 per cent. below the theoretical value, still occurs at the same diameter, but the fall in density on further working is much less marked.

Similar determinations were made on a few samples of Metal B, and the results, which show changes of the same order, are included in Table VI.

TABLE VI.—*Comparison of Densities of Black and Cleaned Wires.*

Material.	Diameter. Mms.	Density.	
		Black.	Cleaned.
Metal A, formed bar	(6·02)	17·29	17·29
,, swaged	5·1	18·34	...
,, ,, ...	3·0	19·10	19·20
,, ,, ...	2·03	19·10	19·22
,, ,, ...	1·4	$19{\cdot}1_2$	$19{\cdot}2_4$
,, ,, ...	1·05	$19{\cdot}0_9$	$19{\cdot}2_1$
,, drawn	0·61	19·00	...
,, ,, ...	0·295	19·05	19·21
,, ,, ...	0·150	$19{\cdot}0_0$	$19{\cdot}2_4$
,, ,, ...	0·072	$18{\cdot}8_0$	...
,, ,, ...	0·048	$18{\cdot}7_5$	$19{\cdot}1_9$
,, ,, ...	0·036	...	$19{\cdot}0_7$
Metal B, swaged	4·7	18·82	18·86
,, ,, ...	2·03	18·95	19·08
,, drawn	0·048	18·54	18·92

These large effects indicate that a comparatively thick layer of low density material exists on the surface of the wire as drawn, and an attempt was made to determine its nature.

It was hoped that by etching off successive small layers from the surface, and determining the change in density, some information about the thickness and density of the surface material might be obtained. This, however, involved an accuracy of about 0·1 per cent., which was found extremely difficult to obtain. The chief errors arose from temperature variations in the benzene due to evaporation, and adsorption of moisture on the wire surface. To obtain sufficiently accurate weighings with wire 0·05 mm. in diameter a coil of 800 metres was necessary, and this has a surface of over 1,000 sq. cms.

A number of determinations were finally made, which, whilst not obtaining such a high accuracy, indicate the nature of the surface layers. These results are summarised in Table VII.

TABLE VII.—*Effect of Progressive Cleaning on the Density of Tungsten Wire.*

Wire.	Diameter. Mms.	Percentage weight removed by etching (Total).	Density of remaining wire.	Average depth (below original surface) of each layer removed. Mms.	(Mean) Density of each layer removed.
*Thoriated ...	2·03	0	$18{\cdot}94_5$		
				0·0030	10
		0·60	19·06		
				0·0090	17
		1·65	$19{\cdot}08_5$		
				0·0132	20
		2·16	19·08		
Metal A ... ... Pure Tungsten	1·4	0·00	$19{\cdot}12_3$		
				0·0016	8·0
		0·38	$19{\cdot}22_3$		
				0·0041	11·5
		0·86	$19{\cdot}23_4$		
				0·0060	13·5
		1·38	$19{\cdot}23_8$		
Metal B ... ... (Thoriated)	0·048	0·00	$18{\cdot}5_4$		
				0·00023	10·0
		$1{\cdot}8_6$	18·80		
				0·00055	15·0
		$3{\cdot}8_7$	18·92		
				0·00101	19·5
		$9{\cdot}3_1$	18·89		
				0·00164	19·0
		14·5	18·91		

Whilst the accuracy obtained is not high, these results show that the outer layers of the wire as drawn have a low density, which increases until the density of the tungsten composing the main part of the wire is reached.

In the case of wire 0·048 mm. in diameter these low density layers are sufficient to reduce the mean density by 2 per cent.

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* This was not of the same batch of metal as B.

In each case the first layer removed evidently contained material, probably oxides of tungsten, of a much higher density than graphite. Still higher densities of 12 and 15, too high for tungsten oxides, suggest the presence of some "loose" tungsten mixed up with the material.

EFFECT OF HEAT TREATMENT.

Considerable modifications in the normal methods of working tungsten are possible, which might account for the differences in density found by different observers. The effect of recrystallising during working was investigated in the case of Metal B. A number of bars were swaged to a diameter of 5·1 mms., and then heated to 3,000°C. in dry hydrogen by the passage of an electric current. It was found that a period of two minutes was sufficient to cause complete recrystallisation. The bars were then worked into wire in the usual way, and the density determined at various stages in drawing. The measurements were made on the wires as drawn, and the results included in Fig. 1. These points lie on the normal curve, and show that no appreciable effect on the density is produced by the recrystallisation.

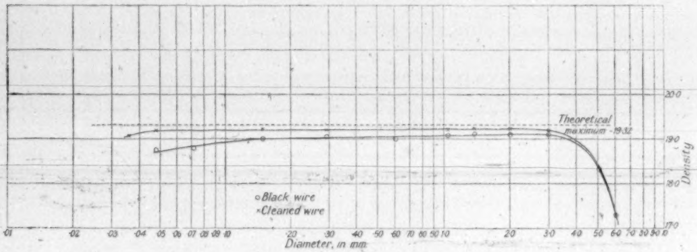

FIG. 2.—DENSITY OF DRAWN TUNGSTEN WIRE BEFORE AND AFTER CLEANING

The effect of variations in the method of heating the wire before it enters the die was investigated for the same metal. Two rods were drawn to a diameter of 0·150 mm. in the usual way; but for the subsequent dies one was heated by the passage of an electric current, and the other by means of a gas burner. Although the maximum temperature was the same in both cases, this could be maintained over a much larger length with gas heating, which is equivalent to heating the wire for a longer time before it enters the die.

Density measurements on both wires showed differences which were within the limits of accuracy of the determinations, and indicate that the general shape of the density curve is unaffected by these changes in heat treatment.

DISCUSSION OF RESULTS.

It appears from these results that when pressed and sintered bars of pure tungsten are reduced by swaging the first effect is to compact the metal and eliminate the voids, the density rapidly reaching a maximum, about 0·5 per cent. less than the theoretical value. Further reduction in diameter by swaging and drawing produces a slight drop in density over a long period, but this increases more rapidly when the diameter is below 0·05 mm.

Tungsten containing thoria behaves in a similar manner, the density throughout being lower on account of the lower density of the thoria. The fall in density as the wire becomes finer is, however, more marked with thoriated tungsten, particularly in the very fine sizes, where the wire is liable to split and break in drawing.

Variations in heat treatment during drawing, covering the extremes likely to occur in practice, produce no appreciable effect on the density of the drawn wire.

These results are not in agreement with those of Geiss and V. Liempt, who found a steady increase in density down to the finest sizes. It is possible that the methods used by them produced wires with different characteristics, although this seems less probable in view of the authors' inability to influence the density by wide variations in the method of drawing.

The fall in density appears to take place in two stages. The slight fall between 3·0 and 0·05 mms. suggests that the metal is deforming by slip on crystal planes, involving break up of the original crystals. The much more rapid decrease in density below 0·05 mm. diameter is undoubtedly due to the incipient formation of longitudinal splits, which eventually become visible under the microscope. Such splits are much more prevalent in thoriated wire, in which the fall in density is most marked.

PART II.—RESISTIVITY.

The resistivity of tungsten wire in the annealed condition has been determined by Worthing,* who obtained the value $5{\cdot}47 \times 10^{-6}$ ohm. cms. at 20°C. Tsukamoto† has measured the resistivity of swaged and drawn tungsten at various stages during the manufacture of fine wire. After an initial fall corresponding to the increase in density which takes place during the first swaging processes, he found an increase in resistivity which was approximately an exponential function of the diameter. The authors have measured the resistivity of the specimens of Metal B used in the density determinations described in Part I of this Paper.

METHOD OF MEASUREMENT.

The method adopted was to measure the potential difference between two points on the specimen, and between the terminals of a standard resistance in series with the specimen and carrying the same current. A Crompton instrument arranged as a millivolt potentiometer was used.

The finer wires were stretched lightly between two terminals on a wooden base and across two razor edges, which acted as potential leads and also defined the length of the specimen. The ends of the thicker rods were soldered to brass terminals, which were connected to the current leads. Fine nickel wires 0·1 mm. in diameter were spot-welded to the specimen near either end as potential leads, the distance between them being accurately determined with a cathetometer.

The temperature was measured with a thermometer placed close to the specimen. The currents used were insufficient to produce any measurable change in the temperature, and no change in the potential difference-current ratio was found when the current was increased 900 per cent. The determinations were made on the drawn wires without preliminary cleaning. The diameter was calculated from the weight of a measured length of wire, using the appropriate value of the density found in the earlier experiments.

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* Worthing, Phys. Rev., Vol. 18, p. 144 (1921).

† Tsukamoto, Nagaoka Anniversary Vol., p. 409 (1925).

Finally, the resistivity at 20°C. was calculated, using the value for the temperature coefficient found by Worthing—namely, $0{\cdot}004\times10^{-6}$ ohm. cms. per degree C. Actually the temperature only varied from 20°C. by one or two degrees during the measurements. The resistivity curve for cleaned wires would have a closely similar form, the values for the finest wires being only 3 per cent. less even if the surface layers were non-conducting. The results obtained with Metal B are given in Table VIII and shown graphically in Fig. 3.

TABLE VIII.—*Resistivity of Worked Tungsten.*

Diameter (mms.).	Resistivity (20° C.) ohm. cms.
(6·24)	$6{\cdot}13\times10^{-6}$
5·1	5·58
4·0	5·47
2·95	5·48
2·03	5·45
1·40	5·47
0·80	5·73
0·69	5·70
0·375	5·92
0·235	6·16
0·150	6·24
0·131	6·52
0·117	6·57
0·100	6·76
0·086	6·48
0·072	6·80
0·060	6·62
0·052	6·99
0·047	6·92

EXPERIMENTAL RESULTS.

As the ingot is worked the resistivity of the metal falls, and reaches a minimum value of $5{\cdot}4\times10^{-6}$ at a diameter of about 2·5 mms. Further working produces a steady increase in the resistivity, the value at 0·05 mm. being about $7{\cdot}0\times10^{-6}$. When the resistivity is plotted against the logarithm of the diameter, as in Fig. 3, the curve is nearly a straight line over the range 1·0 to 0·05 mms.

The initial fall in resistivity follows the increase in density which takes place during the early steps of swaging, and the minimum resistivity occurs at the same point as the maximum density.

The subsequent increase in resistivity is much greater than can be accounted for by the slight fall in density in wires less than 1·0 mm. in diameter, and must be the direct result of the deformation which the metal has undergone. The points representing individual observations on the finer wires lie considerably off the best smooth curve, and greatly exceed the errors of measurement. These variations appear to have a real significance, and must be attributed to the method of working the metal. There are a number of factors involved in drawing fine wires which might account for such variations of which the most important are probably :

Condition of the die.

Temperature of the wire.

Percentage reduction at each draught.

Of these variables, the temperature of the wire can most easily be controlled.

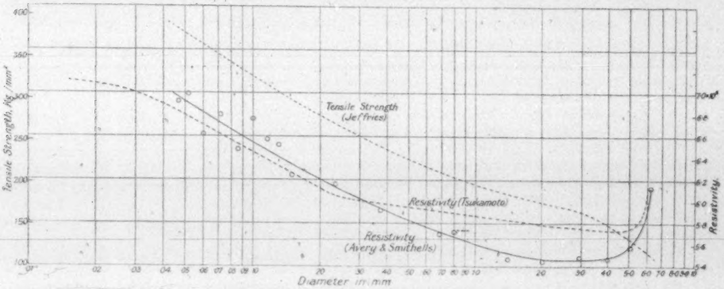

Fig. 3.—Resistivity and Tensile Strength of Drawn Tungsten Wire (0·63% Thoria).

Effect of Drawing Temperature.

Specimens were available which had been drawn using gas and electric heating as already described. The densities showed no appreciable differences. The resistivities are given in Table IX.

Table IX.

Diameter (mm.).	Resistivity.	
	Electric Heating.	Gas Heating.
0·131	$6{\cdot}34 \times 10^{-6}$	$6{\cdot}13 \times 10^{-6}$
0·114	6·28	6·12
0·100	6·32	6·19
0·086	6·43	6·37
0·072	6·60	6·38
0·060	6·79	...
0·051	6·85	6·61
0·048	6·86	6·62

These results show that the electrically heated wires have a considerably higher resistivity than those which had a more prolonged annealing due to the gas heating. The difference is larger than the variations referred to in the previous section, and supports the view that they are due to such uncontrolled variations as might be expected in the wire drawing process.

CONCLUSIONS.

The general shape of the resistivity curve is similar to that found by Tsukamoto for wire containing 1 per cent. thoria.

The increase in resistivity when the metal has suffered a large reduction in cross section cannot be attributed to the change in density, but must be the result of changes in the structure of the metal. It is interesting to note that the curve connecting tensile strength and diameter is of the same shape as the resistivity curve for wires less than 2·0 mms. in diameter. Jeffries* values for the tensile strength of swaged and drawn tungsten wires are plotted in Fig. 3, and the comparison indicates that the changes in both tensile strength and resistivity depend upon the same structural changes in the metal.

DISCUSSION.

Dr. H. BORNS: Mr. Avery has said that water adhered to some spots on the tungsten wires. Had these spots possibly been touched with the fingers?

Dr. E. H. RAYNER said that the drawing of tungsten is a very wonderful process, considering that a 6 in. bar of the metal is pulled out to a length of a mile or two by 50 or 100 successive drawings. At each drawing it is passed through a bath of graphite, and he would like to know what evidence had been found as to the extent to which graphite fails to penetrate into the substance of the metal. The claim that it remains almost entirely on the surface was a surprising one.

Sir RICHARD PAGET enquired whether a relation had been traced between the resistivity and tensile strength of the wires?

Dr. SMITHELLS, in reply to the discussion, said that wires had been tested for carbon after drawing, by chemical analysis, by resistance measurements, by the spectroscope, and by heating to incandescence. It had been found extremely difficult to retain more than 0·3 per cent. of carbon in worked wire, even when carbon was deliberately added to the metal. Analysis of an ordinary wire showed about 0·05 per cent., and this was mainly in the surface layers. If carbon were present as carbide, the fact would at once be apparent from the effect of carbide on the resistivity and ductility of the metal. The tensile strength of the specimens had not been tested, but the upper curve in Fig. 3, which embodies the results obtained by Jeffries for the tensile strength of other specimens, was markedly parallel with the resistivity curves for small diameters. In reply to Dr. Borns' question, the wires were not touched by hand after cleaning. It was not the case that water adhered to some spots on the tungsten wire, but that it clung to certain points on the suspension wire even after every precaution had been taken to clean it.

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* *See* Forsythe and Worthing, Astrophysical J., Vol. 61, p. 148 (1925).

DEMONSTRATION OF SELENIUM CELLS.

By Prof. **Hans Thirring**, Professor of Theoretical Physics in the University of Vienna.

THE sensitivity of a selenium cell depends a great deal upon the thinness of the selenium coating, since only the surface layer is influenced by the light. Experience has shown that with the condenser type of cells the thickness of the selenium coating can be reduced down to any desired value by the capillary action of suitably chosen electrodes. The condenser type is convenient also for making selenium cells of very small light-sensitive area, without unduly increasing the resistance. The following table indicates the approximate resistances of examples of the standard types of cells shown, which have been made according to the author's patents by Messrs. J. Malovich & Co., in Vienna.

Cell No.	Light-Sensitive Area.	Current in microamperes at 12 volts.	
		Dark.	Illuminated 100 metre candles.
195	Circular, 1 mm. diam....	0·2	2·5
753	Rectangular, 0·14 mm. × 10 mm.	0·1	3·8
925	Rectangular, 1 mm. × 10 mm.	0·1	6·0
1,301	Circular, 5 mm. diam....	3·0	24·0
1,312	Rectangular, 7 mm. × 35 mm.	72·0	432·0

The advantage of a small light sensitive surface lies in the fact that a focussing lens which concentrates the light on the cells can be used. Since the ratio between dark resistance and light resistance depends on the intensity of illumination (energy per unit area), a small cell can be used more efficiently than a large one with equal electrical properties.

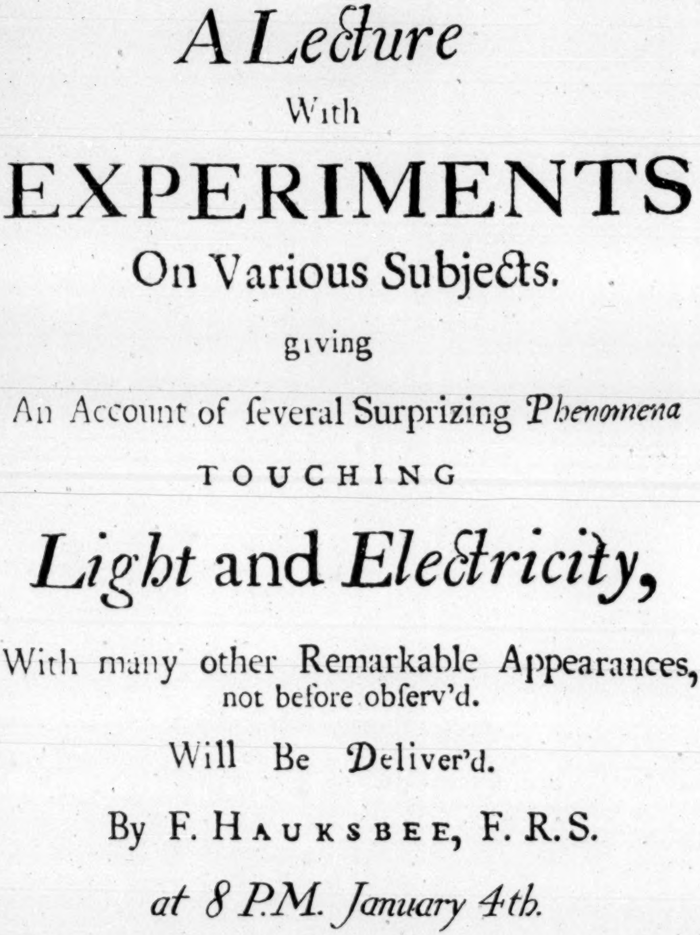
It does not seem to be possible to remove all the deficiencies which are naturally inherent in selenium cells. Among these deficiencies the following may be mentioned:—

(*a*) The resistance is not a linear function of the intensity of illumination;
(*b*) It has a temperature coefficient;
(*c*) The reaction to light is not instantaneous.

Selenium cells should, therefore, be used in an arrangement which compensates as far as possible these deficiencies for the given purpose. A very efficient device is the use of the cells with an intermittent illumination of acoustical frequency, the cell being coupled with an amplifier. It is important in this case that there should be no spontaneous rapid fluctuations of the resistance, for these give rise to grating noises. In this respect also the cells shown are superior to those of the grid type, because of the better electrical contact at the electrode.

One of the experiments shown illustrated an arrangement of this kind. The filament of a small 6-volt lamp was focussed upon a selenium cell with a sensitive area of 1 mm.^{2}, by means of two lenses 4 cm. in diameter, over the length of the lecture table. The light was interrupted by means of a phonic wheel rotating close to the lamp. The fluctuating current in the selenium cell was amplified and made audible throughout the theatre by means of a loud-speaker.

Another experiment illustrated the advantages of balancing two selenium cells in a Wheatstone bridge arrangement. By exposing the cells alternatively to illumination the balance was upset in opposite directions.



A Lecture

With

EXPERIMENTS

On Various Subjects.

giving

An Account of ſeveral Surprizing *Phenomena*

TOUCHING

Light and *Electricity*,

With many other Remarkable Appearances,
not before obſerv'd.

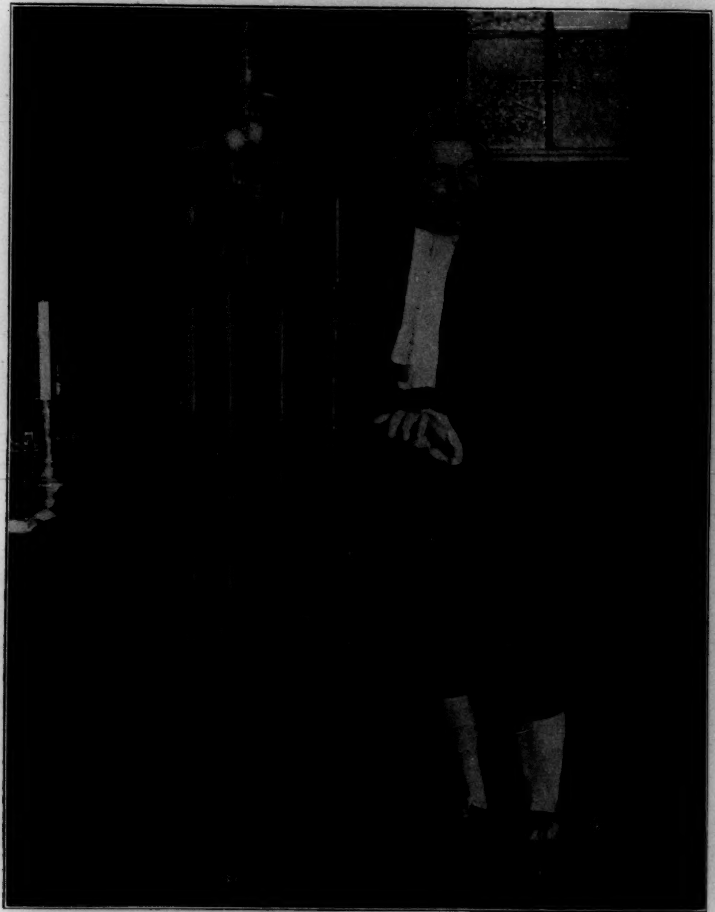
Will Be Deliver'd.

By F. HAUKSBEE, F. R. S.

at 8 P.M. January 4th.

Facsimile of notice of Professor Andrade's lecture, as it appeared in the Catalogue of the Exhibition of the Physical and Optical Societies, January, 1927.

NOTE: A verbatim report of the lecture appears in the Journal of Scientific Instruments.



Professor E. N. da C. Andrade, impersonating Francis Hauksbee, demonstrating the use of the Air Pump, made by Hauksbee about 1705. This was kindly lent by the Royal Society for Professor Andrade's discourse at the Exhibition of the Physical and Optical Societies, January, 1927, and put in working order by Mr. R. W. Paul.

VII.—ELECTRO-ENDOSMOSIS AND ELECTROLYTIC WATER TRANSPORT.

By H. C. Hepburn, *B.Sc.*, Research Student, Birkbeck College.

Received September 25, 1926.

(*Communicated by Prof.* A. Griffiths, *D.Sc.*)

Abstract.

This Paper gives the results of determinations of liquid transport produced by passing an electric current through aqueous solutions of copper sulphate divided perpendicular to the flow of electricity by a diaphragm of powdered glass. The probable factors that determine the liquid transport are investigated, over a wide range of concentrations, by an examination of the dependence of the flow at constant applied voltage and of the electric charge of the diaphragm on the electrolyte concentration, and also by a study of the relation between the flow per faraday and the dilution of the electrolyte.

Introduction.

The nature of the liquid transport which occurs when an electric current is passed through an electrolyte, divided by means of a diaphragm perpendicular to the flow of electricity, has been investigated by Remy(1) over a wide range of concentrations. Remy has shown, by means of a comparative study of data obtained with several electrolytes and with diaphragms of various materials, that two factors fall to be considered: (*a*) electro-endosmosis—i.e., the surface phenomenon which is explained on the classical theory of Helmholtz(2) in terms of the electrical double layer; and (*b*) electrolytic water transport—i.e., transport of water produced by the motion of the ions of the electrolyte, the latter factor becoming important at the higher concentrations.

The present work consists of a series of measurements with a diaphragm of powdered glass and aqueous solutions of copper sulphate over the concentration range 0·0002 normal to 1·0 normal; and the nature of the liquid transport is investigated by an examination of the dependence of the flow at constant applied voltage, and of the electric charge of the diaphragm on the electrolyte concentration, and also by a study of the relation between the flow per faraday and the dilution of the electrolyte. The line of treatment adopted appears to present several novel features; and a simple relation is obtained, at the higher concentrations, between the liquid flow per faraday and the dilution of the electrolyte, which, so far as can be traced from a search of the literature, has not previously been recorded.

The Apparatus.

The apparatus employed was the same as that used by the author in a previous work,(3) except that copper electrodes were used in place of electrodes of silver and silver chloride.

The porous diaphragm consisted of a quantity of easily fusible glass in the form of a 200 mesh powder, securely held in position in the body of the apparatus by means of glass wool plugs. The glass powder was carefully purified as previously described.(3)

The electrodes consisted of lengths of copper wire, 1 mm. in diameter, wound into the form of flat spirals and fixed in the apparatus with the plane of each spiral normal to the axis of the diaphragm.

EXPERIMENTAL.

The liquid flow was measured, as in the previous work, by timing a short air bubble over 5 cm. of a calibrated tube forming part of the circuit of the apparatus, the initial determination being with conductivity water. Readings of current and applied potential difference were taken at the beginning and end of the period of measurement of liquid transport. The apparatus was next emptied, and the diaphragm thoroughly washed out with a considerable quantity of 0·0002 normal copper sulphate solution, following the method previously described by the author.(3) The apparatus was then allowed to stand for about 30 minutes, and a further series of observations made of the current, applied potential difference and time of liquid flow. In a similar manner, measurements were made with solutions of increasing concentration up to 1·0 normal.

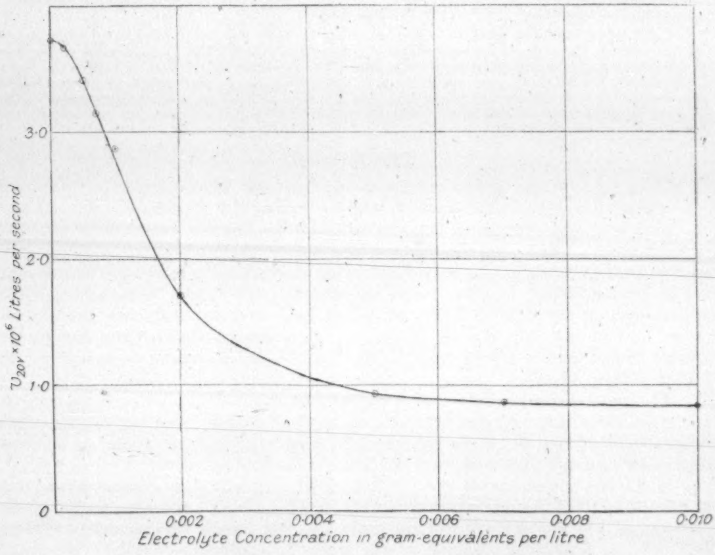


FIG. 1.—LIQUID TRANSPORT AT CONSTANT APPLIED VOLTAGE.

The current was reversed after each determination so as to bring the electrodes, as far as possible, back to their original state. Control experiments by the author have shown that the initial determination, at each concentration, is in general the most accurate, and for this reason measurements on reversal of the current are not given.

The temperature of the room in which the work was undertaken was maintained, as far as possible, constant at 18°C., determinations being made when the solution in the apparatus attained this temperature. At the higher concentrations, however, the current readings indicated a certain amount of heating in the diaphragm. To ascertain the effect of this and other possible disturbing factors, control experiments were carried out with several solutions, within the concentration range of the present work, by timing the bubble over alternate sections of the calibrated tube. The liquid transport, at a given concentration, was found to be approximately proportional to the mean current in each interval, the results indicating that the values given in column 5 of Table I for the liquid transport per faraday exceed the values corresponding to a temperature of 18°C. by about 3 per cent. for 0·5 normal solution, 1·6 per cent. for 0·2 normal solution, and by under 1 per cent. for the lower concentrations.

RESULTS.

(*a*) *Liquid Transport at Constant Applied Voltage.*

The results are given in Table I and shown graphically (up to concentration 0·10 normal) in Fig. 1.

TABLE I.

c.	V.	T.	$v_{20v} \times 10^6$	v_f
Gram. equivs. per litre.	Volts.	Secs.	Litres per sec.	Litres.
0·00	20·50	136·8	3·744	759·0
0·0002	20·76	137·5	3·679	496·3
0·0005	20·18	152·4	3·416	324·1
0·0007	20·18	165·0	3·154	272·2
0·001	20·02	182·2	2·879	202·5
0·002	20·02	306·3	1·713	72·1
0·005	20·20	554·7	0·937	18·34
0·007	20·26	595·8	0·870	12·85
0·01	20·26	614·0	0·844	9·32
0·02	20·22	697·6	0·745	4·754
0·05	20·22	703·7	0·738	2·273
0·1	20·22	735·3	0·706	1·238
0·2	20·02	787·4	0·666	0·680
0·5	20·04	756·7	0·693	0·330

c in Table is the concentration of the solution ; V the potential difference applied at the electrodes ; T the time of flow over 5 cm. of the bubble tube and corresponding to a liquid transport of 0·525 c.c. ; and v_{20v} the liquid transport for an applied potential difference of 20 volts given by $\frac{0{\cdot}525}{10^3} \cdot \frac{1}{T} \cdot \frac{20}{V}$.

With diaphragm 5 cm. in length and electrodes 5·8 cm. apart, as in the present work, the author has shown previously[(3)] that E, the total fall in potential through the diaphragm, is given, within a fraction of 1 per cent., by Vl_d/l_e, where l_d is the length of the diaphragm and l_e the distance between the electrodes ; the values of v_{20v} in column 4 of Table I thus relate to a constant value of E of 17·24 volts.

The direction of liquid flow was from anode to cathode for all concentrations up to 0·5 normal; no perceptible flow was obtained with 1·0 normal solution.

(*b*) *Liquid Transport per Faraday.*

Values of v_f, the liquid transport per faraday, are given in column 5 of Table I. The values are based upon the mean current flowing through the diaphragm during each period of measurement of liquid flow (*see* Table II, column 3).

(*c*) *The Charge of the Diaphragm.*

Smoluchowski(4) has shown, from theoretical considerations, that electro-endosmosis must increase the ordinary voltaic conductivity in consequence of the surface current produced. His result, which relates to the case of a single capillary, is conveniently given in the form $i_\omega = \rho v/q$. i_ω is the surface current, ρ the charge per unit length and q the cross-section of the capillary, and v the liquid transport in unit time. The corresponding expression for a battery of capillaries or porous diaphragm is $i_\omega = Pv/lq$, where P is the total charge and q and l the effective cross-section and length of the diaphragm, giving $P = lq \cdot i_\omega/v$.

In Table II, I_i is the initial total current flowing through the diaphragm as measured during each determination of liquid transport. The values given correspond to the room temperature—i.e., 18°C.—and are in general smaller than the values of the mean current (I_m, column 3); at concentrations not greater than 0·01 normal, however, the differences between the two quantities are inappreciable.

Column 5 of Table II gives the ratio of the initial total current ($I_i \cdot 20/V$), corresponding to an applied potential difference of 20 volts, to the specific conductivity (λ) of the solution, taking the values of λ (column 4 of Table) from the data of Kohlrausch. The values of the ratio over the concentration range 0·005 normal to 1·0 normal differ in general from the mean value for these concentrations by less

TABLE II.

c.	$I^i \times 10^3$	$I_m \times 10^3$	$\lambda \times 10^3$	$\frac{I_i}{\lambda} \cdot \frac{20}{V}$	$i_\omega \times 10^3$	P
Gram. equivs. per litre.	Amperes.		Recip. ohms.	—	Amperes.	Coulombs.
0·00	0·494	0·495	$1\text{–}2 \times 10^{-8}$	483–241	0·471–0·459	
					Mean 0·465	2·19
0·0002	0·741	0·743	0·02159	33·06	0·454	2·17
0·0005	1·024	1·027	0·0518	19·59	0·388	2·00
0·0007	1·126	1·129	0·0710	15·71	0·257	1·43
0·001	1·372	1·374	0·0986	13·90	0·176	1·08
0·002	2·291	2·295	0·1839	12·45	0·061	0·63
0·005	4·978	4·983	0·4049	12·17		
0·007	6·58	6·62	0·537	12·09		
0·01	8·81	8·86	0·717	12·13		
0·02	15·13	15·29	1·248	11·99		
0·05	31·17	31·65	2·558	12·05		
0·1	54·6	55·7	4·385	12·31		
0·2	91·3	94·7	7·532	12·12		
0·5	188·8	202·9	15·40	12·24		
1·0	314·0	...	25·77	12·18		

than 1 per cent. Considering the nature of the diaphragm, such differences are probably no greater than those due to the slight variations in the piling of the glass powder; and the ratio over the concentration range mentioned may reasonably be taken as constant. This result, considered in conjunction with the fact that no liquid transport was perceptible in 1·0 normal solution, indicates that the observed current over the concentration range 0·005 normal to 1·0 normal was practically wholly voltaic—i.e., due almost entirely to the motion of the ions of the electrolyte. The mean value of the ratio over the concentration range mentioned is 12·14, giving the voltaic current for an applied potential difference of 20 volts, and for a total fall in potential through the diaphragm of 17·24 volts (*see* page 101) as $12{\cdot}14\lambda$.

The surface current i_ω (Table II, column 6) is given by the difference between the total current and the voltaic current—i.e., by $I_m\frac{20}{V} - 12{\cdot}14\lambda$—for an applied potential difference of 20 volts. The use of the conductivity data of Kohlrausch in calculating the voltaic current for the solutions of concentration less than 0·005 normal has previously been justified by the author(3) in control experiments embodying conditions similar to those obtaining in the present work.

The value of lq, required to calculate P, the charge of the diaphragm, is obtained as follows:—

It is shown above that the voltaic current for a total fall in potential (E) through the diaphragm of 17·24 volts is given by $12{\cdot}14\lambda$; the voltaic current is also given by $E\lambda q/l$, where q and l are respectively the effective cross-section and length of the diaphragm. Equating these two quantities, we have $q/l=12{\cdot}14/17{\cdot}24$, whence $ql=17{\cdot}60$, taking the effective length of the diaphragm as 5·0 cm.

Values of P, the charge of the diaphragm, given by $lq \,.\, i_\omega/v$, are tabulated in column 7 of Table II, and shown graphically in Fig. 2. The considerations given in paragraph 3 of this Section indicate that the surface current, and consequently the charge of the diaphragm also, are negligible or zero at concentrations 0·005 normal and above.

DISCUSSION OF RESULTS.

It will be seen from Fig. 1 and Table I (column 4) that the liquid transport for a constant applied potential difference across the diaphragm decreases strongly and regularly with the electrolyte concentration up to 0·005 normal—a result that is characteristic of electro-endosmosis—but thereafter, over the concentration range 0·005 normal to 0·5 normal, is not appreciably changed. At the same time the charge P of the diaphragm, as indicated in Fig. 2 and Table II (column 7), decreases with the concentration to a negligible or zero value at concentrations 0·005 normal and above. The values of P given in Table II may require modification if the liquid transport at concentrations below 0·005 normal is not due wholly to electro-endosmosis; but the result that P is negligible or zero at concentrations 0·005 normal and above remains unaffected. The foregoing results, and especially that relating to the charge of the diaphragm, indicate that the liquid transport with the solutions of concentrations 0·005 normal and above is due, in the main, to some factor other than electro-endosmosis.

It will be observed from Fig. 3, within the limit of experimental error, that the liquid transport per faraday, for solutions over the concentration range 0·005 normal to 0·1 normal is related to the dilution of the electrolyte by a linear law, but decreases somewhat more rapidly from concentration 0·1 normal (to a zero or negligible value in 1·0 normal solution) than the linear law requires; at concentrations below 0·005 normal, however, there appears to be no simple relation between the two quantities mentioned. This result, although not recorded by Remy, finds a parallel in his data(1) for a diaphragm of parchment paper and aqueous solutions of potassium chloride. Remy's values for potassium chloride solutions in the concentration range 0·005 normal to 1·0 normal are plotted also in Fig. 3, and give an accurate linear relation, up to concentration 0·1 normal, between the liquid transport per faraday and the dilution; the liquid flow for 1·0 normal solution is less than that required by the linear law.

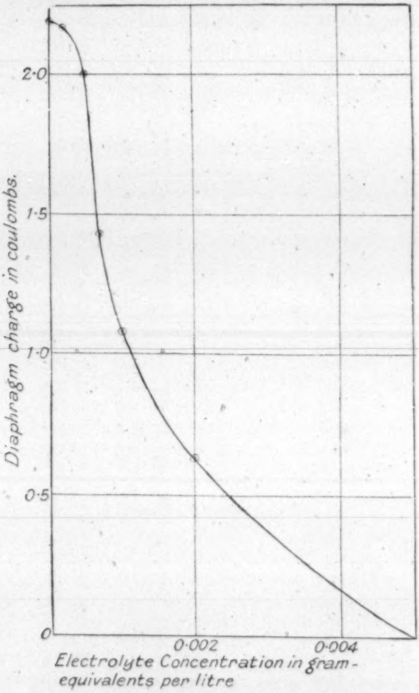


FIG. 2.—THE CHARGE OF THE DIAPHRAGM.

Remy(1) observes from his results that parchment paper appears to form an ideal partition for the measurement of electrolytic water transport, and that the electro-endosmotic effect with this material, even in high dilution, is very small. The relation observed between the liquid transport per faraday and the dilution in the case of the copper sulphate solutions of concentrations 0·005 normal and above thus appears to be characteristic of electrolytic water transport; and at the concentrations mentioned the electric current flowing through the diaphragm appears to be due practically entirely to the motion of the ions of the electrolyte (*see* page 103).

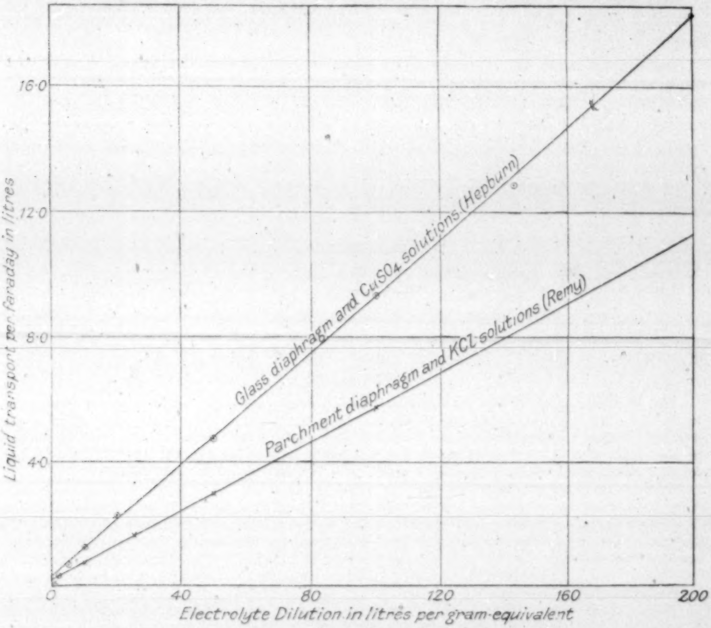


FIG. 3.—LIQUID TRANSPORT PER FARADAY.
(The ordinates represent tenths of a litre as regards Remy's values.)

SUMMARY OF RESULTS.

(1) The liquid transport for a constant applied potential difference, with solutions of copper sulphate in a diaphragm of powdered glass, decreases strongly and regularly with the concentration up to 0·005 normal, but thereafter, over the concentration range 0·005 normal to 0·5 normal, is not appreciably changed.

(2) The electric charge of the diaphragm decreases with the concentration to a negligible or zero value with solutions of concentrations 0·005 normal and above.

(3) The liquid transport per faraday, with solutions over the concentration

range 0·005 normal to 0·1 normal, is related to the dilution of the electrolyte by a linear law, and decreases to a zero or negligible value in 1·0 normal solution. Although not recorded by Remy, a similar result is given by his data for a diaphragm of parchment paper and aqueous solutions of potassium chloride, and with such a diaphragm the liquid transport, even in high dilution, appears to be mainly electrolytic.

(4) With the solutions of concentrations 0·005 normal and above, the electric current flowing through the diaphragm appears to be due practically entirely to the motion of the ions of the electrolyte.

The results of the present work thus indicate, for the case of solutions of copper sulphate in a diaphragm of powdered glass, that electro-endosmosis, with increasing electrolyte concentration, becomes a negligible factor at concentration 0·005 normal; and the data for the solutions of concentrations 0·005 normal and above appear to be characteristic of electrolytic water transport—i.e., transport of water produced by the motion of the ions of the electrolyte.

The author desires to thank Prof. A. Griffiths for facilities provided and for the interest he has taken in this investigation.

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(1) Remy, Zeit. Elektrochem., 29, 365 (1923).

(2) Helmholtz, Wied. Ann., 7, 337 (1879).

(3) Hepburn, Proc. Phys. Soc., 38, 363 (1926).

(4) Smoluchowski, Anz. Akad. Wiss Krakau, A, 182 (1903); Physik. Zeit., 6, 529 (1905).

DISCUSSION.

Dr. D. OWEN pointed out that in effect the author measures the difference between the pressures on the two sides of his apparatus, and inquired whether there is an upper limit to this pressure difference corresponding to the horizontal part of the curve in Fig. 1. As regards the statement that the water transport per faraday is proportional to the dilution, this proportionality would seem to imply that with pure water the rate would become infinite.

Prof. F. L. HOPWOOD said that the data actually observed included the drift of the bubble in the capillary tube, and asked whether the bubble itself was subject to a potential gradient, which would produce a drift apart from that due to the mechanical difference of pressure. In addition to the two causes of water transport mentioned by the author, another possible factor was an action analogous to that which takes place in the Wehnelt interruptor, the rapid opening and closing of a capillary orifice by a bubble, when the current density is high, being capable of producing liquid transport.

Mr. L. HARTSHORN: The author's apparatus may be regarded as a model illustrating the behaviour of certain industrial insulating materials, such as paper, and fibrous materials generally. These materials absorb water from the atmosphere, and, as Evershed* has pointed out, exhibit the phenomenon of water transport when subjected to the action of an electric field. One result of this is that the current through such materials is not directly proportional to the applied voltage. The charge on the diaphragm calculated by the author becomes the charge on the dielectric material in the analogous case in which a sheet of insulating material is placed between two metal plates, across which a constant potential difference is maintained, and in this case, since the material can be easily removed, it might be possible to measure the charge directly. Lübben† has recorded that sheets of telephone cable paper are observed to be charged when treated in this manner.

Modern work on industrial dielectrics suggests that many of them may be regarded as electrolytes, in which the ionic mobility is low, and this, again, suggests that the properties of the author's apparatus might give us some insight into the complex actions occurring in an imperfect dielectric.

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* Evershed, Jour. I.E.E., Vol. 52, p. 51.

† Lübben, Arch. fur Elektrotechnik., Vol. 10, p. 283 (1910).

Mr. A. R. PEARSON said that a further industrial problem connected with the subject of the Paper arose from the tendency of linseed paint to peel off, when wet, from an electrically-charged surface. This behaviour must be ascribed to the water transport which takes place through the film of oxidized linseed oil, the transport being in the same direction as the current in an alkaline medium, but reversible by acidulation. The author might think it worth while to investigate such a film for its industrial interest.

AUTHOR'S reply: In regard to the point raised by Dr. Owen, I may say that the difference in the liquid pressures on the two sides of the apparatus attains a steady value soon after the current is switched on, the value in each case being dependent upon the liquid transport through the diaphragm. In the present work the changes in the liquid levels were almost imperceptible. If, however, the bubble tube is interrupted, the pressure difference rises until equilibrium is reached, when the transport ceases. The proportionality between dilution and liquid transport fails for concentrations below 0·005 normal.

In reply to Prof. Hopwood, I should like to point out that the internal diameter of the bubble tube was about 3 mm.; the bubble was about 1 cm. in length, and, apart from the menisci, occupied the whole section of the tube. A control experiment described by Briggs, Bennett and Pierson,* who devised the form of apparatus employed in the present work, indicates that the potential gradient along the bubble tube produces no perceptible motion of the bubble, even when the potential difference applied at the electrodes is as great as 110 volts.

I am very grateful for the suggestions that have been made as to the possibility of extending the work to industrial problems.

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* Jour. Phys. Chem., 22, 262 (1918).

VIII.—THE INPUT IMPEDANCES OF THERMIONIC VALVES AT LOW FREQUENCIES.

By L. Hartshorn, *A.R.C.S., B.Sc., D.I.C.*, Electrical Department, National Physical Laboratory.

Received October 2, 1926.

Abstract.

It is shown that accurate measurements of input admittance (or of input impedance) under various conditions can be made by means of the Schering Capacity Bridge. The input circuit is regarded as being equivalent to a condenser with a definite phase angle, φ, or "loss angle," $\delta = 90° - \varphi$, and the results are expressed by stating the effective capacity and value of tan δ for each set of experimental conditions.

A series of measurements made on an *R* valve is recorded, and it is shown that the results are in good agreement with the theoretical investigations of Miller and Nichols. The theoretical investigation has been extended to allow for the effect of dielectric losses in the valve, since these were found by experiment to be rather large, and to have an appreciable effect on the capacity and phase angle of the input circuit.

It is shown that the effective capacity may vary from about $10 \mu\mu$F. to $100 \mu\mu$F. for an *R* valve, and the phase angle may vary from about 80° leading to 126° leading, depending mainly on the load in the anode circuit. Values of phase angle greater than 90° correspond to a negative resistance, or negative power factor, and occur when the load in the anode circuit is inductive. The variations of input capacity and phase angle with filament voltage, anode voltage, input voltage, and frequency are also investigated.

The input impedance of a thermionic valve has been investigated theoretically by Miller[*] and Nichols,[†] who showed :—

1. That, owing to the effect of the capacity between the grid and anode, the input impedance varies with the load in the anode circuit.

2. That the input impedance cannot in general be considered as a pure capacity, even when a large negative grid bias is used. It may be considered as a capacity in series with a resistance, or as a condenser with a power factor, which is often quite large, and may be negative when the anode circuit contains an inductive load.

Miller made measurements of the input capacity at audio frequencies, when the load in the anode circuit consisted of a pure resistance, and the results showed that his formula for the input capacity was correct. He was not, however, able to check the formula for the equivalent series resistance, since he found that this was largely due to dielectric losses in the glass of the valve, in the cases he was investigating, and these losses had been neglected by him in obtaining the formulæ.

Morecroft[‡] has made some measurements of input capacity and conductance at a frequency of 500 kilocycles, with both pure resistance and inductive loads in the anode circuit, though he did not attempt a complete analysis of the circuits.

In working with the Schering Bridge at audio frequencies, it was found possible to make measurements of the impedances of valves with considerable precision, so that it seemed to be worth while to attempt to verify the theoretical equations for input impedance for the important case of an inductive load in the anode circuit. Since the electrode capacities were found to be by no means pure, the theoretical treatment was first extended to include the effects of dielectric losses in the valve.

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[*] Miller, Bull. Bureau Stds., Vol. 15, p. 367 (1919).

[†] Nichols, Phys. Rev., Vol. 13, p. 411 (1919).

[‡] Morecroft, Proc. Inst. Radio Eng., Vol. 8, p. 239 (1920) ; or, "The Principles of Radio Communication," p. 428 (1921).

THEORETICAL TREATMENT.

Following Nichols, the valve circuit may be represented by Fig. 1. Here F, G and P represent the filament, grid and plate respectively. The inter-electrode capacities are C_1, C_2 and C_3, and these are not considered to be pure, but to possess loss angles δ_1, δ_2 and δ_3 respectively, or, alternatively, there is associated with C_1 a conductance G_1, such that $\frac{G_1}{C_1\omega}=\tan\delta_1$, and so on. Z_1 represents the external impedance in the grid circuit, and Z_2 is that in the anode circuit. An E.M.F. e_1 is introduced into the grid circuit, and as a consequence we have the mesh currents x_1, x_2, x_3, in the network. The P.D. between the grid and filament is evidently $\frac{x_1-x_2}{\alpha_1}=\delta v$, where α_1 represents the conductance operator of the imperfect condenser C_1, i.e., $\alpha_1=G_1+jC_1\omega$.

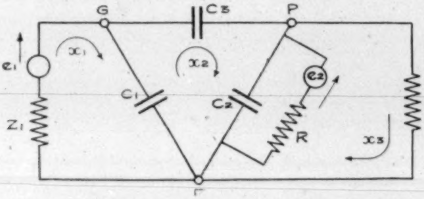

FIG. 1.—THE EQUIVALENT NETWORK OF THE VALVE.

Therefore, we must consider an E.M.F. $e_2=-\mu\delta v=-\frac{\mu(x_1-x_2)}{\alpha_1}$ to be introduced into the plate circuit, μ being the magnification constant of the valve. The direction of the E.M.F. $\mu\delta v$ must be such as to increase the plate current when δv makes the grid more positive. At low frequencies, the capacity C_2 may be neglected, since it is shunted by the resistance R, and its value will only be a few micro-microfarads. Kirchoff's equations for the network may now be represented thus :—

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
Z_1+\frac{1}{\alpha_1} & -\frac{1}{\alpha_1} & 0 & e_1 \\
-\frac{1+\mu}{\alpha_1} & \frac{1+\mu}{\alpha_1}+\frac{1}{\alpha_3}+R & -R & 0 \\
\frac{\mu}{\alpha_1} & -R-\frac{\mu}{\alpha_1} & R+Z_2 & 0
\end{array}$$

Here α_3 is the conductance operator of the imperfect condenser C_3, i.e., $\alpha_3=G_3+j\omega C_3$.

Let D represent the operational determinant of this system of equations, and let D_{11} represent the minor of the element (1, 1), then

$$\frac{x_1}{e_1 D_{11}}=\frac{1}{D}$$

$\therefore$ The input impedance Z_g is given by

$$Z_g=\frac{e_1}{x_1}=\frac{D}{D_{11}} \quad \text{with } Z_1=0.$$

This reduces to

$$Z_g=\frac{1}{a_1}\left[\frac{1+Wa_3}{1+Wa_3+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)}\right]$$

where $$W=\frac{RZ_2}{R+Z_2}\,.$$

The input impedance will be measured as a capacity, and therefore it will be more convenient to consider the admittance

$$\frac{1}{Z_g}=\frac{a_1\left[1+Wa_3+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)\right]}{1+Wa_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At telephonic frequencies a_3 is of the order 10^{-7}; and therefore since W is of the order 10^4, the term Wa_3 may be neglected in comparison with unity as a first approximation. Thus

$$\frac{1}{Z_g}=a_1\left[1+\frac{a_3}{a_1}\left(1+\mu-\mu\frac{W}{Z_2}\right)\right] \quad \text{approx.}$$

$$=a_1+a_3\left(1+\mu-\mu\frac{W}{Z_2}\right) \quad \text{approx.}$$

Substituting in this expression

$$a_1=G_1+jC_1\omega,$$
$$a_3=G_3+jC_3\omega,$$

and $$\frac{W}{Z_2}=\frac{R}{R+Z_2}=\frac{R}{R+R_2+jX_2}=\frac{R(R+R_2)}{(R+R_2)^2+X_2{}^2}-\frac{jX_2R}{(R+R_2)^2+X_2{}^2}\,,$$

(where R_2 and X_2 are the external resistance and reactance in the plate circuit), we find

$$\frac{1}{Z_g}=G_1+G_3\left[1+\mu-\frac{\mu R(R+R_2)}{(R+R_2)^2+X_2{}^2}\right]-\frac{\mu RC_3\omega X_2}{(R+R_2)^2+X_2{}^2}$$
$$+j\omega\left\{C_1+C_3\left[1+\mu-\frac{\mu R(R+R_2)}{(R+R_2)^2+X_2{}^2}+\frac{\mu RC_3X_2\tan\delta_3}{(R+R_2)^2+X_2{}^2}\right]\right\}$$

Separating this into the capacity and conductance components, we see that the input capacity C_g is given by

$$C_g=C_1+C_3+\mu C_3-\frac{\mu RC_3[(R+R_2)-X_2\tan\delta_3]}{(R+R_2)^2+X_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The effective conductance is similarly given by

$$G_g=\left[G_1+G_3+\mu G_3-\frac{\mu RG_3(R+R_2)}{(R+R_2)^2+X_2^2}\right]-\frac{\mu RC_3\omega X_2}{(R+R_2)^2+X_2^2} \quad . \quad . \quad . \quad (3)$$

The phase angle φ_g, or the loss angle δ_g of the input impedance, may now be calculated from

$$\tan\delta_g=\cot\varphi_g=\frac{G_g}{C_g\omega}$$

The following points may be noted :—

1. When the load consists of a pure resistance, the formula for the input capacity becomes

$$C_g=C_1+C_3+\mu C_3-\frac{\mu RC_3}{R+R_2}=C_1+C_3+\frac{\mu R_2C_3}{R+R_2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This is the formula given by Miller for this case. The loss angle of the valve capacities does not appear in this expression, which explains the fact that Miller was able to verify the equation, although he neglected these power losses.

The conductance in this case is given by

$$G_g=G_1+G_3+\mu G_3-\frac{\mu RG_3}{R+R_2}=G_1+G_3+\frac{\mu R_2G_3}{R+R_2}$$

and, therefore, the phase angle is given by

$$\cot\varphi_g=\tan\delta_g=\frac{G_1+G_3+\mu R_2\dfrac{G_3}{R+R_2}}{\omega\left(C_1+C_3+\mu R_2\dfrac{C_3}{R+R_2}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Evidently, if $\tan\delta_1=\tan\delta_3$, then $\tan\delta_g=\tan\delta_1$, and does not vary with the load. In practice $\tan\delta_3$ is often less than $\tan\delta_1$, and consequently $\tan\delta_g$ decreases slightly as the load is increased.

2. When the load in the anode circuit is reactive, the input capacity is a function of this reactance, and is also affected by the dielectric losses associated with the grid-anode capacity ($\tan\delta_3$). Also when the reactance is positive (the load is inductive), the input conductance contains a negative term. This is often much larger than the positive term, which depends almost entirely on the dielectric losses in the valve as explained above. If these losses are such that $\tan\delta_1=\tan\delta_3$, then to a first approximation we may write the expression for the phase angle of the input circuit as

$$\cot\varphi_g=\tan\delta_g=\tan\delta_3-\frac{\mu RC_3X_2}{C_g[(R+R_2)^2+X_2^2]}$$

This equation, and also the capacity equation, have been verified by measurements made on an R valve at audio frequencies.

EXPERIMENTAL INVESTIGATION.

As a preliminary, the electrode capacities and power factors of the valve in its holder were determined. The Schering Bridge* was used for this purpose (*see* Fig. 2). The electrodes, whose mutual capacity was to be measured, were connected in parallel with the standard variable air condenser C_1. The third electrode was connected to earth. Balance was obtained with the valve so connected, by adjusting C_1 and C_4 (or C_3). The electrode which was connected to the point A of the bridge was then disconnected from the lead at X, and then the bridge re-balanced by varying C_1 and C_4 only. The difference between the two readings of C_1 gives the capacity between the two electrodes connected to A and B. (The capacities between these electrodes and the third electrode, which is earthed, do not affect the reading of C_1 appreciably, since one of these capacities is merely a shunt across the whole bridge, and the other is thrown in parallel with C_3, and thus only affects C_4.) The change in the reading of C_4 on disconnecting at X and re-balancing, is due entirely to the change in phase angle of the impedance AB on disconnecting the valve—i.e., it depends only on the phase angle of the valve impedance. (If disconnection were made on the B side of the arm AB, the value of C_4 would be affected, due to changes in earth capacities altering the value of C_3. Earth capacities at A being merely a shunt across the whole bridge, may be altered without affecting C_4.) If now C_4' is the reading of C_4 when the valve is connected to the bridge, and C_4'' when it is disconnected at X; then the power factor of the valve capacity being measured is given by

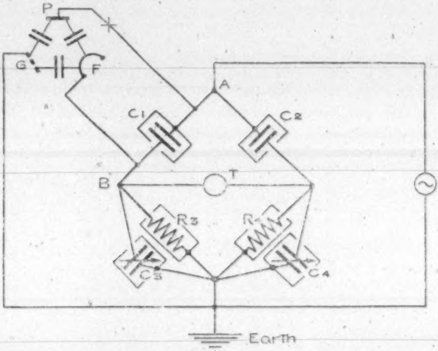

FIG. 2.—BRIDGE FOR THE MEASUREMENT OF ELECTRODE CAPACITIES.

$$\tan \delta = \frac{C_1}{C_v} \omega \,.\, R_4 \,.\, (C_4' - C_4''),$$

where C_1 = value of C_1 when valve is disconnected.

C_v = valve capacity, measured as explained above.

$\omega = 2\pi \times$ frequency.

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* *See* Hartshorn, Beama, Vol. 13, p. 92 (1923), or Proc. Phys. Soc., Vol. 36, p. 401 (1924).

In this way the capacities and power factors of the individual valve capacities were all measured.

The capacities and power factors of the valves may be also measured in pairs, in the manner adopted by Miller, and then the individual capacities and power factors obtained by calculation. The method here described is, however, much more direct, and also takes much less time.

An *R* valve was tested by both methods and the results were found to agree to within $0{\cdot}05\mu\mu$F. In general the uncertainties due to the effect of leads will be greater than this. The results obtained for the *R* valve used are given in Table I.

TABLE I.—*Capacity and Power Factor of Valve R10 in Holder.*
Frequency, 1,000∾.

Electrodes.	Capacity $\mu\mu$F.	Tan δ.
Filament-Plate	$8{\cdot}_6$	0·24
Filament-Grid	$8{\cdot}_7$	0·20
Grid-Plate	$3{\cdot}_3$	0·13

The values of power factor are seen to be very considerable. Probably they are higher than the average for an *R* valve (cf., typical values given in a previous Paper, "Experimental Wireless," p. 263, Feb., 1925), but the fact that such power factors are not uncommon should be recognized.

These high power factors are probably due to absorption effects in the glass, and consequently they are accompanied by changes in the capacity with frequency. The results given in Table II illustrate this.

TABLE II.—*Change of Capacity with Frequency for Valve R10.*

Frequency.	Capacity $\mu\mu$F.	Power Factor.
500 ∾	$9{\cdot}_9$	$0{\cdot}2_0$
1,000	$8{\cdot}_7$	$0{\cdot}2_0$
2,000	$7{\cdot}_6$	$0{\cdot}2_0$
3,000	$7{\cdot}_0$	$0{\cdot}2_2$

The other capacities of this valve, and also those of other valves of the same make, behaved in much the same way.

The input impedance was also measured on the Schering Bridge, the connection being as in Fig. 3, which is self-explanatory. The impedance, which may be regarded as an imperfect condenser, was measured in exactly the same way as the valve capacities and power factors, disconnection being made at *X* (Fig. 3). The calculation of results is exactly the same as before. The case of negative power factor presents no difficulty. It merely means that, when the disconnection is made, the reading of C_4 increases instead of decreasing, and thus a negative sign appears when the usual formula is applied. The physical significance of the negative power factor is as follows. The input circuit behaves like a condenser in which the energy stored when the voltage is increasing is less than that given out when it is decreasing, i.e., there is a net gain of energy, this energy being, of course, received from the anode circuit by way of the grid-anode capacity. The current in the input circuit leads the potential difference between filament and grid, by an angle greater than 90 deg., or, alternatively, the filament-grid impedance contains a negative resistance component. The valve does not, in general, oscillate since the complete input circuit includes the bridge, which has a positive resistance, in general, larger than the negative resistance introduced by the valve.

The alternating voltage applied to the grid in most of these measurements was of the order 0·3 volt. The detector T consisted of an amplifier and telephone. There was no difficulty in obtaining sensitivity to 0·1$\mu\mu$F. A grid bias of -2 or -4 volts was used, the anode voltage being usually of the order of 100.

RESULTS.

(*a*) *Effect of Lighting the Filament on Valve Capacities.*

The input capacity and power factor of a valve at 1,000 ∾ were measured, first, with the filament current switched off, and then with various values of filament current up to the maximum. The circuit was as in Fig. 3, but there was no impedance Z_2 added in the anode circuit, except the resistance of the H.T. battery (about 100 ohms). The results are given in Table III.

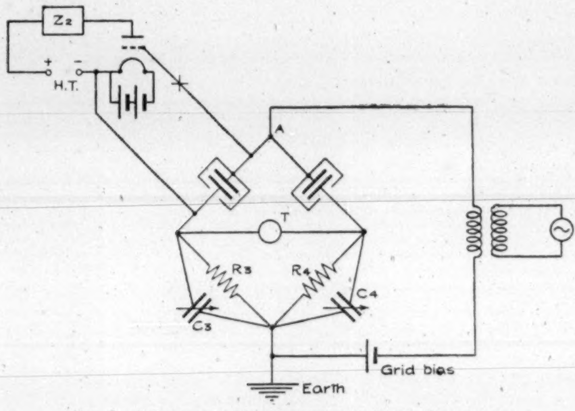

FIG. 3.—BRIDGE FOR MEASUREMENT OF INPUT IMPEDANCE.

TABLE III.—*Input Capacity and Filament Current for Valve R*10.
Frequency, 1,000 ∾. Anode Voltage, 60. Grid Bias, -2 volts.

Voltage Across Filament.	Input Capacity $\mu\mu$F.	Power Factor (tan δ).
0·0	$13\cdot_{0}$	0·21
2·7	$13\cdot_{0}$	0·22
3·2	$13\cdot_{7}$	0·22
4·0	$14\cdot_{8}$	0·38

It is to be noted that the capacity measured when the filament is not alight is $(C_{gf}+C_{ga})$ or (C_1+C_3). When the filament is alight, the capacity measured is given by Equation (4), where $R_2=100$ ohms, which is negligibly small, since R, for a valve of this type, is about 30,000 ohms. Thus Equation (4) gives for the capacity $C_g = C_1 + C_3$, i.e., the capacity measured should not be altered by lighting the filament, unless the electrode capacities are affected by the presence of the electron current. Table III shows that the capacities are practically unchanged by lighting the filament. The small changes which are observed may well be due to the increased dielectric absorption effects which occur when the glass is heated. The increase in the power factor as the filament voltage is increased, supports this view. Similar measurements were made on a second valve of the same type. The results were much the same, except that the increase of power factor was even greater.

(*b*) *Variation of Input Impedance with Filament Voltage, Anode Voltage and Input Voltage.*

Before attempting to verify the various formulæ developed, it is advisable to know what fluctuations in input impedance are to be expected from changes in the experimental conditions. Table IV shows the effect of variations of filament voltage on the input impedance. It is evident that, when the valve is run at its rated voltage, small variations in this voltage do not seriously alter the input impedance.

TABLE IV.—*Input Impedance and Filament Voltage for R Valve.*
Anode Volts, 120. Grid Bias, −4 Volts. Load in Anode Circuit, Inductance Coil of 5 Henries.
Frequency, 1,000 ∾.

Filament Voltage.	Input Capacity C_g	Tan δ_g.
4·0	$64\cdot_7\,\mu\mu$F.	−0·50
3·9	$64\cdot_6$	−0·51
3·8	$62\cdot_3$	−0·55
3·7	$60\cdot_3$	−0·58

Variations of anode voltage were found to cause larger variations in the input impedance. Fig. 4 shows the results obtained with an *R* valve. In subsequent tests made with this valve the anode voltage was maintained at 120, since at this point, the changes of input impedance due to small changes of anode voltage are not large.

As previously stated, the majority of these measurements were made with a P.D. of 0·3 volt applied to the grid circuit of the valve. In order to find out whether the input impedance varied with the applied voltage, a series of measurements was made on one circuit, using various values of input voltage. The results are shown in Table V.

TABLE V.—*Input Impedance and Input Voltage for Valve R*10.
Anode Voltage, 120. Grid Bias, −4 Volts. Filament Voltage, 4. Load in Anode Circuit, Inductance Coil of 5 Henries. Frequency, 1,000 ∾.

Input Voltage.	Input Capacity $C_g\,\mu\mu$F.	Tan δ_g.
0·33	$66\cdot_7$	−0·49
0·16	$66\cdot_6$	−0·49
0·11	$66\cdot_5$	−0·49
0·03	67	−0·49

Equally constant results were obtained when the load in the anode circuit was a pure resistance. Within the experimental error the value of the input impedance is independent of the applied voltage over the range 0·03 to 0·3 volt. The bridge and detecting arrangements used were not sufficiently sensitive to make reliable measurements with an applied P.D. less than 0·03 volt, but it seems probable that the input impedance can be regarded as completely independent of the applied voltage unless this is made very large.

(*c*) *Variation of Input Impedance with Load in the Anode Circuit.*

(A) *Pure Resistance Load.*—Standard wire-wound non-reactive coils of resistances 10,000 to 50,000 ohms were inserted in the anode circuit and measurements of input impedance made for each value used. In each case the H.T. battery voltage was adjusted until the P.D. between the anode and negative filament terminal was 120 volts approximately. The results are shown in Fig. 5. The input capacity is seen to increase as the added resistance R_2 increases. This is in accordance with equation (4),

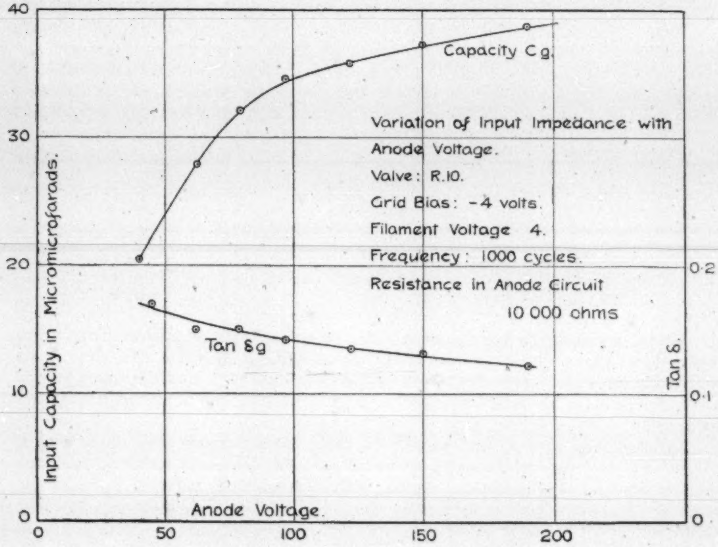

FIG. 4.

$$C_g = C_1 + C_3 + \frac{\mu R_2 C_3}{R + R_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of C_g when the added resistance is zero is $C_1 + C_3$, and thus this quantity is measured directly. The capacity increment $C_g - (C_1 + C_3)$ can therefore be readily obtained for each value of R_2. Equation (4) may be written in the form :—

$$\frac{1}{C_g - (C_1 + C_3)} = \frac{R}{\mu C_3} \cdot \frac{1}{R_2} + \frac{1}{\mu C_3}$$

Thus, if the reciprocal of $C_g-(C_1+C_3)$ is plotted against the reciprocal of R_2, the result should be a straight line. This was found to be the case, and from the slope of this line $\left(\frac{R}{\mu C_3}\right)$, and its intercept on the axis of ordinates $\left(\frac{1}{\mu C_3}\right)$ the following values of the valve constants were obtained :—

$$R=29{,}000 \text{ ohms},$$
$$\mu C_3=63\mu\mu\text{F}.$$

This value of the valve resistance is in good agreement with the value deduced from its characteristic curves. The value of μ for this valve was found to be $8{\cdot}3$, which

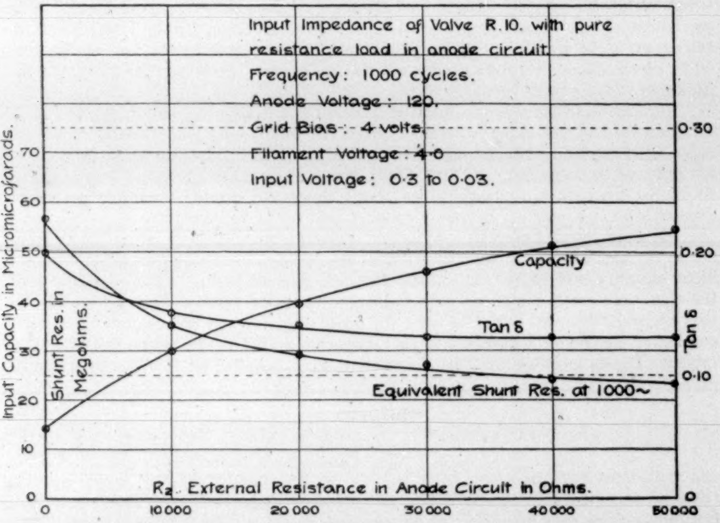

FIG. 5.

gives a value of $7{\cdot}6\mu\mu$F. for the grid-anode capacity C_3. The value for C_3 obtained by direct measurement (Table I) was $3{\cdot}3\mu\mu$F. This represents the value for the valve in its holder, excluding any connecting leads. Evidently the effective value of C_3 when the valve is connected in circuit with other apparatus is higher than this. In the present case there is an additional $4{\cdot}3\mu\mu$F. which probably represents the capacity between the leads to the grid, and the leads to the anode, including the load Z_2 (Fig. 3). In Fig. 5 the observed points are marked with small circles. The capacity curve has been drawn through points calculated from equation (4), using the above values of the constants μC_3 and R. The agreement between the observed and calculated capacities is very close. Considering now the phase angle, the value of tan δ was found to decrease from the value 0·20 when the added resistance R_2 was zero to a constant value of 0·13 for values of R_2 greater than 30,000 ohms. This is explained by equation (5). When R_2 is zero the value of tan δ_g is $\frac{G_1+G_3}{\omega(C_1+C_3)}$—i.e., it is the value for the two condensers C_1 and C_3 in parallel. This is evidently 0·20. If now R_2 is increased until it becomes comparable with R, tan δ_g approaches its limiting value $\frac{G_3}{\omega C_3}$ or tan δ_3, since in equation (5) the first term in numerator and denominator then becomes small compared with the second and third terms. Thus when R_2 exceeds 30,000 ohms the value of tan δ_g is 0·13, which is the value given in Table I for the tan δ of the grid plate capacity C_3. This agreement is partly accidental, since the values given in Table I exclude leads capacities. The existence of these would reduce the power factor, but the heating effect of the filament current increases the losses, and so brings the power factor back more or less to its previous value.

The curve for the equivalent shunt resistance at 1,000 ∼ is also shown in Fig. 5. This has been obtained by calculation from the values of C_g and tan δ_g. For this valve with a pure resistance load the equivalent shunt resistance at 1,000 ∼ is of the order of 30 megohms, and varies with the load in much the same way as tan δ. At higher frequencies the equivalent shunt resistance would, of course, be much smaller, since it is roughly inversely proportional to the frequency.

(B) *Load Purely Inductive.*—Standard self-inductance coils of various sizes from 0·1 to 31 henries were in turn inserted in the anode circuit, and measurements of input capacity and phase angle made for each case as before. The resistances of the coils were small compared with their reactances at 1,000 cycles per second, so that the resistance was neglected, and the input capacity and phase angle measured, considered merely as a function of the reactance added in the anode circuit. The capacity results are shown in Fig. 6 and those for tan δ_g in Fig. 7. The variation of capacity should be given by equation (2), viz.,

$$C_g=C_1+C_3+\mu C_3-\frac{\mu R C_3[(R+R_2)-X_2\tan\delta_3]}{(R+R_2)^2+X_2^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where X_2 represents the added reactance and R_2 the added resistance, which is there taken as negligible. When X_2 is zero the value of C_g given by the above equation is C_1+C_3. Thus this quantity can be measured directly. In the present case

$$C_1+C_3=15\mu\mu F.$$

When the added reactance X_2 is very large, we find from equation (2) that $C_g=C_1+C_3+\mu C_3$. Thus the input capacity C_g approaches this limit as the added reactance is increased indefinitely. In Fig. 6 the capacity is seen to approach a limiting value of $104\mu\mu F$. Hence

$$C_1+C_3+\mu C_3=104\mu\mu F.$$

and $$\therefore\ \mu C_3=104-15=89\mu\mu F.$$

This value for μC_3 is higher than that obtained for a pure resistance load. This is doubtless due to a small increase in the component due to leads, etc., of the capacity C_3. The quantity C_1+C_3 is also correspondingly larger in this case.

Taking now the previously determined value of R—viz., 29,000 ohms—the values of C_g for various values of X_2 were calculated from (2). The value of tan δ_3 is 0·13 when the valve is cold (Table I). The value $0{\cdot}1_5$ was used in these calculations. The curve drawn in Fig. 6 represents the values of C_g obtained by calculation in this way. The points marked with small circles represent the observed values. The agreement is well within the experimental error, since slight changes in C_3 are almost inevitable when the added inductance is changed—i.e., when a larger or smaller coil is added. The part played by power losses in the valve in increasing the input capacity is made clear by the dotted curve in Fig. 6. This represents the calculated values of C_g neglecting the term involving tan δ_3. The vertical distance between the dotted and continuous curves thus represents the capacity increment due to power losses.

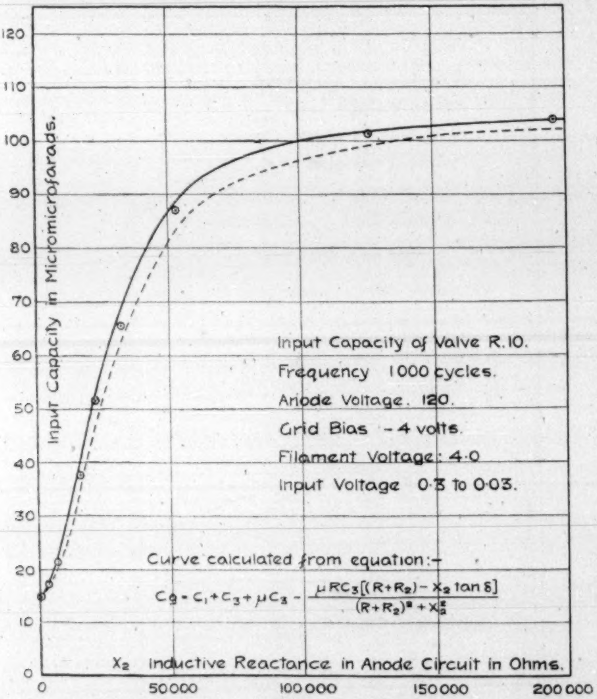

FIG. 6.

Turning now to the phase angle φ_g of the input circuit, which is represented by tan δ_g [=cot φ_g] in Fig. 7, we see that when the added reactance is zero the value of tan δ_g is 0·21—i.e., the input circuit behaves like a condenser with a power factor of approximately 20 per cent.—i.e., the current leads the voltage by an angle of 79 deg. As the added reactance is increased tan δ_g rapidly decreases, reaching a minimum value of $-0{\cdot}7_5$ when $X_2=12{,}000$ ohms (approx.). At this point the input circuit behaves like a condenser with a negative power factor (sin δ_g) of 60 per cent.—i.e., the current in the grid circuit leads the grid-filament P.D. by 126 deg.—since $\tan^{-1}(-0{\cdot}7_5)=\sin^{-1}(-0{\cdot}60)=36$ deg. As the value of X_2 is increased still further the value of tan δ_g increases, becoming positive again for very large added reactances. The full expression for tan δ_g obtained from (2) and (3) is rather cumbersome, and in order to simplify it, a further approximation has been made. It

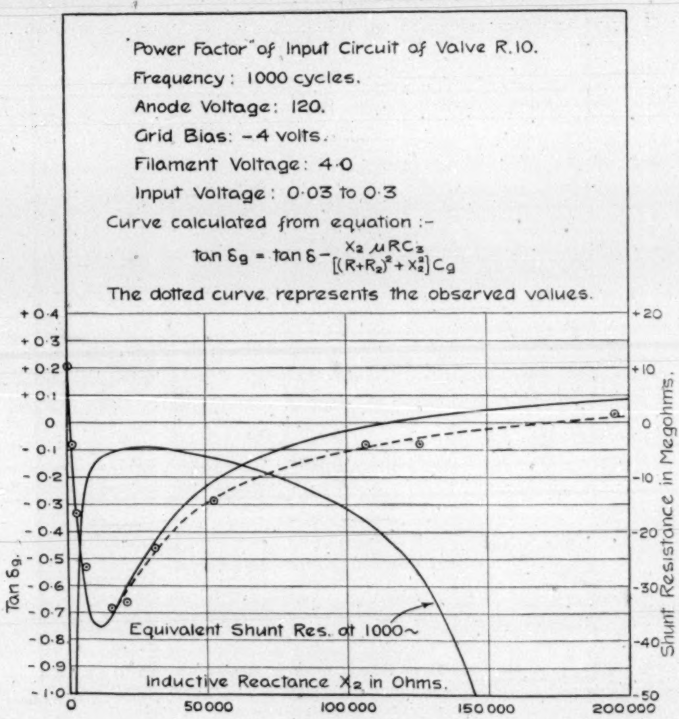

FIG. 7.

is evident from (3) that the full expression for tan δ_g will contain two terms, the first being a function of the power losses in the valve, and the second being a function of the added reactance. In general, the first term will be the smaller, and it varies comparatively little with the added load. It has therefore been assumed constant, and has been denoted by tan δ. We then have

$$\tan \delta_g = \tan \delta - \frac{X_2 \mu R C_3}{[(R+R_2)^2+X_2^2]C_g} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

When $X_2=0$ is inserted in this equation we have tan δ_g=tan δ, and therefore tan δ has been taken as 0·21, which is the measured value of tan δ_g when $X_2=0$. Since all the other quantities in (6) are known, the curve showing the relation between tan δ_g and X_2 can be plotted. This is the continuous curve in Fig. 7. The observed points lie on the dotted curve. The agreement between the two is as close as can be expected. The divergence is evidently due to the fact that the term tan δ is not strictly speaking constant. As a matter of fact, the value of tan δ will decrease with added load, in much the same way as it does in Fig. 5, as may be seen by a comparison of the equations for the two cases. The calculated value for tan δ_g will therefore be slightly high for large added reactances, which explains the divergence of the two curves. We may therefore conclude that the equations given, represent

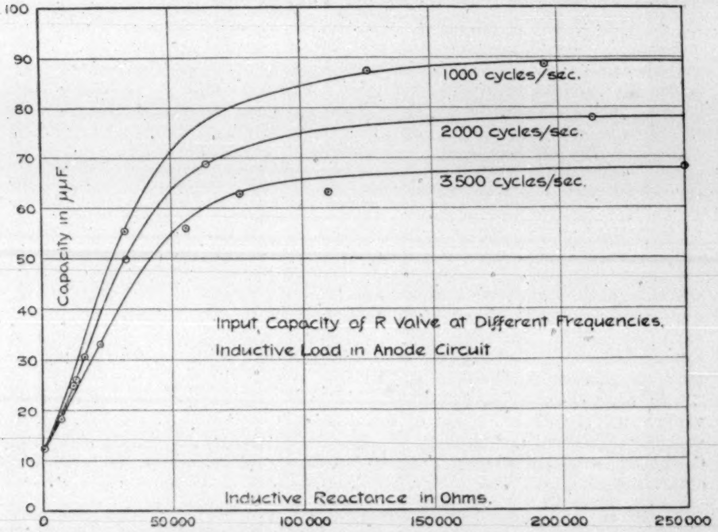

FIG. 8.

the input impedances of valves at low frequencies with sufficient accuracy for all practical purposes.

In this case, also, the equivalent shunt resistance curve at 1,000 ∾ has been calculated from those of capacity and tan δ. It is shown in Fig. 7. The smallest value of the equivalent negative shunt resistance is 5 megohms at 1,000 ∾. This quantity is, of course, enormously dependent on the frequency—e.g., at 10,000 cycles it would be one tenth of this amount if the value of tan δ were unchanged—and to a first approximation tan δ is independent of frequency for a given added reactance.

(C) *Variation of Input Impedance with Frequency.*—The present investigation applies only to low frequencies (the telephonic range, say), and for this case the equations indicate that for a given added impedance in the anode circuit the input impedance is independent of frequency, provided that the valve constants μ, R, C_3, C_1 are independent of frequency. For the valve used in these experiments the values of C_1 and C_3 were found to vary with frequency (*see* Table II). Exactly corresponding variations were found in the input impedance. Fig. 8 shows curves giving the variation of input capacity with added reactance in the anode circuit for a number of different frequencies. From the curves we may deduce the values of the valve constants C_1+C_3, μC_3, and therefore C_3, in the manner previously indicated. The following results were obtained in this way :—

Frequency.	C_1+C_3	μC_3	C_3
1,000	$12\cdot_5$	76	$9\cdot_2$
2,000	$12\cdot_0$	66	$8\cdot_0$
3,500	$11\cdot_3$	58	$7\cdot_0$

The changes are of the same order as those shown in Table II. The curves in Fig. 8 were calculated from the formula (2), using these values of C_1+C_3 and μC_3 and the same values of R. The agreement is sufficient to show that the change in input capacity is due solely to the change in valve capacities. Similar results were obtained for a pure resistance load in the anode circuit.

In carrying out this investigation, I have had the privilege of discussing various points with a number of my colleagues, and I wish to thank them for valuable suggestions, particularly Messrs. D. W. Dye, T. I. Jones, and F. M. Colebrook. My thanks are also due to Mr. D. A. Oliver for help in making the observations.

DISCUSSION.

Mr. J. NICOL said that the Paper had interested him very greatly. Some of the details mentioned by the author in presenting it to the meeting might advantageously be added to the Paper for publication in the Proceedings ; for instance, the diagram of the Schering Bridge or a reference to a published description of this, and the method of increasing the sensitivity of the bridge by using a two-valve amplifier in the detector arm. Would not the mathematical expressions be simplified if admittances were used throughout instead of impedances ?

Prof. C. L. FORTESCUE said that it was a new and gratifying experience to meet with valve-impedance measurements which agree with theory. The Paper had an important bearing on a subject not mentioned in it—namely, the behaviour of valve voltmeters. It now appeared that variations in the load impedance of the valve had important effects on the effective input conductance, and might render this negative, so that the reaction on the measured circuit might be important. Fig. 7 of the Paper bore out the contention that the behaviour of the valve voltmeter is by no means simple. In amplifiers the input conductance was of greater practical importance than the input capacity, which was small, and usually swamped by larger shunt capacities. Thus equation (3) of the Paper, not equation (2), was of prime importance in connection with the howling which arises from negative resistance. The theory in the Paper was presented in the American manner, the ouput voltage variations being expressed as a fraction of the input variations; possibly the alternative method, in which the parameters of the valve are employed, would give the same results in a simpler and more direct manner. In conclusion, he asked for information as to the lower limit of the voltage range of which the bridge could take account.

Mr. T. G. **Hodgkinson**: I should like to join in congratulating Mr. Hartshorn on the careful measurement of some of the quantities we talk about so flippantly in these days. It is interesting to attack the problem from another point of view—that is, the point of view of the design of capacity coupled oscillators for the shorter waves. For oscillations below, say, 350 metres, with quite ordinary coil dimensions, the capacity between the electrodes (grid and anode) is sufficient to maintain the system in oscillation, and the part this capacity plays in both the determination of the oscillation frequency and the power output is important. For example, a valve system with inductances in the grid and anode circuits, not coupled magnetically, has at least two free periods, a longer period and a shorter period, which can be predetermined in terms of the inductances, resistances, capacities and inter-electrode capacity of the combination. The longer period (I call it the sum mode) is supported in oscillation by the grid-anode capacity, and I can confirm the order of the author's results from cases of this kind which I have predetermined and measured.

Dr. E. H. **Rayner** said that he thought the author should put on record the terminal points across which his measurements were taken. Did he include in the valve impedance the wire leads, the holder and the cap of the valve? The insulating composition in the cap, if not removed for the purpose of the experiments, would account for the power factor to a large extent.

Mr. J. **Nicol** (subsequent contribution): With reference to Prof. Fortescue's remarks on errors introduced by valve voltmeters, I believe that in the Moullin voltmeter a large condenser connects the plate and filament legs together, bridging the microammeter—there is no H.T. battery. Though the author does not consider the case of a capacitative loading, a reference to his curves will show that when the load impedance is very low the input impedance of the valve will be simply that of the grid filament condenser, and therefore very high, so that the voltmeter should in most cases have very little effect on the voltage it is used to measure. Of course, if the bridging condenser is omitted great errors may easily be introduced, as Prof. Fortescue suggests.

Author's reply: Replying to Mr. Nicol, the two-stage transformer-coupled amplifier used in the detector arm was not specially designed for this work, and presents no novel features. Almost any amplifier will do. Convenient values for the resistance arms of the bridge are 1,000 or 5,000 ohms. The variable air condensers had maximum values of 0·001 or 0·002 microfarad. Impedances were used for the parts of the circuit external to the valve, since in practice the unit of impedance, the ohm, is much more convenient to handle than the unit of admittance.

I quite agree with Prof. Fortescue as to the importance of the input conductance. I have no information as to the minimum input voltage required to give sufficient sensitivity for these bridge measurements. It will, of course, depend largely on the amplifier used as detector and the degree of freedom from stray disturbances. I have little doubt that an amplifier could be designed which would give sufficient sensitivity with input voltages considerably less than 0·03, the smallest used by me in these experiments.

I am very interested to hear that Mr. Hodgkinson can confirm the order of my capacity results from observations on capacity coupled oscillators.

In reply to Dr. Rayner, I may say that all the measurements were made with the valve mounted in its holder, so that the effects of capacity and power loss in the leads, holder and cap of the valve are included in the impedances measured.

IX.—A PRINCIPLE GOVERNING THE DISTRIBUTION OF CURRENT IN SYSTEMS OF LINEAR CONDUCTORS.*

By FRANK WENNER, *Ph.D.*, Physicist, Bureau of Standards.

Received October 21, 1926.

ABSTRACT.

A brief résumé is given of the procedures which have been developed for determining the distribution of direct current in systems of linear conductors. In this connection reference is made to practically all the laws, theorems, principles and procedures generally considered to pertain to this particular field of investigation. Consideration is then given to a principle which when employed usually leads more directly to the solution of problems than does any of the procedures commonly used.

This principle applies to systems of linear conductors in which the currents are proportional to the impressed electromotive forces ; the electromotive forces may be any function of time, and may be distributed in any manner throughout the system ; and the branches may contain resistance, inductance, capacitance or any two or all of these in series, may be so arranged as to move with respect to a permanent magnet, thus developing counter electromotive forces, and may be connected by contacts or mutual inductances or both of these. For such a system of conductors the current in any branch is that which would result if all impressed electromotive forces were replaced by a single impressed electromotive force, located in the particular branch and equal to the drop in potential which originally would have appeared across the break had this branch been opened. While this principle is a logical consequence of well-known laws, it has been used but very little and seems to be practically unknown. It is shown here that it may be used to advantage in all or practically all cases in which the conductors form a series-parallel combination or a network which may be changed to a series-parallel combination by opening the branch in which it is desired to determine the current.

1. INTRODUCTION.

IN electrical measurements and the distribution of electric power it is often necessary to estimate or determine the current to be expected in one or more branches of a system of conductors in advance of assembly or construction, or when conditions are such that a direct measurement is not practicable. A case in point is the determination of the current in the galvanometer branch of a Wheatstone bridge network for a given set of conditions.

In the discussion of this subject it will be presumed, at least for the greater part, that the currents are direct and constant, so that currents and electromotive forces may be added algebraically, and the resistances only of the branches need be considered. However, it should be understood that the same procedures may be followed when the currents are alternating, and also when the currents are transient, provided due consideration is given to those electromotive forces resulting from changes of the current in branches containing inductance or capacitance, motion of branches free to move with respect to the field of a magnet, etc.

A straightforward and obvious procedure for the solution of the problem stated above, and similar problems, originally proposed by Kirchhoff,(1) is to apply Ohm's law, or Kirchhoff's second law, to $n-m+1$ of the enclosed meshes or circuits of the

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system, and Kirchhoff's first law(2) to $m-1$ of the branch points, where m is the number of branch points and n the number of branches. This procedure gives n simultaneous equations from which the current in each of the n branches may be determined in terms of the electromotive force of the battery and the resistances. However, since for the simplest bridge n is 6, it will be seen that for many cases the algebra involved in the solution of the n simultaneous equations will be long and tedious. Therefore attempts have been made to develop less complicated procedures, and these have led to a marked improvement in the situation.

By the use of mesh or cyclic currents, which is a special application of the principle of superposition,(3) instead of Kirchhoff's first law, Maxwell(4) has shown that a partial solution may be obtained from $n-m+1$ simultaneous equations, and that a complete solution may be obtained simply by adding expressions given by the partial solution. A very thorough discussion of the subject, based mainly on the contributions made by Kirchhoff and Maxwell is given by Feussner.(5)

Further, it has been noted that in some cases a knowledge of the current in one branch in terms of the current in another branch and the resistances will serve as well as a knowledge of the current in terms of the electromotive force of the battery and the resistances. In such cases the number of unknowns is $n-1$, and consequently a partial solution may be obtained from $n-m$ simultaneous equations.(6) Therefore a partial solution for the Wheatstone bridge network may be obtained from two simultaneous equations, whereas the procedure proposed by Kirchhoff requires six. A discussion of the subject, including the contributions made by Kirchhoff, Maxwell and Callender, and extended so as to apply when the current is alternating, is given by Hague.(7)

In still other cases it is sufficient if the currents are known only approximately, or when some special relation exists between the resistances. In many such cases it is possible to avoid entirely the use of simultaneous equations. For example, in a particular case it may be obvious that the current in one branch is small in comparison with the currents in other branches, and that if this branch were removed the system would be changed from a network to a group of series-parallel conductors. The current in all branches except the one may then be calculated as though this one were not a part of the system, and the values thus obtained will not be materially in error. In some cases, too, it is possible to determine to a close approximation the current in that branch in which the current is small in comparison with the currents in other branches. Illustrations of this are the determination of the sensitivity of the Wheatstone and other bridges, potentiometers, etc., by Smith,(8) and the determination of the sensitivity of the Thomson bridge by Northrup.(9) In these determinations, both considered the current through the galvanometer branch when the bridge is very nearly balanced to be that which would result from an electromotive force in one of the arms equal to the current in this arm times the change in resistance in this arm necessary to reduce the current in the galvanometer branch to zero. A proof of this principle, given by Smith, is based on Kirchhoff's reciprocal theorem,(10) a theorem which may be used to advantage in the solution of many problems in current distribution.

Another theorem of importance in this connection pertains to the equivalence of stars and delta in systems of linear conductors. Kennelly(11) has shown that any three-point star (three-way branch-point) may be replaced by a delta or triangle, and, conversely, any delta may be replaced by a three-point star. Further, Rosen(12) has shown the equivalence of any n-point star and a delta or network in which there are $1/2\ n(n-1)$ conductors connecting each of the n points to every other point. The use of this theorem leads to material simplification in the calculation of the resistance or impedance between any two points of a network. The theorem also has other applications.

In this brief résumé of the subject reference has been made to most of the laws, principles, theorems and procedures which pertain to the distribution of direct currents in systems of linear conductors.

There is, however, another principle which in some cases is of material assistance in the solution of problems of this type. This principle may have been applied to direct current bridges(13) by Jaeger, Lindeck and Diesselhorst. It has been applied to alternating-current bridges(14 and to other problems(15) by myself, and with direct currents has been applied to a general network(16) by Harrison and Foote. However, except as applied to simple circuits, it seems to be practically unknown. The purpose of this Paper is to state this principle, to prove its validity, to show to what types of problems it is applicable, and to show the advantages to be gained by its use.

II. STATEMENT AND PROOF.

Let it be assumed that the system of conductors is one in which

1. Each branch is linear.
2. Ohm's law is applicable to each branch.
3. Impressed electromotive force may be distributed in any manner throughout the system.

For such a system of conductors, or that described in an appendix to this Paper, *the current in any branch is that which would result should an electromotive force, equal to the potential difference which would appear across the break were the branch opened, be introduced into the branch and all other electromotive forces be removed.*

This principle may be developed from well-known laws and principles as follows :—

1. From Ohm's law and the principle of superposition it follows that each electromotive force causes a current in each branch proportional to itself and independent of the currents caused by other electromotive forces, and the current in any branch is the algebraic sum of the currents in that branch caused by the various electromotive forces.

2. From 1 it follows that if in any branch there is introduced an electromotive force of the proper sign and magnitude to reduce the current in this branch to zero, the current in this branch caused by this electromotive force is equal in magnitude but opposite in sign to the current in this branch caused by the other electromotive forces in the system.

3. With the current in any branch equal to zero, the branch may be opened without changing the current in any branch or the potential difference between any two points in the system. Consequently, for the conditions given in 2, opening the branch causes no potential difference to appear across the break.

4. With the branch open, any or all of the electromotive forces in it may be removed without changing the current in any branch of the system ; but with the

appearance of a potential difference across the break equal in magnitude but opposite in sign to the electromotive forces removed.

5. Consequently, when a branch is opened, the potential difference which appears across the break is equal in magnitude but opposite in sign to the electromotive force which, if placed in this branch, would reduce the current to zero.

6. Therefore, if an electromotive force equal to the potential difference which would appear across the break should a branch be opened, were introduced into the branch under consideration and all other electromotive forces regardless of their location removed, leaving all branches closed, the current in it would be the same as that which is caused by the other electromotive forces.

As a result of discussions with my colleagues, independent proofs of this principle have been developed by Dr. A. S. McAllister and Dr. Chester Snow. These unpublished proofs differ from each other, from that given by Harrison and Foote,(16) and from that given above. Additional proof is contained in the solutions of a number of the problems which are considered later.

From the principle as stated and the principle of superposition it follows that—

1. For systems of conductors such as those described above or in the appendix, and in which there may be any number of electromotive forces, the change in current in any branch brought about by the introduction of an additional electromotive force is equal to the change in current which would be produced by the introduction into the particular branch of an electromotive force equal to the change which would occur in the potential difference across the break if such a break were present at the time the additional electromotive force is introduced.

2. For systems of conductors such as are described above, or in the appendix, the change in the current in any branch caused by the addition of a branch containing no electromotive force is equal to the change in current which would be produced in the particular branch by an electromotive force in the added branch equal to the potential difference which existed between the points to which the branch is connected, before the connection was made.

3. For systems of conductors such as those described above or in the appendix, except that in one branch the current is not proportional to the impressed electromotive force, the current in this branch is that which would result from an electromotive force, located in it, equal to the potential difference which would appear across the break were the branch opened.

III. APPLICATIONS AND ADVANTAGES.

To show something of the conditions under which the principle is applicable and the advantages to be gained by its use, it will be applied in the solution of a number of problems. As these are to serve as illustrations only, no effort will be made to get the solutions into a convenient form for use, or to explain their significance.

Problem 1.—To measure the potential difference V between two terminals of a system of conductors by means of a voltmeter when conditions are such that the connection of the voltmeter causes an appreciable lowering of the potential difference. One of two procedures for making such a measurement, proposed by Brooks,(17) is (*a*) connect a voltmeter to the terminals and note the reading V_1; (*b*) insert in series

C

with the voltmeter a resistance equal to the resistance of the voltmeter and note the reading V_2; (c) calculate V from the equation

$$V=V_1V_2/(V_1-V_2).$$

To show that this procedure gives a correct result, let

i_1 be the current through the voltmeter when the reading is V_1,

i_2 be the current through the voltmeter when the reading is V_2,

r be the resistance of the voltmeter, and

R be the resistance of the system of conductors between the terminals, between which the potential difference is to be determined.

According to the principle under consideration

$$i_1=V/(R+r) \text{ and } i_2=V/(R+2r),$$

while, presumably, the instrument is so graduated that

$$i_1=V_1/r \text{ and } i_2=V_2/r.$$

Therefore

$$V=V_1(1+R/r)=V_2(2+R/r),$$

from which it follows on the elimination of R/r that

$$V=V_1V_2/(V_1-V_2).$$

Problem 2.—One of two two-wire feeders supplying power from the same source to adjacent territories is found to be overloaded, and it is desired to know what shift in load might be expected were a two-wire tie to be made between these. The conditions of the problem are as follows :—

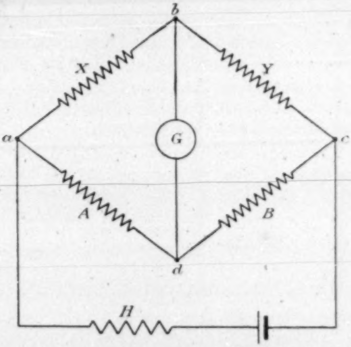

FIG. 1.—WHEATSTONE BRIDGE.

(*a*) Each feeder is transmitting power nominally at 13,000 volts.

(*b*) At what seems to be a favourable place for making the tie, the voltage of the excessively loaded feeder, found by measurement, is 260 volts less than that of the other feeder.

(*c*) The resistance (or impedance) to an electromotive force located in the tie, estimated from the size and length of the conductors proposed for the tie and the size and length of the feeders is 3·25 ohms.

Therefore, according to the principle under consideration, making the tie would cause a shift of 260/3·25, or 80 amperes from one feeder to the other, and would relieve the excessively loaded feeder to the extent of 80 × 13,000 volt-amperes, or 1,040 k.v.a.

As a simple independent analytical method for solving this problem was not evident, the principle was applied to other problems of the same type and the results obtained checked experimentally.

Problem 3.—To determine the current through the galvanometer branch of a Wheatstone bridge network. Referring to Fig. 1, let

E be the potential drop from a to c with the galvanometer branch open.

X, Y, A and B be the resistance of the arms of the bridge.

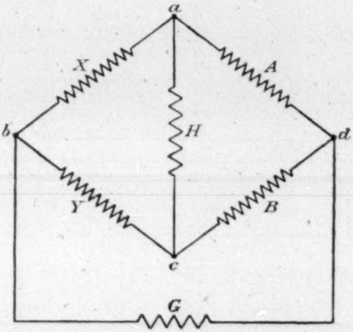

FIG. 2.—WHEATSTONE-BRIDGE ARRANGED FOR SHOWING RESISTANCE OF COMPLETE GALVANOMETER CIRCUIT.

G be the resistance of the galvanometer branch.

H be the resistance of the battery branch.

R be the resistance of the galvanometer circuit—that is, the resistance to an electromotive force in the galvanometer branch.

i_g be the current in the galvanometer branch.

With the galvanometer branch open it may be seen by inspection that the potential drop from a to b is

$$EX/(X+Y),$$

and from a to d is

$$EA/(A+B)$$

The potential difference across the break in the galvanometer branch then is

$$EX/(X+Y)-EA/(A+B) \text{ or } E(XB-YA)/(X+Y)(A+B).$$

Therefore, if the galvanometer branch is closed

$$i_g = E(XB - YA)/(X+Y)(A+B)R.$$

If conditions are such that the drop in potential from a to c is not appreciably lowered by closing the galvanometer branch, it may be seen by inspection of Fig. 1 that

$$R = G + XY/(X+Y) + AB/(A+B) \text{ approximately.}$$

This expression for R applies either in case the bridge is approximately balanced or in case the resistance of the battery branch is very small in comparison with the resistance of the galvanometer circuit.

If an exact expression for R is desired, it may be obtained readily by transforming one of the deltas to a star, as suggested by Kennelly.⁽¹¹⁾ Replacing the delta *bac* of Fig. 2 (which is the same as Fig. 1, except that the battery and galvanometer are considered to be replaced by conductors having equivalent resistances) by a star gives the arrangement shown in Fig. 3. Here

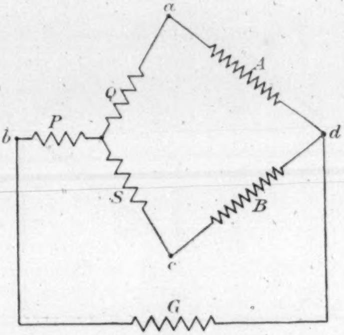

FIG. 3.—WHEATSTONE BRIDGE WITH THE DELTA *bac* OF FIG. 2 REPLACED BY THE EQUIVALENT STAR.

$$P = \frac{XY}{X+Y+H}, \qquad Q = \frac{HX}{X+Y+H}, \qquad \text{and } S = \frac{HY}{X+Y+H}$$

Therefore it may be seen by inspection of Fig. 3 that

$$R = G + P + (Q+A)(S+B)/(Q+A+S+B)$$

or

$$R = G + \frac{XY}{X+Y+H} + \frac{\left(\frac{HX}{X+Y+H} + A\right)\left(\frac{HY}{X+Y+H} + B\right)}{\frac{HX}{X+Y+H} + A + \frac{HY}{X+Y+H} + B}$$

Problem 4.—To determine the current through the galvanometer branch of the Brooks model 5 potentiometer (18) for which the circuit arrangement is that shown in Fig. 4.

To solve this problem, consider first that the galvanometer branch and the r_1 branch are open. Then the current in the r_6 branch will be

$$\frac{e_1}{r_3+r_6}$$

and the potential difference across the break in the r_1 branch will be this current times the resistance r_6, or

$$\frac{e_1r_6}{r_3+r_6}$$

Now consider the r_1 branch closed, then the current in it will be this potential difference considered as an electromotive force divided by the resistance to an electromotive force in the r_1 branch, or

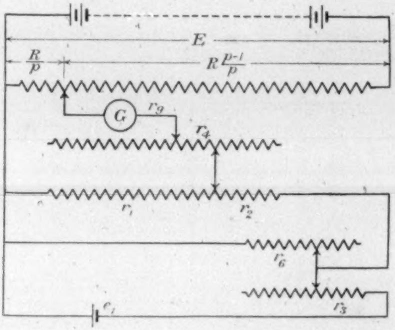

FIG. 4.—CIRCUITS OF BROOKS MODEL 5 DEFLECTION POTENTIOMETER.

$$\frac{\dfrac{e_1r_6}{r_3+r_6}}{r_1+r_2+r_3r_6/(r_3+r_6)}$$

The potential difference across the break in the galvanometer branch caused by the electromotive force e_1 then will be this current times r_1, and that caused by the electromotive force E, which is of opposite sign, is the current E/R times the resistance R/p. The potential difference across the break in the galvanometer branch therefore will be

$$\frac{\dfrac{e_1r_6r_1}{r_3+r_6}}{r_1+r_2+r_3r_6/(r_3+r_6)}-\frac{ER}{Rp}$$

With the galvanometer branch closed, the current in the galvanometer branch will be this potential difference considered as an electromotive force divided by the resistance of the galvanometer circuit, or

$$i_g=\frac{\dfrac{\dfrac{e_1r_6r_1}{r_3+r_6}}{r_1+r_2+r_3r_6/(r_3+r_6)}-\dfrac{ER}{Rp}}{r_g+r_4+\dfrac{r_1[r_2+r_3r_6/(r_3+r_6)]}{r_1+r_2+r_3r_6/(r_3+r_6)}+\dfrac{R/p\times R(p-1)/p}{R/p+R(p-1)/p}}$$

In the development of this equation terms once written down have not been changed even in those cases in which simplifications might readily have been made. Consequently it might have been written simply from an inspection of the diagram of connections, and, in slightly different form, was so written on first consideration of the problem. The equation may readily be reduced to the simpler form given by Brooks.

Problem 5.—To determine the values for the resistances of a shunt box for a galvanometer to satisfy the following conditions: (*a*) The current drawn from the source of the electromotive force after the deflection has reached a constant value shall be the same as though no shunt were used. (*b*) The resistance to the electromotive force developed by the motion of the coil of the galvanometer shall be the same as though no shunt were used. (*c*) The current through the galvanometer after the deflection has reached a constant value shall be $1/n$ times what it would be if no shunt were used.

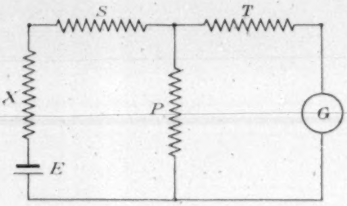

FIG. 5.—SHUNT FOR REDUCING SENSITIVITY OF GALVANOMETER WHILE KEEPING THE DAMPING AND CURRENT DRAWN FROM SOURCE CONSTANT.

Volkmann[19] has shown that these conditions may be fulfilled by the arrangement shown in Fig. 5, in which X represents the resistance and E the electromotive force of the source of the current, G the resistance of the galvanometer, and S, P and T the resistances of the added conductors. Then if I is the current drawn from the source and i_g is the current through the galvanometer, it follows from condition (*a*) that

$$I=\frac{E}{X+S+P(T+G)/(P+T+G)}=\frac{E}{X+G} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Therefore

$$S+P(T+G)/(P+T+G)=G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From condition (*c*) it follows that

$$i_g=\frac{E}{X+S+P(T+G)/(P+T+G)}\times\frac{P}{P+T+G}=\frac{E}{n(X+G)} \tag{3}$$

and dividing equation (3) by equation (1) gives

$$P/(P+T+G)=1/n \tag{4}$$

From the principle under consideration and condition (*b*) it follows that

$$i_g=EP/(P+S+X)(X+G)$$

and substituting the expression for i_g given by equation (3) gives

$$P/(P+X+S)=1/n \tag{5}$$

From equations (4) and (5) it follows that

$$T=S+X-G \tag{6}$$

and from equation (5) it follows that

$$P=(S+X)/(n-1) \tag{7}$$

These expressions for T and P substituted in equation (2) give

$$S=(Gn-X)/(n+1) \tag{8}$$

This expression for S substituted in equation (5) gives

$$P=n(X+G)/(n^2-1) \tag{9}$$

and this expression for P substituted in equation (4) gives

$$T=(nX-G)/(n+1).$$

These expressions for S, P and T are the same as those given by Edler,(20) who showed that the expressions given by Volkmann are unnecessarily complicated.

Problem 6.—To design a multiplier for a deflection instrument so that it may be used as a voltmeter with a range of 1·5 volts, when the only information available concerning its constants is that when 20 ohms is placed in series the reading is proportional to the applied potential difference and a full scale deflection is obtained for an applied potential difference of 0·15 volt. The problem, therefore, is to devise a system of conductors with the instrument connected into one branch, such that the current in this branch for any potential difference applied between a particular pair of terminals will be the same as for one-tenth this potential difference applied to the instrument with 20 ohms in series. A possible arrangement is that shown in Fig. 6, in which values for the resistances A and B are so chosen that with the instrument branch open the potential difference across the break is one-tenth the potential drop from m to n, and the resistance as measured from the position of

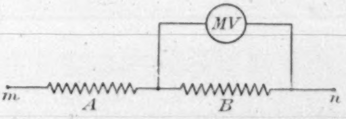

FIG. 6.—MULTIPLIER FOR MILLIVOLTMETER.

the instrument with the instrument removed and the terminals *m* and *n* connected by a conductor of negligible resistance is 20 ohms. This gives

$$B/(A+B)=0{\cdot}1 \text{ and } AB/(A+B)=20$$

or

$$A=200{\cdot}0 \text{ and } B=22{\cdot}22.$$

Here the potential difference between *m* and *n* has been considered as though it were produced by a source of electromotive force having no resistance. However, as it is the potential difference, and not the electromotive force, which produces it that is to be measured, the resistance or impedance of the source of the electromotive force does not enter into the problem.

In this case, if the potential difference is alternating, the impedance of the instrument may be presumed to increase with the deflection; consequently the current will not be proportional to the electromotive force. However, no limit need be stated or implied as to how nearly the current must be proportional to the applied potential difference, provided only that any increase in the potential difference causes an increase in the current.

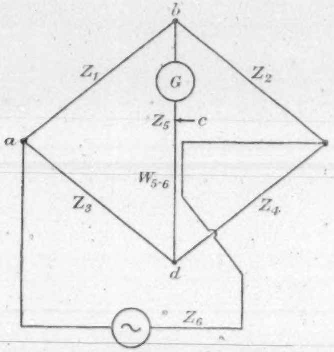

FIG. 7.—HUGHES BALANCE.

To show the applicability of the principle under consideration to somewhat more complicated problems it will be assumed that the currents are alternating. Here the symbolic notation will be used—that is, $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, etc., will be referred to as currents; $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, etc., will be referred to as electromotive forces or potential drops; $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., will be referred to as impedances; and $\mathbf{W}_{1-2}$, $\mathbf{W}_{1-3}$, $\mathbf{W}_{2-3}$, etc., will be referred to as conjunctances; but it should be understood that each is a complex quantity, or what in alternating-current theory is often called a vector.$^{(21)}$ Further, it should be understood that $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., each includes the "motional impedance,"$^{(22)}$ if present, as well as the electrical impedance, and $\mathbf{W}_{1-2}$, $\mathbf{W}_{1-3}$, $\mathbf{W}_{2-3}$, etc., each includes the effects of all types of coupling between the branches to which it pertains.

The solution of the problems will be left in the complex form, since to rationalize them would add a considerable amount of detail which is not required for the purpose at hand.

Problem 7.—To determine the current through the detector branch of the Hughes' balance(23) for which the circuit arrangement is that shown in Fig. 7. It is to be understood that the detector branch 5 and the generator branch 6 are connected by a mutual inductance, and that each arm may be inductive.

Let $\mathbf{E}$ be the electromotive force developed by the generator,

i_g be the current in the detector branch,

$\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, $\mathbf{Z}_5$ and $\mathbf{Z}_6$ be the impedances of the various branches ; and

$\mathbf{W}_{5\text{-}6}$ be the conjunctance between branches 5 and 6.

With the detector branch open between *b* and *c* the total current is

$$\frac{\mathbf{E}}{\mathbf{Z}_6+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)/(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the drop in potential from *d* to *c* is this current times $\mathbf{W}_{5\text{-}6}$, or

$$\frac{\mathbf{E}(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)\mathbf{W}_{5\text{-}6}}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

The current through branches 3 and 4 is

$$\frac{\mathbf{E}(\mathbf{Z}_1+\mathbf{Z}_2)}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the potential drop from *a* to *d* is this current times $\mathbf{Z}_3$. The current through branches 1 and 2 is

$$\frac{\mathbf{E}(\mathbf{Z}_3+\mathbf{Z}_4)}{\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)}$$

and the potential drop from *a* to *b* is this current times $\mathbf{Z}_1$.

The potential difference across the break between *b* and *c* is the potential drop from *a* to *d* plus the potential drop from *d* to *c* minus the potential drop from *a* to *b*, and, with the detector branch closed, the current is that which would be produced by an equal electromotive force in the detector branch, or

$$i_g=\frac{\mathbf{E}}{\mathbf{Z}_m\mathbf{Z}_n{}^2}[(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)\mathbf{W}_{5\text{-}6}+\mathbf{Z}_2\mathbf{Z}_3/\mathbf{Z}_1\mathbf{Z}_4] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\mathbf{Z}_m$ is the impedance of the detector circuit, and

$$\mathbf{Z}_n{}^2=\mathbf{Z}_6(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4)+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_3+\mathbf{Z}_4)$$

It should be noted that to this point no approximations have been made or conditions imposed. However, $\mathbf{Z}_m$ can be expressed readily in terms of $\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, etc., only when the bridge is very nearly balanced, or when the impedance of the generator circuit is very high in comparison with the impedance of the arms of the bridge. Then $\mathbf{Z}_m$ may be considered to be the same as though the generator branch were open, in which case

$$\mathbf{Z}_m=\mathbf{Z}_5+(\mathbf{Z}_1+\mathbf{Z}_3)(\mathbf{Z}_2+\mathbf{Z}_4)/(\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4).$$

Should a case arise in which the approximation made would not be justifiable it probably would be possible to measure $\mathbf{Z}_m$ directly.

Problem 8.—To determine the current through the detector branch of the Anderson bridge [(24)] when each branch is inductive. Referring to Fig. 8, let

$\mathbf{E}$ be the potential drop from *a* to *b* with the detector branch open,

$\mathbf{i}_g$ be the current through the detector branch, and

$\mathbf{Z}_1$, $\mathbf{Z}_2$, $\mathbf{Z}_3$, $\mathbf{Z}_4$, $\mathbf{Z}_5$, $\mathbf{Z}_6$ and $\mathbf{Z}_7$ be the impedances of the various branches.

With the detector branch open, it will be seen by inspection that the potential drop from *a* to *c* is

$$\mathbf{e}_1=\frac{\mathbf{E}\,\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}$$

The current through $\mathbf{Z}_4$ is

$$\frac{\mathbf{E}}{\mathbf{Z}_4+\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)/(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}=\frac{\mathbf{E}(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}{\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)+\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)}$$

and the part of this current passing through $\mathbf{Z}_7$ is

$$\mathbf{Z}_3/(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)$$

Therefore the current through $\mathbf{Z}_7$ times $\mathbf{Z}_7$, which is the drop in potential from *a* to *d*, is

$$\mathbf{e}_2=\frac{\mathbf{E}\,\mathbf{Z}_7\mathbf{Z}_3}{\mathbf{Z}_3(\mathbf{Z}_6+\mathbf{Z}_7)+\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}$$

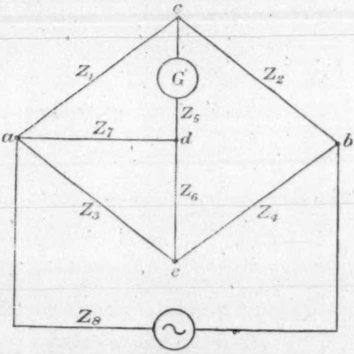

FIG. 8.—ANDERSON BRIDGE.

With the detector branch closed the current in it is e_1-e_2, considered as an electromotive force, divided by the impedance $\mathbf{Z}_m$ of the detector circuit, or

$$\mathbf{i}_g=\frac{\mathbf{E}}{\mathbf{Z}_m}\left[\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}-\frac{\mathbf{Z}_7\mathbf{Z}_3}{\mathbf{Z}_3(\mathbf{Z}_7+\mathbf{Z}_6)+\mathbf{Z}_4(\mathbf{Z}_3+\mathbf{Z}_6+\mathbf{Z}_7)}\right]$$

If the bridge is very nearly balanced, which is the only case of practical importance, or if the impedance of the generator branch is very small in comparison with the impedance of the arms of the bridge, in writing an expression for $\mathbf{Z}_m$ the branch points *a* and *b* may be considered to be connected by a conductor of negligible impedance. Referring to Fig. 9, it will be seen by inspection that

$$\mathbf{Z}_m=\mathbf{Z}_5+\frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}+\frac{\mathbf{Z}_7[\mathbf{Z}_3\mathbf{Z}_4/(\mathbf{Z}_3+\mathbf{Z}_4)+\mathbf{Z}_6]}{\mathbf{Z}_7+\mathbf{Z}_3\mathbf{Z}_4/\mathbf{Z}_3(+\mathbf{Z}_4)+\mathbf{Z}_6},$$

The suggestion made with reference to the determination of the impedance of the detector circuit in connection with Problem 7 also applies here. Further, it would be possible in any case by transforming both the delta *c a b* and the delta *a e d* to equivalent three-point stars to write from inspection an expression for the impedance of the detector circuit. In case **E** represents the electromotive force developed by the generator instead of the potential drop between terminals *a* and *b* of the bridge, it may be necessary to take into consideration the impedance of the generator branch. This adds some complications to the equations without serving any very useful purpose.

To show the applicability of the principle under consideration to a still more complicated problem, let it be assumed that the currents are transient or discontinuous functions of time.

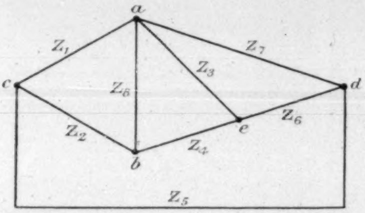

FIG. 9.—ANDERSON BRIDGE ARRANGED FOR DETERMINING IMPEDANCE OF DETECTOR CIRCUIT.

Problem 9.—To determine the average current in the galvanometer branch of a Maxwell commutator bridge[(25)] for which the circuit arrangement is that shown in Fig. 10. Let

C be the capacitance of the condenser,
A, *B* and *Y* be the resistances of the bridge arms,
G be the resistance of the galvanometer branch, and
H be the resistance of the battery branch.

Also let

E be the electromotive force of the battery, and
n be the number of times the condenser is charged and discharged (by the action of the commutator) per second, or the number of cycles per second.

With the galvanometer branch open, if conditions are such that steady states, following changes in position of the commutator, are reached in times less than half the period, it may be seen that at the point in each cycle at which the condenser branch is opened the condenser is charged to a potential difference of

$$E(A+B)/(A+B+H)$$

Therefore the average current through the condenser and Y branches is

$$nCE(A+B)/(A+B+H)$$

This current causes an average potential drop in the Y branch

$$e_y=YnCE(A+B)/(A+B+H) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

and in the battery branch of

$$HnCE(A+B)/(A+B+H)$$

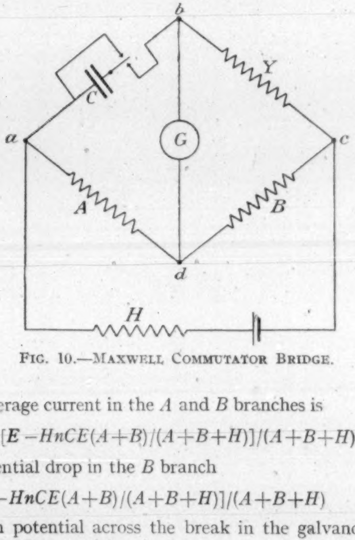

FIG. 10.—MAXWELL COMMUTATOR BRIDGE.

Consequently the average current in the A and B branches is

$$[E-HnCE(A+B)/(A+B+H)]/(A+B+H)$$

and the average potential drop in the B branch

$$e_b=B[E-HnCE(A+B)/(A+B+H)]/(A+B+H) \quad . \; . \; . \; . \quad (2)$$

The average drop in potential across the break in the galvanometer branch, considering the direction dGb as positive, is

$$e_g=e_b-e_y$$

or

$$e_g=\frac{E}{A+B+H}\left\{B-nCY(A+B)\left[1+BH/Y(A+B+H)\right]\right\} \quad . \; . \; . \; . \quad (3)$$

while during the greater part of each cycle, including the instant when the condenser branch is opened by the commutator, the potential drop across the break is

$$e_g'=EB/(A+B+H) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

To determine the average current in the galvanometer branch resulting from an electromotive force in this branch, equal to the potential drop which would appear across a break were the branch opened, it will be convenient to replace the delta ***dca*** of Fig. 11, which is equivalent to Fig. 10, by a star, as shown in Fig. 12. In this transformation (11)

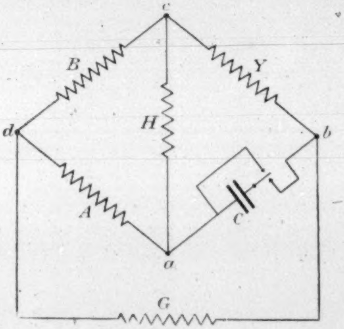

FIG. 11.—MAXWELL COMMUTATOR BRIDGE ARRANGED FOR DETERMINING THE CURRENT RESULTING FROM AN ELECTROMOTIVE FORCE IN THE GALVANOMETER BRANCH.

$$\left.\begin{aligned} P&=BA/(A+B+H)\\ Q&=HB/(A+B+H)\\ \text{and}\qquad S&=HA/(A+B+H)\end{aligned}\right\}\quad\ldots\ldots\ldots\ (5)$$

Referring to Fig. 12, it will be seen that at the instant in each cycle at which the condenser branch is opened an electromotive force e_g' in the galvanometer branch would cause a current

$$e_g'/(G+P+Q+Y)$$

in Q and Y, and therefore cause the condenser to be charged to a potential difference of

$$e_g'(Q+Y)/(G+P+Q+Y)$$

The average current through the condenser and S branches which would result from an electromotive force e_g' in the galvanometer is

$$i_c=nCe_g'(Q+Y)/(G+P+Q+Y)\quad\ldots\ldots\ldots\ (6)$$

and this causes an average potential drop in G and P of

$$(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)$$

Therefore the average current in Q and Y which would result from an average electromotive force e_g in the galvanometer branch which at the instant the condenser branch is opened is equal to e_g' is

$$i_y=\frac{e_g-(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)}{G+P+Q+Y}\quad\ldots\ldots\ldots\ (7)$$

The total average current in the galvanometer branch would be $i_g = i_c + i_y$, so that

$$i_g = \frac{nCe_g'(Q+Y)+e_g-(G+P)nCe_g'(Q+Y)/(G+P+Q+Y)}{G+P+Q+Y}$$

or

$$i_g = \frac{e_g+nCe_g'(Q+Y)^2/(G+P+Q+Y)}{G+P+Q+Y} \quad \ldots\ldots\ldots\ldots \quad (8)$$

Replacing P and Q by their equivalents as given by equation (5) and e_g and e_g' by their equivalents as given by equations (3) and (4) gives

$$i_g = E\frac{\left\{B-nCY(A+B)\left[1+\frac{HB}{Y(A+B+H)}-\frac{B^3H^2/(A+B+H)+2B^2HY+BY^2(A+B+H)}{Y(A+B)[(G+Y)(A+B+H)+B(A+H)]}\right]\right\}}{(G+Y)(A+B+H)+B(A+H)} \quad (9)$$

This gives as the condition of balance—that is, for $i_g=0$,

$$C = \frac{B}{nY(A+B)\left\{1+\frac{BH}{Y(A+B+H)}-\frac{B^3H^2/(A+B+H)+2B^2HY+BY^2(A+B+H)}{Y(A+B)[(G+Y)(A+B+H)+B(A+H)]}\right\}} \quad (10)$$

which differs in form only from the equation given by Thomson.(26) However, the supposition that a steady state is reached in a time less than half a cycle deserves consideration. The deflection of the galvanometer neither follows the current nor remains stationary. Consequently, there is an electromotive force generated in the galvanometer branch. The effect of this generated electromotive force is not included in equation (10), though the method of analysis followed lends itself readily to its determination. Further, there are reasons why another supposition generally made—namely, that a zero average deflection of the galvanometer is an indication that the average current through it is zero—should not be accepted without experimental proof.

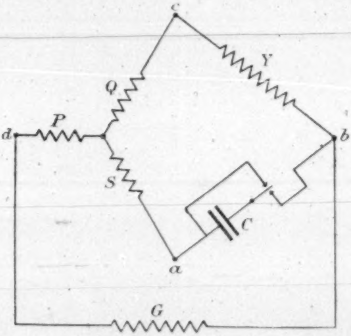

FIG. 12.—MAXWELL COMMUTATOR BRIDGE WITH THE DELTA *dca* OF FIG. 11 REPLACED BY THE EQUIVALENT STAR.

The problems considered should furnish a fairly definite idea as to the possible applications of the principle, while a comparison of the solutions given here with the solutions for the same problems as given in text-books, or as derived by any one of the more usual procedures, should show the advantage to be gained by its use.

Before bringing this Paper to a close, I wish to state a personal point of view regarding it. As I consider the matter, there is only one law, principle or theorem giving the relation between the electromotive forces and the currents in a system of conductors, if of the usual type, which should be considered as fundamental. This was first definitely stated by Ohm approximately 100 years ago, and is generally known as Ohm's law. This Paper was written not so much for the purpose of setting forth a new or little understood principle as of describing a procedure which I have been using in the application of Ohm's law to various problems, and of showing that this procedure possesses certain advantages in comparison with other procedures followed in the application of the same law to the same problems. The form in which the subject is presented here was adopted in the hope of bringing this procedure to the attention of undergraduate students of physics, engineers and those of us who either never have had or have forgotten that expert knowledge of determinants necessary for the expeditious handling of such problems by the "classical" method.

IV. SUMMARY.

1. A brief résumé is given of the procedures which have been proposed for determining the distribution of current in systems of linear conductors. In this connection reference is made to practically all of the laws, principles and theorems which pertain to this field of investigation.

2. A new, or at least not generally known, principle is discussed. This principle may be stated as follows: In a system of linear conductors in which the current in every branch is proportional to the impressed electromotive force, the current in any branch is that which would result should an electromotive force, equal to the potential difference which would appear across the break were the branch opened, be introduced into the branch and all other electromotive forces be removed.

3. A proof of this principle is given, and it is shown that—in some cases, at least—its use leads more directly to a solution for the current in a branch of a network than does the procedure generally referred to as the use of Kirchhoff's laws.

V. APPENDIX.

Specification for the System of Conductors.

The following is a more complete specification for the system of conductors than that given on page 126.

1. Each branch may contain a resistance, an inductance or a capacitance, or any two or more of these in series; a capacitance between parts of itself; may be inductively related to one or more of the other branches of the system; may be of sufficient cross-section so that the distribution of the current depends upon the time function of the impressed electromotive force; and may be so arranged as to move with respect to a constant magnetic field, thus developing a counter electromotive force. An example of the latter is a branch containing a galvanometer, a telephone receiver or similar apparatus.

2. The impressed electromotive forces may be arbitrary functions of time, and may have an arbitrary distribution throughout the system. For example, in one branch there may be a direct and constant electromotive force, in a second there may be an alternating electromotive force, in a third there may be an alternating electromotive force having an arbitrary phase relation to that in the second or a different frequency, in a fourth there may be a transient electromotive force, etc., or any two or more of the electromotive forces may be the same function of time or may be located in the same branch.

3. The current in each branch and the potential difference between each pair of points depends linearly upon all the applied electromotive forces. By this it is to be understood that should the introduction of an electromotive force $EF(t)$ into a particular branch cause changes in the currents amounting to $I_1F_1(t)$, $I_2F_2(t)$, $I_3F_3(t)$, etc., in the various branches, and cause changes in the potential differences amounting to $V_1f_1(t)$, $V_2f_2(t)$, $V_3f_3(t)$, etc., between various pairs of points, the introduction of an electromotive force $nEF(t)$ into the particular branch would cause changes in the currents amounting to $nI_1F_1(t)$, $nI_2F_2(t)$, $nI_3F_3(t)$, etc., in the same branches, and cause changes in the potential differences amounting to $nV_1f_1(t)$, $nV_2f_2(t)$, $nV_3f_3(t)$, etc., between the same pairs of points. However, one branch, if it is the only one in which the current is to be determined, may be of such a type as to cause a departure from proportionality between currents, potential differences and the applied electromotive forces.

For such a system of conductors, the principle and the proof for it applies as readily as they do when each branch contains resistance only, and all currents and electromotive forces are direct and constant.

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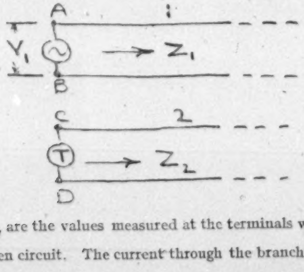
DISCUSSION.

Mr. **Rollo Appleyard**: Dr. Wenner's paper is a further elucidation of a theorem due to L. Thévenin (*Comptes Rendus* Vol. 97, 1883), and explained and generalised in relation to modern problems by K. S. Johnson (*Transmission Circuits for Telephonic Communication*, p. 79). The examples given by Dr. Wenner are valuable testimony of the simplicity of the method. To obtain confidence it is useful to begin with the case of a circuit consisting of a battery connected to a shunted galvanometer. First express the current in the battery-branch of resistance r, as the product of the galvanometer-current and the multiplying-power $(g+s)/s$, of the shunt. Then express the same current directly from Ohm's law—i.e., $E\Big/\left(r+\frac{gs}{g+s}\right)$. Equate these currents, and derive the galvanometer-current as the ratio of $E[s/(g+s)]$ to $[r+gs/(g+s)]$, which is the same as the ratio of $E[s/(r+s)]$ to $[g+rs/(r+s)]$. This is seen to be the ratio of the potential-difference that would exist across the galvanometer-terminals if the galvanometer-branch were cut, to the resistance made up of the galvanometer and the shunted battery circuit when the battery is replaced by a resistance equal to the battery-resistance, the electromotive-force of the battery being imagined suppressed. By adding branches one at a time, a complex network can be treated in like manner, and assurance is gained that to find the current in a given branch it suffices (1) to suppose the branch cut; (2) to express the potential-difference between the terminals of the cut branch; (3) to express the total resistance made up of the branch itself, and of the resistance between its terminals when the branch is cut and the electromotive-forces are suppressed; (4) to divide the result of (2) by the result of (3). In the generalisation, capacities, inductances and leakances can be treated as resistances by the well-known device.

Mr. G. W. **Sutton** questioned the value of the method for teaching purposes. The old methods were admittedly more laborious, but it was in practice necessary to limit the number of principles which have to be explained to an ordinary physics class.

Dr. A. **Russell** said that in his opinion the author's method gives certain classes of results in a tenth of the time taken by direct methods.

Dr. A. **Rosen** (communicated): The theorem given by Dr. Wenner can be stated in a slightly different form as follows: The current in any branch of a network may be determined by replacing the remainder of the network by a generator whose E.M.F. is V_g and internal impedance Z_g; V_g and Z_g are the values measured at the terminals when the branch is opened, i.e., the generator is on open circuit. The current through the branch of impedance Z is $\frac{V}{Z+Z_g}$.



In this form the case is analogous to that of a real generator, which helps to make it easier to remember and apply. The theorem is useful in the theory of telephone transmission, and I am indebted to Mr. K. Haemers, of the Belgian Post Office for the following illustration :—

The interference between the circuits 1 and 2 in the figure is caused by a complex series of electrostatic and electromagnetic couplings, and is measured by the ratio of the power in the disturbed to that in the disturbing circuit when the former is closed by an impedance equal to its own, i.e., Z_2. The interference may be expressed as an attenuation factor b, where $\frac{P_2}{P_1} = e^{-2b}$. It is required to find the current in the telephone T when there is a voltage V_1 across AB. Let the p.d. at the terminals of circuit 2 when the branch is open be V_0. Then when closed by Z_2 the current is $\frac{V_0}{2Z_2}$. Hence $P_1 = \frac{V_1^2}{Z_1}$, $P_2 = \frac{V_0^2}{4Z_2}$ and $e^{-2b} = \frac{V_0^2}{4V_1^2} \cdot \frac{Z_1}{Z_2}$. When closed by a telephone of impedance Z_t, the current is $I_t = \frac{V_0}{Z_2+Z_t} = \frac{2V_1}{Z_2+Z_t} \cdot \sqrt{\frac{Z_2}{Z_1}} \cdot e^{-b}$

Mr. A. Campbell (communicated) : All who are interested in electrical measurements are deeply indebted to Dr. Wenner for the many illuminating and helpful papers which he has already published ; the present Paper adds to the list and it is clear, from the many interesting examples which he deals with, that his system of working will be most useful to many workers who do not care for long lines of simultaneous equations. (Could Dr. Wenner kindly apply it to the Carey-Foster network ?) To test the working of the system I looked into Problem 6. It may be worked out in the ordinary way as follows :—

Let g = resistance of instrument,
i = current through instrument,
and I = total current through A.

We are given that

$$(g+20)i = 0{\cdot}15 \qquad (1)$$

and

$$AI + gi = 1{\cdot}5 \qquad (2)$$

We have also

$$\frac{i}{I} = \frac{B}{B+g} \qquad (3)$$

By eliminating i and I we obtain

$$AB - 200B = g(9B - A).$$

If this condition is to be true *for all values of g* then

$$AB - 200B = 0$$

and

$$9B - A = 0$$

or

$$A = 200 \text{ ohms},$$

and

$$B = 22{\cdot}22 \text{ ohms},$$

the result that Dr. Wenner obtains in two lines ! The result in itself is interesting, for it holds good no matter what resistance the instrument may have.

X.—A CAPACITANCE BRIDGE OF WIDE RANGE AND A NEW INDUCTOMETER.

By Albert Campbell, M.A.

Received November 16, 1926.

Abstract.

A bridge is described by which quick measurements can be made of capacitances covering a range of from $1\mu\mu F$ up to $30\mu F$, the power factor also being indicated. The unknown capacitance C is put in parallel with a resistance P, and the effective self-inductance of the combination, which is approximately equal to $-P^2C$, is read on a mutual inductometer forming part of the bridge. By giving P a series of suitable values scale multipliers providing for a very wide range of capacitance are obtained. The inductometer used is of a new type, having a circular scale extending to about 260°, the percentage accuracy of reading being almost constant over the greater part of the range. This novel scale system allows the lower readings to be taken with good accuracy. A small rheostat allows the power loss in the condenser to be balanced, and enables the power factor to be deduced.

§ 1. Introductory.

For the accurate measurement of capacitance the author many years ago[*] introduced Heydweiller's modification of Carey Foster's method, using as standard a mutual inductometer calibrated against a mutual inductance calculated from its dimensions. This method still holds its place as one of the best for precision measurements of capacitance and power factor; but it requires a rather large set of resistances if a wide range of values are to be dealt with. For tests which do not require the highest accuracy the author has designed a simple self-contained bridge which affords a quick and convenient means of measuring capacitances over a long range ($1\mu\mu F$ up to $30\mu F$), also indicating the power factor with fair accuracy except in extreme cases.

§ 2. Theory of the Method.

The method consists in placing the unknown condenser C in parallel with a known non-inductive resistance P, and reading the effective self-inductance and resistance of the combination by means of a bridge containing a mutual inductometer. Let the combination be represented as in Fig. 1, where the series resistance x causes the power loss in the condenser.

Thus, if the pulsatance $\omega \doteq 2\pi f$, where f is the frequency, the effective resistance P' and inductance L' are given by the equations

$$P'=\frac{P+Px(P+x)C^2\omega^2}{1+(P+x)^2C^2\omega^2} \quad \ldots\ldots\ldots\ldots (1)$$

and

$$L'=\frac{-P^2C}{1+(P+x)^2C^2\omega^2} \quad \ldots\ldots\ldots\ldots (2)$$

In most cases in practice all the terms in ω^2 are much smaller than 1, and then the approximate values are

$$P' \doteq P[1-P(P+x)C^2\omega^2] \quad \ldots\ldots\ldots\ldots (3)$$

and

$$L' \doteq -P^2C[1-(P+x)^2C^2\omega^2] \quad \ldots\ldots\ldots\ldots (4)$$

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[*] Proc. Phys. Soc., Vol. 20, p. 626 (1907).

When ω, P, P' and L' are known, these equations suffice to determine C and x, and hence the power factor $Cx\omega$. By suitably choosing ω and P the term in ω^2 in equation (4) can usually be made negligible (except for very leaky condensers), and then we have

$$L' = -P^2C, \qquad (5)$$

or,

$$C = L/P^2, \qquad (6)$$

where

$$L = -L'.$$

Thus the capacitance C is obtained directly by multiplying the reading L of the inductometer by the factor $1/P^2$. This factor is conveniently chosen to be some power of 10, so making the scale read the capacitance practically directly.

Using the approximation of equation (6) we have from equation (4)

$$\text{Power factor} = Cx\omega,$$

$$\doteq \frac{P - P'}{L\omega} - CP\omega \qquad (7)$$

Unfortunately, as in some of the other methods (e.g., Carey Foster's), the power factor, if very small, comes out as the difference of too much larger quantities.

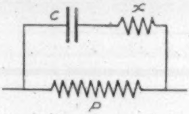

FIG. 1.

The present method, however, has been designed for the convenient determination of capacitance, and treats power factor only as a secondary consideration.

If the resistance P is not non-inductive, but has a small self-inductance l, then, when $s = o$, we have

$$P' = \frac{P}{(1 - \omega^2 lC)^2 + P^2C^2\omega^2} \qquad (8)$$

and

$$L' = \frac{l(1 - \omega^2 lC) - P^2C}{(1 - \omega^2 lC)^2 + P^2C^2\omega^2} \qquad (9)$$

In the actual bridge, however, l can be kept so small that these equations are unnecessary in general.

§ 3. GENERAL ARRANGEMENT OF THE BRIDGE.

The bridge (in the present model) is arranged, as in Fig. 2, with equal ratio arms ($R = S$), P and Q being equal resistances in the other arms, which also include H and K, the two sections of the secondary winding of the mutual inductometer, which are made identical in resistance and self-inductance. The primary circuit of the inductometer enters the upper corner of the bridge through the slider of the potentiometer rheostat B, which has sufficient range to balance the small change in effective resistance when the condenser C is put across the terminals of P (or of Q). The inductometer scale is marked to read L directly, usually extending from -10 to $+105$ microhenries. In the actual instrument the resistances P and Q can be

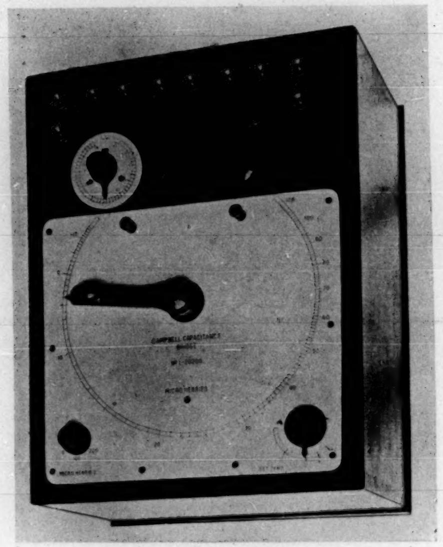
CAMPBELL CAPACITANCE BRIDGE
MICROHENRIES

FIG. 3.—CAMPBELL CAPACITANCE BRIDGE

simultaneously altered by a step-switch, giving a series of values, 3·162, 10, 31·62, 100, 316·2, and 1,000 ohms, which give scale ranges of 100$\mu\mu$F, 1,000$\mu\mu$F . . . up to 10 microfarads.

Two fixed steps on the inductometer carry each range to three times the above, giving a total range from (say) 1$\mu\mu$F to 30μF. The rheostat B has its scale marked to read directly changes in P or Q, and can be shunted so as to increase the accuracy of reading tenfold. To make a measurement, the range-setting switch is turned to give the desired range, and a balance (in the telephone or vibration galvanometer G) is obtained by setting B and a small zero-setting L-dial, the main L-dial reading zero. The condenser C is then put across P, and a new balance obtained by altering B and the main inductometer. Then the multiplied inductometer reading gives C, and the change in the rheostat reading gives $(P-P')$, and hence the power factor. In many cases the junction of the ratio arms is a convenient point to earth.

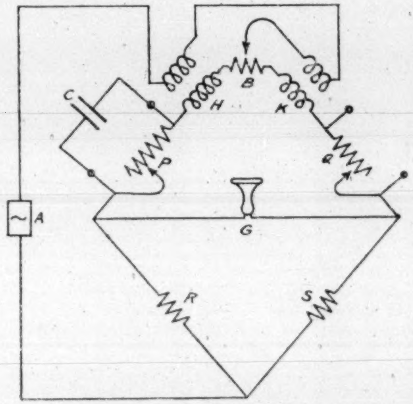

Fig. 2.—Capacitance Bridge.

Two condensers not very unequal can be compared by putting the larger across P and the smaller across Q.

The actual instrument, which is made by Messrs. The Cambridge Instrument Co., is illustrated in Fig. 3.

§ 4. A NEW INDUCTOMETER.

The accuracy of measurement in this capacitance bridge has been greatly increased by the use of a new type of mutual inductometer designed by the author.* The scale of this new type embodies two improvements :—

(1) Its angular extent is very great, being about 260°, older types giving only about 160°.

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* A. Campbell, British Patent Specifications, 244,596 and 252,990.

(2) The scale gradually opens out towards the zero, in such a way that over the greater part of the range *the percentage accuracy of reading is* ***nearly constant.*** From Fig. 4 it will be seen how nearly this condition is fulfilled from 10 up to 100 μH; there is a slight intentional opening towards the top, in order not to have the divisions too close there, the minimum length being 1·8 mm. per division.

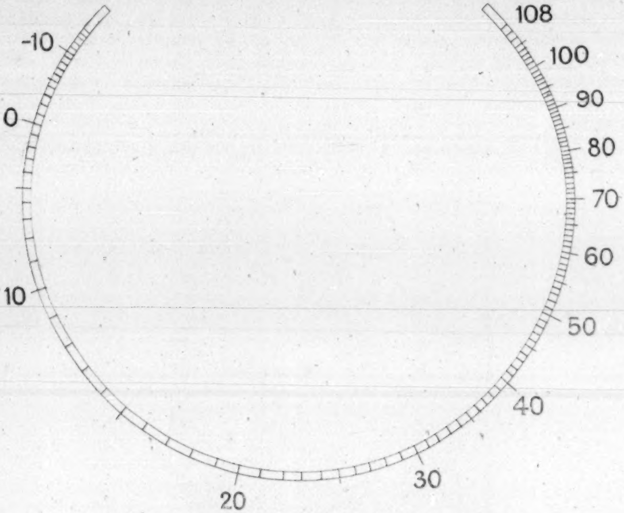


Fig. 4.—Scale of New Campbell *M* Inductometer.

The importance of constant percentage accuracy of reading does not seem to have been realised by many instrument makers and users, and it is very often difficult of attainment. The equation for a scale of constant percentage accuracy is

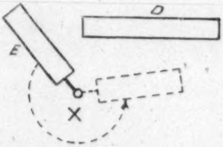


Fig. 5.

$$M = a\varepsilon^{\theta},$$

where M is the mutual inductance at angular reading θ, ε the base of natural logarithms, and a a constant; but such a scale could never get down to zero until

$\theta = -\infty$. So, for a practical scale, we must be content to close it in gradually at the lower end so as to reach zero and negative values.

In the present instance, the ideal scale has been attained by constructing the inductometer as shown in plan in Fig. 5, where the fixed and moving circuits are the vertical coils D and E respectively, E being mounted on a vertical axle X carrying the pointer.

§ 5. Limits of Accuracy.

In the Capacitance Bridge, except at the beginning of the very lowest range, the accuracy of reading is within 2 parts in 1,000 throughout. With the two highest ranges (or for very large power factors) the correction shown in equation (3) is sometimes required. For example, with frequency of 800∿ per second for $C=1\mu\mathrm{F}$, the correction is about 2 in 1,000, and for $C=30\mu\mathrm{F}$ it becomes quite excessive. This difficulty with the very large condensers can be avoided by making the tests at a lower frequency, such as 100∿ per second. In conclusion, it may be remarked that the system of shunting used in the bridge involves considerable loss of sensitivity, but this disadvantage appears to be well outweighed by the convenience and ease of working of the instrument.

DISCUSSION.

Mr. D. W. Dye said that it seemed paradoxical at first sight that a high impedance could be more accurately measured when diluted with a shunt, but, as in a method described by himself in which the impedance to be measured is shunted by a resistance of 100,000 ohms, the dilution had the effect of making an accurate measurement practicable. The new scale of the instrument, which was substantially that of a slide rule, was particularly valuable, and might be applied with advantage to other instruments. Would it be possible to make the inductometer astatic, in order to avoid the errors introduced by variation of the earth capacity of the telephones ?

Dr. E. H. Rayner said that the proposed method of measuring power factor was particularly interesting in view of the importance of the power factor in determining the behaviour of cables under high voltages. The volt-amperes applied to an ordinary cable circuit amount to about 200 times the real power, the power factor being nearly 90 degrees, and in measuring this factor a sensitivity of 0·0001 radian at 20,000 volts is desirable.

Mr. Rollo Appleyard said that the instrument would be of great value to electricians if it could be had at a moderate price. The scale was that of the aneroid barometer, and in effect it was the same scale as that originally used by Cavendish when he first compared the capacity of a battery of Leyden jars with that of a small plate condenser by repeated sharing of charges.

Mr. G. W. Sutton asked whether it would be possible to design the instrument so as to avoid the effect of capacity between the primary and secondary of the mutual inductance.

Author's Reply.—In reply to Mr. Dye: The procedure in my method of testing condensers is quite the same as that employed in Mr. Dye's method of testing large self-inductances (which, I may remark, is probably the most accurate of all known methods for that purpose) ; but the two methods depend on quite different formulæ. Thus, the condenser method is practically direct reading and independent of frequency, while in the self-inductance method the results have to be deduced by calculation involving the frequency.

With regard to astaticism, it is not very difficult to obtain moderate astaticism in an inductometer of the new type, but I think that astaticism almost always involves loss of efficiency (e.g., by increasing the weight of wire used). In the present model of the capacitance bridge, I have not found the effect of stray fields troublesome, so long as the generator is kept sufficiently distant. In reply to Mr. Sutton, I would point out that in the capacitance bridge the maximum mutual inductance used is of the order of $50\mu\mathrm{H}$, and hence the internal capacitances of the instrument have very little effect. In higher-reading inductometers of the new type the troublesome effects of capacitance are minimised by suitable design of the coils. It is interesting to learn from Mr. Appleyard's remarks that the ordinary aneroid barometer has a scale of constant percentage accuracy ; this appears to be due to its inherent law of working, while my inductometer scale is the result of design involving a great number of experiments. I would thank all the speakers for their kind remarks of appreciation.

XI.—ON THE SPECTRA OF DOUBLY-IONISED GALLIUM AND INDIUM.

By K. R. Rao, *M.A.*, Madras University Research Scholar.

Received October **20, 1926.**

(*Communicated by* **Prof. A. Fowler**, *F.R.S.*)

Abstract.

The Paper gives an account of experimental work which has led to an extension of our knowledge of the second spark spectra of In and Ga, into the region of long wave-lengths. After making a careful study of the spark spectra of the elements in hydrogen and air under different conditions of excitation the author has identified the lines corresponding to the second series in the spectra of doubly-ionised Indium and Gallium. The principal features of the spark spectra are now known, and all these structures are in beautiful accord with the quantum theory of spectral line emission.

Introduction.

According to Bohr's theory of spectra and atomic structure the arc and spark spectra of an element are quite distinct, arc lines being emitted by the interorbital transitions of the valence election in the neutral atom, while spark lines result in the energy changes in atoms that have lost one or more of their outermost electrons. Recent researches of Fowler and Paschen have shown that, in passing from arc to spark lines, the series constant is changed from N to $4N$, $9N$, etc., quite in keeping with Bohr's theory.

One of the main approaches to the problem of atomic structure is by the identification of wave-lengths in arc and spark spectra of different elements, and their representation by formulæ. The main difficulty in this work is that in general the experimental arc and spark spectra of any particular element contain many lines in common; the true spark lines are often excited in an ordinary electric arc, while arc lines appear also in high potential sources such as condensed sparks.

According to the spectroscopic displacement law the doubly-ionised spectra of Ga and In are expected to consist of series of doublets with a Rydberg constant $9N$. Owing to the high value of the constant the primary series fall in the extreme ultra-violet, while the secondary series lines fall in the region above $\lambda 2200$. After the publication of the work of Weinberg(1) on the spark spectra of Ga and In, and of Lang(2) on Tl, a study of the series in the spectra of doubly-ionised Ga,(8) In and Tl was undertaken. When a partial analysis of the second spark spectra of Ga and In had been made, Carroll communicated a Paper(6) on "Vacuum Spark Spectra of some of the heavier elements and series classifications in the spectra of ionised atoms homologous with Cu, Ag and Au." As the classifications of Carroll were mainly confined to the primary series, the author's efforts were concentrated on the study of the spark spectra in the visible and ultra-violet regions up to $\lambda 2200$, to identify the wave-lengths of Ga III and In III, and to establish, if possible, other series relationships. Though only a few leading members of the series have so far been detected, it is thought desirable to publish the present account, as it might possibly facilitate the investigation of series in the spark spectra of succeeding elements of the periodic table.

EXPERIMENTAL PROCEDURE.

To provide data likely to be useful in identifying the spectra of these elements at higher stages of ionisation, a study was made of the spark spectra of the elements in hydrogen, in air, and in vacuum, under different conditions of excitation. In the case of Indium the arc in vacuum also was examined. The metal in each case was contained in a small cavity made at the end of an aluminium rod, which served as one electrode, while the other electrode was a second aluminium rod made to taper at the end. The spectra were photographed before and after the introduction of the metal, and these helped in the elimination of lines due to aluminium or other impurities. To obtain the enhanced spectra of these elements a large X-ray coil capable of giving a 10 in. spark in air was used, employing a mercury interrupter in the circuit, and the primary was fed with a current of 15 amps. The secondary contained a battery of large Leyden jars of capacity about 0·02 mfd. in parallel with the spark-gap. An auxiliary spark-gap in air was also placed in series with this. In order to classify the lines and assign them to the various stages of ionisation of the element in question the degree of excitation was largely varied. This was attained by inserting a varying self-induction in the secondary circuit or changing the condenser capacity, or adjusting the pressure of the hydrogen in the spark chamber. Certain strong lines cease to be produced with the smaller intensities of discharge, and, as such, are assigned to higher stages of ionisation. The effect of self-induction is clearly seen from the Plates 1 and 2. In general the lines are intense and diffuse in the spark in air spectra, while in an atmosphere of hydrogen they are sharp and well-defined.

To investigate the region from $\lambda 6000$ to $\lambda 3500$ an ordinary Hilger constant deviation spectrograph and a plane grating spectrograph of about 15000 lines to the inch were used, while for the ultra-violet a large Hilger quartz spectrograph was utilised.

For measuring the wavelengths, the iron arc and Zn-Cd spark were used as standards. In the case of Indium, the wave-length measurements obtained by the author were found to differ from the values given by Schulemann(4) consistently by about 0·1 A.U. As Schulemann worked with a 10 ft. concave grating his wave-lengths were taken for the lines referred to. After the completion of this work Carroll's Paper containing wave-lengths for Gallium and Indium appeared. In the case of Ga these recent measurements of Carroll have been substituted in place of the writer's measurements.

NOTATION AND NUMERATION OF TERMS.

The notation adopted in the present Paper is generally similar to that used by Prof. Fowler(5) in his Paper on ionised oxygen, the respective inner quantum numbers being used as subscripts in the case of multiple terms. The numeration of series terms is given on the new system introduced by Bohr on theoretical grounds. In the consideration of series, however, the azimuthal component of the total quantum number is sufficiently represented by the symbols s, p, etc. These symbols are adopted here for convenience of printing, as all the terms are doublets, but it is to be understood that in the generalised notation, s is equivalent to $^2S_1$, p_2 to $^2P_2$, and so on.

The Series System of In III.

In accordance with the displacement law the spectrum of doubly-ionised Indium bears a general resemblance to the series of Ag I and Cd II. The lines which are considered to belong to this series are brought together in the table below with their wave-numbers in vac., intensities and separations. The last column gives the designation assigned to each line.

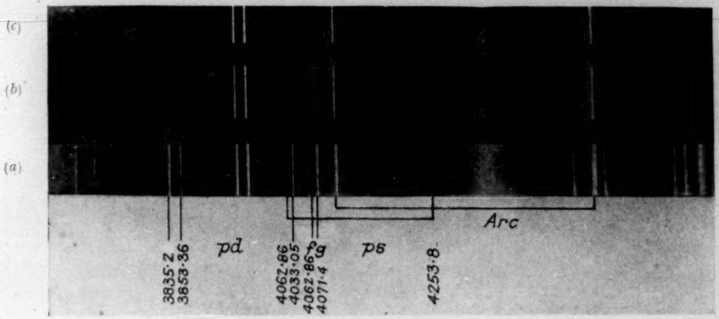
λ I.A.	Int.	ν (vac.)	$\Delta\nu$	Series designation.
5644·86	6	17710·3	1337·4	$6s-6p_1$
5248·52	8	19047·7		$6s-6p_2$
4253·88	5	23501·4	1337·4	$6p_2-7s$
4024·83	4	24838·8		$6p_1-7s$
4062·86	4	24606·3	181·8	$6p_2-6d_2$
4033·05	8	24788·1	1337·8	$6p_2-6d_3$
3853·36	7	25944·1		$6p_1-6d_2$
5918·54	5	16891·4	184·5	$6d_3-5f$
5854·58	3	17075·9		$6d_2-5f$
3008·30	10	33232·0	281·0	$5d_3-4f$
2983·04	8	33513·0		$5d_2-4f$
4071·40	6	24554·0		$4f-5g$

The work of Fowler, Paschen, Millikan and Bowen on series in the spectra of ionised atoms indicates that there is generally a regular displacement of corresponding lines towards the region of shorter wave-lengths in passing from Na I to Cl VII. A comparison with the already located corresponding members of Ag I and Cd II suggested that the pair $6s-6p_{21}$ of the second principal series falls in the region λ5400. Further, an analogy between the doublet separations of Na I, Mg II, etc., and those of Ag I and Cd II led to the following values in the case of In III,

$$5p_1-5p_2=4600 \qquad 6p_1-6p_2=1400$$

Repeated observations of the spark spectra of Indium in the visible region indicated the presence of the pair λ5645 (6) and λ5249 (8) with $\triangle\nu=1337{\cdot}4$. The pair is intense and prominent under the maximum excitation and is easily suppressed by the insertion of self-induction, and, as such, is believed to belong to In III. The relative order of the intensities show that it must clearly be a principal pair. The detection of two other pairs having the above frequency separation and persisting under the same conditions as the above doublet confirmed the identification of the principal pair. The magnitude of the intensities and the presence of the satellite shows that the third doublet in the above table is the first member of the diffuse secondary series. This fixes the separation of the $6d_{32}$ terms of the series. A further search revealed the pair $6d_{32}-5f$ with this separation.

The first fundamental series is generally very prominent in the spectra of ionised atoms, and is perhaps the easiest to fix upon. The f term being "hydrogen-like" is calculable and the sequence of $5d$ terms in Ag I, Cd II, fixes the position of the member $5d_{32}-4f$ in the spectrum of In III. This led to the identification of the only


(c)
(b)
(a)
Arc
3835·2
3853·36
pd
4062·86
4033·05
fg
4062·86
4071·4
ps
4253·8

(*a*) (*b*) (*c*)—Spectra with increasing self-induction.

FIG. 1.

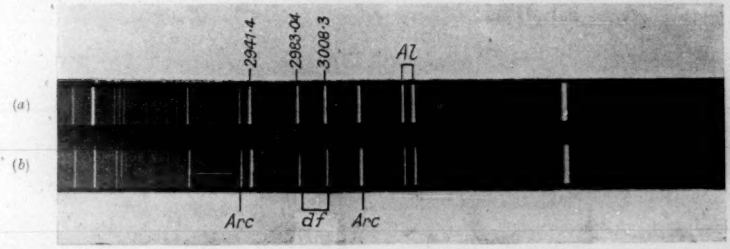

2941·4
2983·04
3008·3
Al
(a)
(b)
Arc
df
Arc

FIG. 2.

PLATE I.—SPARK SPECTRUM OF INDIUM.

To face page 152]

strong pair λλ3008, 2983, in this region, which, from an examination of the various spectra, seems definitely to belong to In III. No satellite has, however, been detected. The f level may be single or, if double, the levels must be too close to be resolved on these plates. The $4f-5g$ line is next considered. In the appropriate region there is a line λ4071, which is clearly of the second stage of ionisation. If this be $4f-5g$ a probable value of 39600 for $5g$ will lead to a value of the $4f$ term, which, perhaps, may be a little too high in the sequence of terms from Ag I to In III. Still as this line appears along with the other classified pairs, as there is no other line within about 20 A.U., particularly on the long wave-length side of it, which may be said to belong to In III, and as corresponding lines in the case of the elements (Al III and Ga III) of the same sub-group of the periodic table have been excited with sufficient intensity under similar conditions, the above identification has been adopted. This line has not been recorded by Schulemann, but is present on all the Indium plates taken here. The wave-length is carefully measured by taking the oxygen triplet λλ4075·94, 4072·40, 4069·93 as standards.

CALCULATION OF SERIES LIMITS.

Owing, perhaps, to the rapid fall in intensity in the successive members of the series, no observed series is long enough to permit the use of any fairly accurate series formula for the calculation of limits. They are, therefore, determined on the assumption of the values for the remote levels. As Carroll did not fix the super-fundamental series line $4f-5g$, he assumed the value of f for the evaluation of the remaining terms. But as this member is now identified a recalculation of the term values is made by using the value of the more remote level g. The table below gives the term values and the effective quantum numbers.

Designation.	Term Value.	Effective Quantum No.
$5s$	226132	2·089
$6s$	101051	3·126
$7s$	58501	4·108
$5p_2$	164605	2·449
$5p_1$	168950	2·417
$6p_2$	82003	3·469
$6p_1$	83340	3·442
$5d_3$	97387	3·184
$5d_2$	97664	3·180
$6d_3$	57214	4·145
$6d_2$	57396	4·138
$4f$	64154	3·913
$5f$	[40320]	4·955
$5g$	39600	4·999

The value of the limit of $6p$ of the stronger member of the series $6p_{21}-6s$ is calculated from the first two members by the use of a simple Rydberg formula. The adoption of $9N$ for the series constant in the calculation leads to term values which are consistent with the sequence of terms of Ag I and Cd II. This justifies that the series system belongs to the second stage of ionisation.

	ν
$6p_2-6s$	−19047·7
$6p_2-7s$	23501·5

From these, $6p_2=81102$.

This limit agrees fairly with the value of $6p_2$ obtained as above on the assumption of g.

The terms $6s$ and $5p_{21}$ give for the positions of the pair of the first sharp series :—

	ν	Δν
$5p_2-6s$	63554	4345
$5p_1-6s$	67899	

A search for this pair is made among the published wave-lengths of vacuum-spark spectra of Indium. No suitable pair is, however, found in the calculated position. The lines λ1571·5 (1) and λ1472·4 (5) are in the approximate position, but the intensities and the separation are not satisfactory. The pair,

λ	Int.	ν	Δν
1599·5	4	62520	4365
1495·1	2	66885	

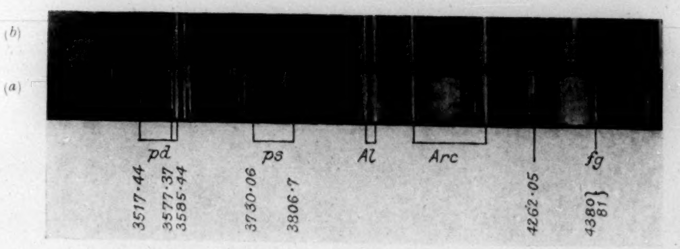
has perhaps the appropriate separation, but it is a little too remote from the calculated position. It may be that the equality of separation is fictitious.

GALLIUM.

The identification of the series of In III considerably facilitated the working out of the series of doubly ionized Ga. The separation $5p_{21}$ of the principal pair is now approximately estimated to be about 520. The sequence of corresponding members of Cu I, Zn II, and the position of the pairs in In III suggested the classification of the first two pairs in the following table, which is self-explanatory. All the lines are found to belong to Ga III. The diffuse pair and satellite long eluded detection. There are two lines λλ3577 and 3517, exactly in the required position, having the separation $\triangle\nu=476$. The intensities, their position and behaviour suggested that they may be classed as $5p_2-5d_3$ and $5p_1-5d_2$. If this be correct, the satellite $5p_2-5d_2$ must have a frequency 27883. This was then searched for.

λ I.A.	Int.	ν (vac.)	Δν	Series Designation.
−4994·15	7	−20018·0	538·5	$5p_1-5s$
−4863·19	5	−20556·5		$5p_2-5s$
3806·72	5	26261·9	539·7	$5p_2-6s$
3730·06	4	26801·6		$5p_1-6s$
(3585·44)*	...	(27882·7)	62·9	$5p_2-5d_2$
3577·37	7	27945·6	539·0	$-5d_3$
3517·44	6	28421·7		$5p_1-5d_3$

* Taken from Carroll's data.


(b)
(a)
pd
ps
Al
Arc
fg
3517·44
3577·37
3585·44
3730·06
3806·7
4262·05
4380
81

(a)—Without self-induction. (b)—With self-induction.

FIG. 1.

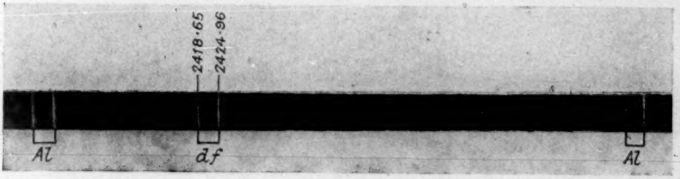

2418·65
2424·96
Al
df
Al

FIG. 2.

PLATE II.—SPARK SPECTRUM OF GALLIUM

To face page 154]

But the line λ3586, being the first fundamental series line of Al II, is found on all plates to be very intense and diffuse and is, therefore, considered to have masked the presence of the faint satellite. The recently published measurements of Carroll⁽⁶⁾ have confirmed this view. The satellite is included in the table on p. 154.

SERIES LIMITS.

As in the case of Indium, no series is extended far enough to justify the use of any accurate formula. The term values are calculated from the frequencies of the members identified by assuming the value of 5*s* obtained by Carroll from a series of pairs from 4*s*–4*p* to 4*f* – 5*g*. The term values and effective quantum numbers are tabulated below.

Designation.	Term Value.	Effective Quantum Number.
5*s*	107064	3·037
6*s*	60245	4·048
$5p_2$	86507	3·378
$5p_1$	87046	3·368
$5d_3$	58561	4·106
$5d_2$	58624	4·104

The value of $5p_2$ thus obtained agrees fairly with the value 87092 got by the application of an approximate Rydberg formula to the first two members of the second sharp series. The adoption of 9*N* for the value of the series constant and the representation of the series system are justified.

Details as to the various series of In III and Ga III which have so far been traced (including the work of Carroll) are given in Tables A and B at the end, arranged on the plan adopted by Fowler in his "Report on Series."

COMPARISON OF Ag I, Cd II, In III AND Cu I, Zn II, Ga III.

Doublet Separations.—The table contains the doublet separations for the series of each of these groups of elements of similar electronic structure.

Element.	Atomic Number.	1st *p* Separation.	2nd *p* Separation.
Cu I	29	248·0	...
Zn II	30	872·2	245·35
Ga III	31	1716·0	539·0
Ag I	47	920·6	202·9
Cd II	48	2482·8	672·82
In III	49	4345·0	1337·4

The curve connecting log $\triangle\nu$ and log (squares of net nuclear charges) is found to be very approximately linear.

Millikan and Bowen's application of Sommerfeld's fourth-power law for X-ray **doublets** to the field of optics shows that the separation is given by $\Delta\nu = \xi\left(\frac{Z-s}{n}\right)^4$ with the usual notation. The variation of the screening constant "s" as we pass from Ag I to In III is illustrated below.

Z	Element.	$\Delta\nu$	$\sqrt[4]{\Delta\nu/0\cdot0135}$	s
47	Ag I	202·9	11·08	35·82
48	Cd II	672·82	14·94	33·06
49	In III	1337·4	17·74	31·26

Corresponding Lines.—The regular displacement of corresponding lines in the case of elements of like electronic structure is shown by a comparison of these spectra :—

	$6s-6p_2$	Diff.	$6p_2-6d_3$	Diff.
Ag I	5946	6447	5715	9149
Cd II	12393	6655	14864	9924
In III	19048		24788	
	$5s-5p_2$	Diff.	$5p_2-5d_3$	
Cu I	6246	6928	6008	10374
Zn II	13174	7374	16382	11564
Ga III	20556		27946	

Term Values.—The s and p terms are found to diminish in these sequences as in the case of Na I to Cl VII. But the d terms also are found to regularly decrease.

	$6s$	$7s$	$6p_2$	$5d_3$	$6d$	$4f$
Ag I	18540	9209	12596	12371	6881	6892
Cd II	13347	(7300)	10248	11633	6532	6986
In III	11228	6500	9111	10821	6357	7128
	$5s$	$6s$	$5p_2$	$4d_3$	$5d$	
Cu I	19171	9460	12925	12366	6917	
Zn II	14114	7598	10820	11983	6725	
Ga III	11896	6694	9612	11512	6507	

COMPARISON OF Ga III AND In III.

As elements belonging to the same sub-group of the periodic table, they exhibit progressive spectral differences. The doublet separations are roughly proportional to the squares of their atomic numbers. The limits of the sharp and principal series, as is evident from the foregoing tables, are displaced towards the red with increase of atomic number.

TABLE A.—INDIUM.

First Principal Series : $6s=101051$

Classification.	λ I.A.	Int.	ν	Δν	*m*	mp_{21}
$6s—6p_2$	5248·52	8	19047·7	1337·4	6	82003
$6s—6p_1$	5644·86	6	17710·3			83340

Second Sharp Series : $6p_2=82003$
$6p_1=83340$

						ms
$6p_2—6s$	−5248·52	8	—19047·7	1337·4	6	101051
$6p_1—6s$	−5644·86	6	—17710·3			
$6p_2—7s$	4253·88	5	23501·5	1337·4	7	58501
$6p_1—7s$	4024·83	4	24838·8			

Second Diffuse Series : $6p_2=82003$
$6p_1=83340$

						md_{32}
$6p_2—6d_2$	4062·86	4	24606·3	181·8	6	57214
$6p_2—6d_3$	4033·05	7	24788·1	1337·8		57396
$6p_1—6d_2$	3853·36	8	25944·1			

Second Fundamental : $6d_3=57214$
$6d_2=57396$

						mf
$6d_3—5f$	5918·54	5	16891·4	184·5	7	[40320]
$6d_2—5f$	5854·58	3	17075·9			

First Principal Series : $5s=226132$

	λ (vac.)					mp_{21}
$5s—5p_2$	1625·3*	15	61527	4345	(5)	164605
$5s—5p_1$	1748·8*	15	57182			168950

First Diffuse Series : $5p_2=164605$
$5p_1=168950$

						md_{32}
$5p_2—5d_2$	1494·0*	3	66934	284	(5)	97387
$—5d_3$	1487·7*	7	67218	4338		97664
$5p_1—5d_2$	1403·1*	5	71272			

* As classified by Carroll.

First Fundamental : $5d_3=97387$
$5d_2=97664$

Classification.	λ I.A.	Int.	ν	Δν	*m*	*mf*
$5d_3—4f$	3008·3*	10	33232	281	(4)	64154
$5d_2—4f$	2983·04*	8	33513			

Super-Fundamental : $4f=64154$

						mg
$4f—5g$	4071·4	6	24554	...	(5)	(39600)

TABLE B.—GALLIUM.
First Principal : $4s=247797$

Classification.	λ (vac.)	Int.	ν (vac.)	Δν	*m*	mp_{21}
$4s—4p_2$	1495·36*	(12)	66874	1713	(4)	180923
$4s—4p_1$	1534·65*	(14)	65161			182636

First Sharp : $4p_2=180923$
$4p_1=182636$

						ms
$4p_2—4s$	−1495·36	(12)	−66874	1713	(4)	247797
$4p_1—4s$	−1534·65	(14)	−65161			
$4p_2—5s$	1353·98*	(4)	75576	1720	(5)	107064
$4p_1—5s$	1323·17	(4)	73856			

First Diffuse : $4p_2=180923$
$4p_1=182636$

						md_{32}
$4p_2—4d_2$	1295·29*	(1)	77203	109	(4)	103609
$—4d_3$	93·46*	(6)	77312	1712		103721
$4p_1—4d_2$	1267·19*	(6)	78915			

First Fundamental : $4d_3=103609$
$4d_2=103721$

						mf_{43}
$4d_3—4f$	2424·96*	(7)	41237·8	107·6	(4)	62370 62376
$4d_2—4f$	2418·65*	(6)	41345·4			

* As classified by Carroll.

Super-Fundamental: $4f_4 = 62370$
$4f_3 = 62376$

Classification.	λ I.A.	Int.	ν (vac.)	Δν	*m*	*mg*
$4f_4-5g$	4380·67*	(5)	22821·2	6·2	(5)	(39555)
$4f_3-5g$	4381·66*	(5)	22815·0			

* As classified by Carroll.

Second Principal: $5s = 107064$

						mp_{21}
$5s-5p_2$	4863·19	...	20556·5	538·5	(5)	86507
$5s-5p_1$	4994·15	...	20018·0			87046

Second Sharp Series: $5p_2 = 86507$
$5p_1 = 87046$

						ms
$5p_2-5s$	−4863·19	...	−20556·5	538·5	(5)	107064
$5p_1-5s$	−4994·15	...	−20018·0			
$5p_2-6s$	3806·72	...	26261·9	539·7	(6)	60245
$5p_1-6s$	3730·06	...	26801·6			

Second Diffuse Series: $5p_2 = 86507$
$5p_1 = 87046$

						md_{32}
$5p_2-5d_2$	3585·44	...	27882·7	62·9	(5)	58561
$-5d_3$	3577·37	...	27945·6	539·0		58624
$5p_1-5d_2$	3517·44	...	28421·7			

The spectrograms of Gallium and Indium were found to contain some new lines previously unrecorded, the wave-lengths of which are under measurement; and the author hopes to give a complete list of wave-lengths for Ga and In in the glass and quartz regions in a future communication.

In conclusion, I gladly take this opportunity of expressing my deep gratitude to Prof. A. L. Narayan for suggesting this problem and for his continued interest and advice throughout the work, and also to Prof. A. Fowler for his interest and encouragement in the pursuit of this investigation.

The author's best thanks are due to Mr. Carroll for having kept him informed of his work before the publication of the Paper in the Trans. Roy. Soc., and also for kindly sending the author a reprint of his Paper soon after its publication.

EXPLANATION OF PLATES.

Plate I.

Fig. 1 is the spark spectrum of Indium from λ4700-λ3650 ; (*a*), (*b*) and (*c*) showing the spectra taken with increasing self-induction.

Fig. 2 is the spectrum of Indium from λ3500-λ2700 ; (*a*) and (*b*) representing spectra taken without and with self-induction.

Plate II.

Fig. 1 is the spark spectrum of Gallium in the region λ4500-λ3400 ; (*a*) and (*b*) showing the spectra taken without and with self-induction.

Fig. 2 is the spectrum of Gallium from λ2600-λ2300.

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XII.—ON THE SPECTRUM OF IONISED TIN (Sn III).

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Received September 13, 1926.

(*Communicated by* Prof. A. L. Narayan, *M.A.*, *D.Sc.*)

Abstract.

To help in the detection of series in In II attempts have been made to find regularities in Sn III. Visual observations under varying conditions of excitation revealed a strong triplet in the region λ5100. The detection of two more triplets, particularly a diffuse multiplet, confirmed the above identification. A member of the first fundamental series has also been found with the aid of the corresponding members in In II and Ga II. A possible triplet of the first sharp series is suggested, and evidence of its correctness is sought by correlating it with corresponding members of related spectra.

Continuing the work on series in the spectra of doubly-ionised indium and gallium, attempts have been made to discover, if possible, series relations in these spectra corresponding to the singly-ionised stage. By analogy with the spectrum of Al II, it is inferred that in these cases the primary series occur in the extreme ultra-violet, while the principal members of the secondary series fall in the red, and as such do not readily yield themselves for visual examination. Attention is therefore directed, as a preliminary to this, to the detection of series in the spectrum of doubly-ionised tin, which, in general characteristics, should resemble, according to the spectroscopic displacement law, the spectrum of singly-ionised indium.

The spectrum of tin, like that of silicon, is of special interest to spectroscopists, for it must provide another illustration of the possibility of obtaining different types of spectra from a single atom. Occurring in the fourth group of the Periodic Table with silicon, germanium and lead, it has been most refractory as regards the resolution of its spectra into series. In the spark spectrum only the principal doublet of Sn IV has till now been recognised. The present Paper contains a few of the leading members of the spectrum of Sn III, and the investigation of other spectra is in progress.

The experimental procedure adopted is that of examining visually and photographically the spark spectrum of tin in air, in vacuo and in an atmosphere of hydrogen at varying pressures, and also of the "arc in vacuo" between electrodes of tin. To distinguish between lines due to the successive stages of ionisation of the element, the spectrum is observed under different experimental conditions by varying the intensity of discharge, which is done by including in the secondary circuit a variable self-induction and capacity. Great care is taken to make sure that a line belongs to a particular stage of ionisation before it is included into the series system of that stage, and it may be observed that almost all the lines here taken into the series of doubly-ionised tin are found among the list of lines tabulated by Kimura and Nakamura(1) under Sn III, who, by photographing the "cathode spectrum" of tin, have grouped some of the spark lines of tin under successive stages—Sn II, Sn III, Sn IV—as they appear with increasing stimulus.

The notation adopted in the present Paper is that used by Prof. Fowler(2) in "The Spectrum of Ionised Oxygen (O II)," the subscripts denoting the inner quantum numbers. The number denoting the group multiplicity is, however, omitted, as only one system is being considered throughout.

Observations have been made in the visible region, with a constant deviation spectrometer, to find the triplet of the principal (secondary) series which is likely to be observed in this range. The result is the detection of the prominent and intense triplet shown below, and this served as the starting point for the identification of the other members.

Hemsalech.(4) λ	Hemsalech.(4) Int.	Eder and Valenta.(5) λ	Eder and Valenta.(5) Int.	Exner(4) and Haschek. λ (I.A.)	Exner(4) and Haschek. Int.	Author. Int.	ν (vac.)	Δν
4858·2	6	4858·4	4	4858·12	5	9	20578·4	
								977·6
5100·8	7	5100·56	3	5100·48	2	8	19600·8	
								465·7
5224·85	5	5225·09	2	5224·53	1	7	19135·1	

The wave numbers are calculated from the accurate measures of Exner and Haschek. The order and magnitude of the intensities indicate that the triplet is inverted, and is, perhaps, the first member of a sharp series. The position of the corresponding triplet of Si III(3) confirmed this suggestion. The separation $\triangle \nu_{p21}$ agrees with that of C III and Si III, as shown.

Atomic number (N).	Element.	Δν	Δν/N²
6	C III	12·8	0·356
14	Si III	73·2	0·372
50	Ge III	...	...
	Sn III	977·6	0·391

The ratio of the separations 1 : 2·1 is approximately in keeping with Landé's interval rule.

Further examination of the spectrograms, taken with a quartz spectrograph, revealed two more triplets, which, within limits of experimental error, exhibit the same separations. One of these, as is shown below, is accompanied by satellites, and it is therefore inferred that it must be a member of the diffuse series. The region is approximately that where the multiplet *bp—bd* is to be expected. It is a group of six lines, and arises from transitions of the electron between triplet *p* and triplet *d* terms. The second is presumably the second member of the sharp series.

The Diffuse Series Multiplet.

Hemsalech. λ	Hemsalech. Int.	Hartley and Adeney(4). λ	Hartley and Adeney(4). Int.	Exner and Haschek. λ	Exner and Haschek. Int.	ν (vac.)	Δν
...	...	...	...	...	...	[26713]	
							86·7
3730·2	3	...	...	3730·2	1	26801·7	
							159·7
3708·2	5	3707·6	...	3708·1	2	26961·4	
							976·9
		3609·3	...	3610·3*	...	27691·9	
							86·7
3599·05	2	3598·3	...	3599·02	2	27778·6	
							464·7
		3549·7	...	3550·7	1	28156·6	

* Measured by the writer.

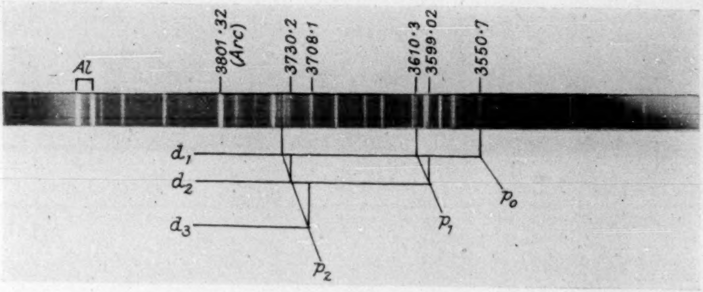
Al
3801·32 (Arc)
3730·2
3708·1
3610·3
3599·02
3550·7
d_1
d_2
d_3
p_0
p_1
p_2

FIG. 1.

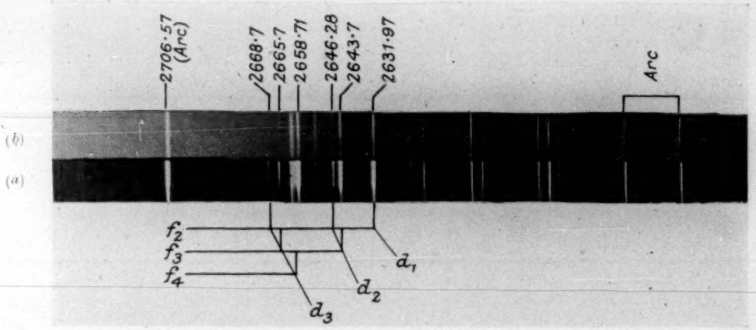
2706·57 (Arc)
2668·7
2665·7
2658·71
2646·28
2643·7
2631·97
Arc
(b)
(a)
f_2
f_3
f_4
d_1
d_2
d_3

FIG. 2.

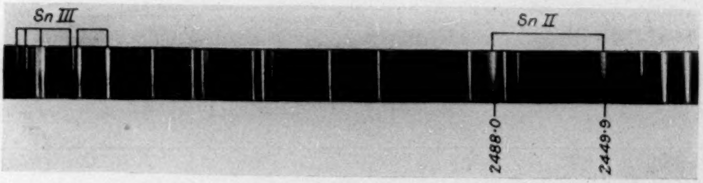
Sn III
Sn II
2488·0
2449·9

FIG. 3.

PLATE I.—SPARK SPECTRUM OF TIN

The multiplet differs from that of the member of the diffuse series in Si III inasmuch as the satellites are normal, and appear on the less refrangible side of the main lines, so that the *d* terms are not inverted as in the case of Al II or Si III.

The second triplet is

λ(I. A.)	Int.	ν (vac.)	Δν
2878·5	4	34730·2	976·0
2800·6	3	35696·2	467·4
2764·4	2	36163·6	

The first line of this triplet has been previously measured by Hartley and Adeney, who record λ2877·4. The limit bp_2 obtained as usual with a 9 *N*-constant from this member and the first triplet is $\nu=101565$.

FUNDAMENTAL SERIES.

The first member of the fundamental series in several spectra is, perhaps, the easiest to fix upon, as the intensity of this member is usually very high, and its position can be approximately located from the values of *d* and *f* terms inferred from a spectral sequence.

There is a very prominent triplet, in the ordinary range of observation, in the spectrum of indium, which from its position, intensity and the value of the separations of the lines is easily seen to be $(ad_{321}-af)$ of In II.

——	λ(6) (I. A.)	Int.	ν	Δν
ad_3-af	4682·00	10	21352·4	116·2
ad_2-af	4656·66	10	21468·6	82·2
ad_1-af	4638·9	10	21550·8	

The detection of a corresponding narrow triplet in the spectrum of gallium, shown below, substantially supported this assignment.

λ(7)	Int.	ν	Δν
4262·05	10	23456·2	34·8
4255·75	9	23491·0	25·3
4251·18	9	23516·3	

These led to the identification of the corresponding member in the spectrum of Sn III. In the case of In II and Ga II the *f* levels, presumably triple, are perhaps too close to be resolved with the grating employed, so that a group of only three lines is observable. In the case of tin, however, the *f* levels are also resolved, and the following six-line multiplet, resulting from the triple *d* and *f* levels, is most prominent and is easily detected.

—	ad_3		ad_2		ad_1
af_4	(15) 2658·71 37602·5		... ...		... ...
	98·6				
af_3	(7) 2665·7 37503·9	*312·1*	(15) 2643·7 37816·0		... ...
	36·9		*36·9*		
af_2	(3) 2668·7* 37467·0	*312·1*	(6) 2646·28 37779·1	*205·4*	(15) 2631·97 3798 45

* This satellite which completes the multiplet is measured by the writer.

THE PRIMARY SERIES.

The primary series—the first principal, sharp and diffuse series—fall in the extreme ultra-violet, and attempts are here made to fix these, if possible, first by extrapolation and then to seek for confirmation by correlating the corresponding members of the spectra of related elements. This led to the suggestion of the probable principal triplet of the sharp series. But, in spite of such confirmation, it may be observed that it can only be tentative, and requires experimental confirmation.

The usual Rydberg relation and formula applied to the members of the second sharp series lead to $\lambda 1020$ as the approximate position of the principal triplet $ap_{210}-as_1$, with the larger separation $\triangle\nu_{21}=3200$. A more correct idea of the separation is perhaps obtained from the relation that in the spectra of elements of the same vertical group of the Periodic Table $\triangle\nu/N^2$ is approximately constant. This gives for $\triangle\nu_{21}$ a value of about 4500.

The only available values of wave-lengths of spark lines of tin in the required region are those of Lang.(8)

A careful search was made for a possible triplet among these lines of λ below 1400 A.U., having in view the relative order and magnitude of the intensities and the probable ratio of the intervals between the lines. This suggested the triplet

λ	Int.	ν	Δν
1251·3	60	79917	
			4138
1189·7	40	84055	
			2286
1158·2	60	86341	

Evidence for the possibility of this being regarded as the member $ap_{210}-as_1$ of Sn III is now sought by searching for the corresponding triplet in the spectrum of In II with the aid of the relativistic irregular and regular doublet laws, and also in the spectrum of Ga II, the spectra of gallium being found to resemble very closely the corresponding spectra of indium.

The method as shown below is that adopted by Millikan(9) in the identification of lines of C III.

I.—Irregular doublet law, with the usual notation,

$$\frac{\nu'}{R}=\{(n_2{}^2-n_1{}^2)Z^2-2(n_2{}^2\sigma_1-n_1{}^2\sigma_2)Z+(n_2{}^2\sigma_1{}^2-n_1{}^2\sigma_2{}^2)\}/n_1{}^2n_2{}^2$$

In the case under consideration the line ap_2-as_1 (or using total quantum numbers, $5p_2-6s_1$) is due to an electron jump between two orbits of different total quantum numbers, n_2 being different from n_1.

Hence, transposing

$$\nu'-\frac{R(Z-A)^2(n_2{}^2-n_1{}^2)}{n_1{}^2\,n_2{}^2}=C'Z+D'$$

A being a constant introduced for convenience of calculation. The whole expression on the left side varies linearly with the atomic number Z for any particular set of values of n_2, n_1, σ_1, σ_2.

Here $n_2=6$ and $n_1=5$.

$$\therefore\quad \frac{R(n_2{}^2-n_1{}^2)}{n_1{}^2n_2{}^2}=\frac{R(36\text{-}25)}{36\times25}=1340{\cdot}5$$

The first column in the table below gives the value of ν of Cd I, and the above value tentatively supposed to be that of Sn III.

Putting $A=47$, for Cd I we have the expression on the left equal to $\nu-1340{\cdot}5\times1^2=18316{\cdot}3$

For Sn III

$$=\nu-1340{\cdot}5\times3^2$$
$$=79917-1340{\cdot}5\times3^2=67852$$

At. No.	Element.	$\nu(5p_2-6s_1)$	$[\nu-1340{\cdot}5(Z-A)^2]$	Diff.
48	Cd I	19656·8	18316	24415
49	In II	[48093]	[42731]	25121
50	Sn III	79917	67852	

Interpolation gives a value for In II of $\nu-1340{\cdot}5\times2^2$, equal to about 43300, or $\nu=48662$.

II.—Regular doublet law: $\{(5p_1-5p_2)=0{\cdot}0234\ (Z-s)^4\}$.

At. No.	Element.	$\Delta\nu\ (5p_1-5p_2)$	$4\sqrt{\Delta\nu/0{\cdot}0234}$	s.
48	Cd I	1171	14·96	33·04
49	In II	[2481]	[18·05]	[30·95]
50	Sn III	4138	20·5	29·5

Interpolation gives about 31 for the screening constant " s " for In II, which leads to

$$\triangle \nu = 0{\cdot}0234(49\text{-}31)^4$$
$$= 2457.$$

The following strong triplet in the spectrum of indium is found to be exactly in agreement with the calculated values. The respective values, adopting this triplet, are shown in the above tables enclosed in brackets. It is seen that these laws are exactly satisfied. For purposes of comparison, the values of λ and intensity by different observers are given below.

Triplet (5 p –6s) *of In II.*

Saunders.(10)		Weinberg.(11)		Eder & Valenta.(4)		Carroll.(7)			
λ	Int.	λ	Int.	λ	Int.	λ	Int	ν (vac.)	Δν
2079·28	7	2079·6	5	2078·8	8	2079·3	4	48093	2481
1977·44	5	1977·8	6	1976·8	7	1977·3	3	50574	1058
		1936·5	4	1935·9	5	1936·8	3	51632	

The wave-numbers are calculated from the recent accurate measurements of Carroll. The exact coincidence of observed and calculated values of this triplet perhaps lends good support to the correctness of the identification.

Further support is given by the detection of the corresponding triplet, shown below, in the spark spectrum of gallium; which is very prominently seen in the beautiful spectrograms accompanying Carroll's Paper, and agrees excellently with that of In II as regards position, frequency, interval, etc.

Triplet (5p –6s) *of Ga II.*

Weinberg.		Carroll.			
λ	Int.	λ	Int.	ν (vac.)	Δν
1845·0	8	1845·28	9	54201	932
1813·8	9	1813·91	9	55133	450
1799·1	6	1799·31	7	55583	

There is a displacement of the line towards the region of shorter wave-length as we pass from In II to Ga II. Values of $\triangle \nu / N^2$ for the elements of the same vertical group Al II, etc., are shown in the following table.

At. No. N.	Element.	Δν (5p_1 –5p_2)	Δν/N²
13	Al II	125·5	0·743
31	Ga II	932·0	0·97
49	In II	2,481·0	1·034

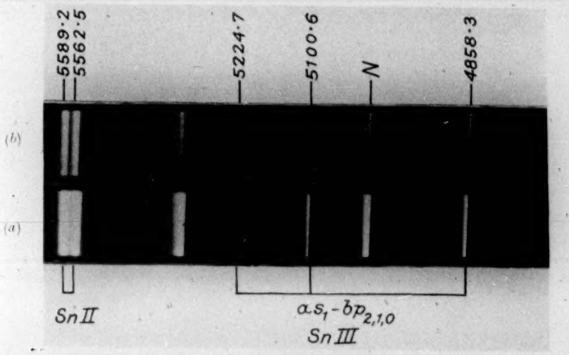
5589·2
5562·5
5224·7
5100·6
N
4858·3
(b)
(a)
Sn II
$as_1-bp_{2,1,0}$
Sn III

FIG. 4.

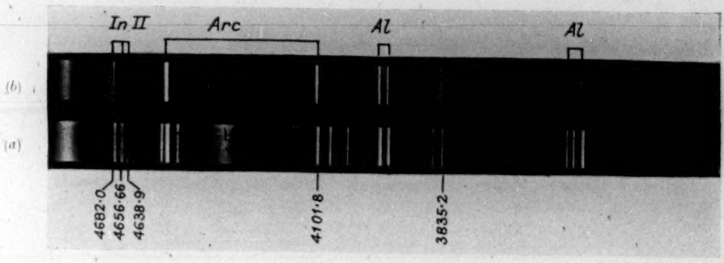
In II
Arc
Al
Al
(b)
(a)
4682·0
4656·66
4638·9
4101·8
3835·2

FIG. 5.

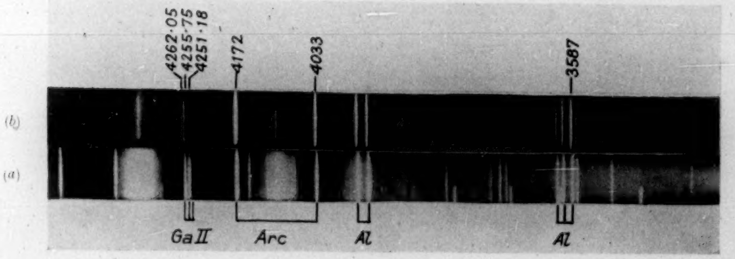
4262·05
4255·75
4251·18
4172
4033
3587
(b)
(a)
Ga II
Arc
Al
Al

FIG. 6.

PLATE II.—FIG. 4.—SPARK SPECTRUM OF TIN.
FIG. 5. ,, ,, ,, INDIUM.
FIG. 6. ,, ,, ,, GALLIUM.

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For comparison, a similar table for the group of spectra Al III, etc., is also added.

N.	Element.	$\Delta\nu$	$\Delta\nu/N^2$
13	Al III	238	1·408
31	Ga III	1714	1·7
49	In III	4347	1·81

Term Values.—The frequency values of the various energy levels in the spectrum of doubly-ionised tin cannot be determined at this stage. For the first member of the first diffuse series ($ap-ad$) has not yet been located for want of accurate data of the spark spectrum of tin in the region of short waves below about λ1000 A.U., and this alone would give, with the aid of the above determined members, a connected system of terms from af to ap.

SPECTRUM OF Sn II.

It may not be out of place here to include two pairs of the same separation which are found in the spark spectrum of tin and definitely belong to the doublet system of Sn II.

λ	Int.	ν (vac.)	$\Delta\nu$
3352·47	10	29821·5	625·4
3283·6	9	30446·9	
2488·0	5	40180·6	626·3
2449·9	4	40806·9	

It is difficult now to conclude definitely about the series to which these pairs belong. But the lines are very intense and diffuse under all the conditions examined, except in the "arc in vacuo," where they appear with great sharpness. And no satellite could be detected accompanying any of these lines.

Applying the usual Rydberg formula with a four-fold constant, they give a limit equal to 58232 and term values 28408 and 18048. These latter suggest that the members probably form a combination series of the type $x_{12}-mf$, a series of exactly this nature being found in the spectrum of Si II by Prof. Fowler (loc. cit.).

EXPLANATION OF PLATES.

Plate I.

Fig. 1 indicates the spectrum of tin showing the diffuse multiplet at about λ3700.

Fig. 2 shows the prominent six-line multiplet of the first fundamental series; (*a*) and (*b*) are photographs taken without and with inductance in the secondary circuit.

Fig. 3, the spectrum (spark) of tin in the region λ2500, showing the pair λ2488 and 2449 of Sn II.

Plate II.

Fig. 4, (*a*) and (*b*), spark spectra between tin electrodes without and with inductance respectively.

Figs. 5 and 6, spectra of indium and gallium indicating the triplets of the first fundamental series of In II and Ga II respectively ; (*a*) and (*b*), as before, are photographs taken without and with inductance respectively.

In all these the metal is contained in Al cups.

In conclusion, I wish to express my great indebtedness to Prof. A. L. Narayan, M.A., D.Sc., F.Inst.P., for giving me valuable guidance throughout the course of the investigation.

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(2) Fowler, Spectrum of Ionised Oxygen (O II), Proc. Roy. Soc., p. 476, Vol. A 755 (1926).
(3) Fowler, Phil. Trans., A, Vol. 225, pp. 1-48 (1925).
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(5) Atlas Typ. Spect., 1924 edition.
(6) Schulemann, Zeits. f. Wiss. Phot., 10, 263 (1912).
(7) Carroll, Phil. Trans., A, Vol. 225, p. 357 (1925).
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(11) Weinberg, Proc. Roy. Soc., A, Vol. 107, p. 138 (1924).

DEMONSTRATION OF THE BEHAVIOUR OF BODIES WITH NON-CONDUCTING SURFACES IN ELECTROSTATIC FIELDS.

By L. G. VEDY.

THE rotation of a piece of dielectric material when suspended between the poles of a Wimshurst machine was observed, about a year ago, by S. W. Richardson, Esq., M.A., D.Sc., F.Inst.P., Chief Physics Master at Whitgift Grammar School, Croydon, who kindly allowed me to investigate this phenomenon in the school laboratory. The experiments which I have performed up to the present, in which I have been materially assisted by Mr. G. Gowlland, indicate, in general, the following results :—

1. Any body consisting of or covered with dielectric material, and free to turn about an axis, rotates when placed between the poles of a Wimshurst machine. This effect increases with the thickness of the surface layer to a certain extent, and is stopped if the body is covered with conducting material.

2. Rotation occurs only when a discharge takes place from either or both of the poles of the machine. It occurs in either uniform or non-uniform fields, increases with the amount of the discharge, masking the orientation effect of dielectrics, and usually occurs in either direction. If, however, the knobs are pointed tangentially towards the surface rotation proceeds more easily in the direction in which they point. If, instead of knobs, points are used the rate of rotation is much less, but the directive effect is more marked.

3. The effect also occurs, but to a much less extent, with partial conductors (e.g., cork) if there is sufficient discharge present. Also in the case of a body covered with a thin dielectric surface layer, the rate of rotation is greater when the interior is a poor conductor (e.g., cork) than when it is a metal.

4. In general a body is charged during rotation. The charge is the same kind as that of the pole from which it receives the greater discharge. When mounted systematically between two similar knobs the body has no detectable charge.

5. These effects are completely stopped if the body is shielded by a thin plate of dielectric substance. The other effects, however, of electrostatic fields such as orientation of dielectrics, etc., are unaffected.

The phenomenon appears to be due to the effect of the discharge on the non-conducting surface layer of the body, and does not appear to be associated with the nature of the dielectric (as are the orientation effects, etc.), and the experiments performed seem to show that local charging of part of the surface and the subsequent repulsion of this maintain the rotation.

These effects were illustrated by the following experiments :—

Pieces of paraffin wax, ebonite, sealing wax, and a glass beaker rotated when hung from threads between the knobs of a Wimshurst machine, but not when covered with tin or copper foil. Brass spheres, etc., covered with paraffin wax, rotated, the greatest rate being attained by the sphere with the thickest wax layer. Better results were obtained with bodies mounted on bearings (steel needles in glass sockets), as this prevented the body from swaying and knocking the knobs. Rotation of a paraffin sphere occurred about any axis, provided this did not make a large angle with the plane perpendicular to the line joining the centres of the knobs.

By using an alternative gap it was shown that rotation did not occur in the absence of a brush discharge. Also sparking usually decreased the effect. The directive effect of knobs and points was shown by the direction in which the rotation of a paraffin wax sphere (about 10 cms. diameter) started, when the knobs or points were directed tangentially towards it.

In the presence of a large brush discharge a cork (about 10 cms. diameter, and mounted on bearings) rotated. When covered with paper the rate of rotation was increased, though it was considerably decreased if a layer of tinfoil was placed under the paper.

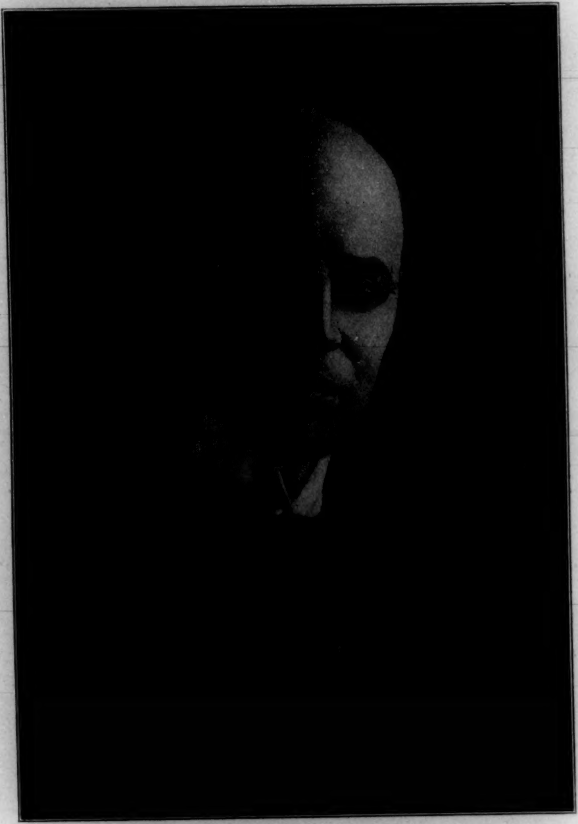
A cardboard tube covered partially with paraffin wax commenced rotation in such a direction that the paraffined portions moved away from the knobs, thus indicating a repulsive effect.

The presence of a charge on the bodies was shown by using a proof plane and an ordinary gold-leaf electroscope. When the paraffin sphere was mounted symmetrically between two nearly similar knobs the charge was shown to be very small.

When the bodies were suspended inside a glass beaker, or between ebonite plates, which shielded them from the discharge, rotation did not occur, but the ordinary orientation effects were obtained. This was confirmed by the charges found to be present on the parts of the body near to the knobs. In particular, a sphere remained in the position in which it was originally placed, and if displaced oscillated about this position. If, however, a charge was communicated to it, by means of a conductor touching one of the knobs, rotation commenced.

DISCUSSION.

Prof. F. L. Hopwood (communicated): I should like to call the attention of those interested to a record of similar experiments to those shown, which is contained in the chapter on Electrostatic Motors in V. E. Johnson's book "Modern High Speed Influence Machines" (Spon, 1921).



O. W. Richardson.

PRESIDENT, 1926–1928.

XIII.—THE PRESENT STATE OF ATOMIC PHYSICS.

(*Presidential Address delivered on February* 11, 1927, *by* Prof. O. W. RICHARDSON, M.A., D.Sc., F.R.S.)

§ 1.

THE discovery of the electron by J. J. Thomson at the close of the last century, followed by Planck's solution of the contradictions involved in temperature radiation phenomena by proposing the quantum theory, and later by Bohr's theory of the behaviour of the nuclear atom, to which Rutherford had been led by his studies of α particle deflections, have resulted in an insight into the physics of atoms which would have been incredible fifty years ago. They have turned spectroscopy from a heterogeneous collection of miscellaneous facts into a rapidly advancing science, and have created several other new branches of science of varying degrees of importance. Whilst these ideas suitably grafted on to the general body of earlier physical doctrine have been able to co-ordinate vast stores of existing facts, and to predict, perhaps, even vaster stores of new ones, they do not appear to be sufficiently wide in scope to embrace all that is required in atomic physics. There are certain groups of phenomena or facts which do not fall into the general framework which I have indicated, or, if they are forced in, the result is found to be a misfit. It would be very interesting to discuss these obdurate phenomena in some detail, but I shall have to content myself with doing little more than making a list of some of them. It is as follows :—

(1) The contradiction between the quantum theory and the wave theory of light. The contradiction is that the same light behaves as a wave when it is travelling through a telescope and is occupied with refraction or dispersion, and as a projectile when it is engaged in emission, absorption or photo-electric action. The energy of the radiation behaves as though it possessed at the same time the opposite properties of extension and localisation. I have pointed out the grave nature of this contradiction a good many years ago.*

(2) According to any reasonable interpretation of Bohr's theory, the spectra should be governed by the various kinds of quantum numbers which we should expect to take the various integral values 0, 1, 2, . . . ∞. But in actually working out the spectra spectroscopists find that they cannot in general get along with this set of numbers.† Half integral numbers insist on turning up. The case of band spectra furnishes very simple and convincing examples. On the theory (which is quite straightforward and fully borne out in other respects) the levels of the vibration bands should be $nh\nu_0$ ($n=0, 1, 2, \ldots$), where ν_0 is the fundamental vibration frequency at infinitesimal amplitude. As a matter of fact, they are found to be at $(n+\frac{1}{2})h\nu_0$ in the one case, the band spectrum of boron oxide, where the evidence is conclusive. There is no doubt of this matter, as the centre of the bands can be determined with certainty by using bands of isotopic molecules (cf., Mulliken, Phys. Rev., Vol. 25, p. 259, 1925).

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* Electron Theory of Matter, 2nd Edn., p. 507 (1916).

† *See* for example Sommerfeld. Three Lectures on Atomic Physics (1926).

(3) The Landé splitting factor, $g=1+\{j(j+1)+j_s(j_s+1)-j_a(j_a+1)\}/2j(j+1)$, determines the Zeeman effect in multiplets.[*] This is the formula required by the facts. The calculated formula is similar with the $j(j+1)$s replaced by j^2s. There are similar discrepancies between the calculated and found intensities of the components of multiplets.

(4) The remarkable effect discovered by Ramsauer[†] in Germany, and independently, but later, by Townsend and Bailey[‡] at Oxford, and by Chaudhuri[§] in my laboratory and working under my direction, each of these experimenters using a different method, that the mean free path of an electron in the inert gases, and especially in argon, becomes exceedingly long when the velocity of the electrons is reduced. This phenomenon seems incomprehensible on the Rutherford-Bohr picture of the atom. Dempster[||] has observed something similar with slowly moving charged hydrogen atoms in helium.

(5) If we contemplate a more complicated atom, the way in which the orbits interpenetrate and yet maintain their identity and stability seems very curious. One has to postulate a very harmonious intercoupling of the orbits. Even though this may be given a plausible justification on the quantum principles, this hardly destroys its mysteriousness.

(6) The calculated structures for the helium atom and the hydrogen molecule do not agree with the facts. It may be urged that these bodies furnish problems so complicated that the results must be regarded as endowed with an element of uncertainty ; but it must at least be considered a matter for suspicion that all the calculations lead to ionisation potentials which are much too high.

(7) The magnetic anomalies. There are two of these—spectroscopic and gyromagnetic. According to the classical dynamics, as well as to the quantum theory, there is a general relation between the angular momentum j and the magnetic moment μ of any electron orbit—viz.,

$$j=\frac{2mc}{e}\mu$$

The gyromagnetic anomaly is that when a suspended rod is caused to spin by suddenly magnetising it along the axis, or if an unmagnetized rod is magnetized by spinning, the relation between the magnetic moment M and the angular momentum U is found, experimentally, to be

$$U=\frac{mc}{e}M$$

i.e., the factor has half the expected value. The spectroscopic anomaly is something similar.[¶] For the excited electron the proportion between what may be identified as the angular momentum and the magnetic moment has the normal value, but for the unexcited core (ground term) the magnetic moment is twice as large as would appear to correspond to the angular momentum. A collection of various proposals for explaining away the gyromagnetic anomaly has been given by Stoner.[*] I will add to it another which is a specialisation of one I have already given. Suppose the nucleus has a quantized spin with angular momentum equal in magnitude to one half of, and opposite in sign to, that of all the electrons in the atom.[†] To get some figures we need to make some hypothesis about the structure of the nucleus. Fortunately we only need the order of magnitude, so that I shall assume that we shall be somewhere near the mark if we treat the nucleus as a uniformly charged non-conducting spherical surface of radius R kept from exploding by a uniform non-electromagnetic tension. The magnetic moment of this structure is

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[*] *See* E. C. Stoner, "Magnetism and Atomic Structure," p. 238 (1926).

[†] Ramsauer, Ann. der Physik., Vol. 64, p. 451 (1921).

[‡] Townsend and Bailey, Phil. Mag., Vol. 43, p. 593 (1922).

[§] Chaudhuri, Phil. Mag., Vol. 46, p. 461 (1923).

[||] Dempster, Nature, Vol. 116, p. 900 (1925) ; Phys. Rev., Vol. 27 (1926).

[¶] cf., Sommerfeld. Three Lectures, p. 41.

$$M=\frac{eR^2}{3c}\omega$$

if ω is the angular velocity.[‡] The moment of momentum is

$$U=I\omega=\frac{e^2R}{18\pi C^2}\omega$$

The mass of the sphere is $m=e^2/8\pi c^2R$ (electromagnetic) $+4/3\pi R^3\varepsilon/c^2$ (non-electromagnetic). The non-electromagnetic energy is $4/3\pi R^3\varepsilon$. If the electron forces are balanced by a normal non-electromagnetic tension $\varepsilon=e^2/32\pi^2R^4$, so that $m=e^2/6\pi C^2R$ and $M/U=e/cm$. If the suffixes a, e and n denote atom, electron and nucleus we have now

$$j_a=\Sigma j_a-j_n$$

$$\mu_a=\frac{e_e}{2m_ec}\Sigma j_e+\frac{e_n}{m_nc}j_n$$

By hypothesis $$\Sigma j_e=2j_n,\quad \frac{e_n}{m_n}\div\frac{e_e}{2m_e}=10^{-3}, \text{ about,}$$

so that $$j_a/\mu_a=\frac{\Sigma j_e-j_n}{\Sigma j_e}\cdot\frac{2mc}{e}=\frac{mc}{e}$$

as accurately as it has been measured.

(8) Space Quantisation and the Selection Rules. Whilst these can be presented as a fairly consistent body of doctrine which unquestionably expresses something very real in the actual phenomena, the physical interpretation seems to present considerable difficulties.

(9) The excitation of soft X rays.[§] Whilst there is much experimental evidence of doubtful value in this field, some of the inflexions which seem to have no ascertainable connection with the Bohr scheme of levels are in my judgment well established experimentally.

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[*] Stoner, "Magnetism and Atomic Structure," p. 192. *See* also Richardson, Phys. Rev., Vol. 26, p. 248 (1908); 1921 Solvay Conference, loc. cit., and Proc. Roy. Soc., Vol. 102, p. 538 (1922).

[†] O. W. Richardson, Nature, Vol. 117, p. 652 (1926).

[‡] O. W. Richardson, 1921 Solvay Physics Conference, p. 219; Gauthier-Villars, Paris (1923).

[§] *See* Richardson and Chalklin, Proc. Roy. Soc., A, Vol. 110, p. 247 (1926); and K. T. Compton and C. H. Thomas, Phys. Rev., Vol. 28, p. 601 (1926).

(10) The peculiar distribution in angle of the electrons of medium speeds reflected by various solid elements discovered by Davisson.* These interesting phenomena also seem to lie outside the scope of the theories to which I have referred. There are other difficulties also concerning the detailed interpretation of spectra. These are mostly of an intricate character, and I shall not enter into them; in fact, I am hardly sufficiently familiar with them to be well fitted to do so.

§ 2. THE NEW QUANTUM MECHANICS.

Whilst the foregoing difficulties are by no means all of equal weight, and many of them if standing alone might be treated, perhaps, with indifference, yet taken together they form a solid obstacle. It is the consideration of such questions as these which in the last year or two has led atomic physicists to look for some radical departure from the points of view hitherto used by Bohr, Sommerfeld and their followers. The revolt, in fact, seems to have been started by Bohr himself.

The first attempt in this direction was made by taking the most obvious road. I do not, however, by this mean to imply that there was any really obvious road. What seemed solid in the situation were the two following things. The body of physical doctrine conveniently termed classical holds good for large scale phenomena where the frequencies involved are low. The physics of atoms in which very high frequencies are involved is governed by an entirely different body of doctrine, which has been found empirically, and is called the quantum theory. The distinction between a low and a high frequency is merely one of degree, not of kind, so that we should expect the quantum doctrine to extrapolate to the classical when the frequencies are sufficiently reduced, and this implies also a general correlation between the two. The groundwork of this was formulated by Bohr, and is known as the Correspondence Principle. This correlation is peculiar. For the solvable problems the co-ordinates in the classical motion (i.e., the description of an electron orbit) can be expressed as a trigonometrical series, such as

$$x = \sum_{p_1 p_2 \ldots p_k} A_{p_1 p_2 \ldots p_k} \cos\left[(p_1\omega_1 + p_2\omega_2 + \ldots + p_k\omega_k)b + a_{p_1 p_2 \ldots p_k}\right]$$

In this case the following electron "jumps" are possible,

$$n_1' - n_1'' = p_1 \, n_2' - n_2'' = p_2 \ldots n_k' - n_k'' = p_k$$

and no others. Also the real frequency of the radiation emitted as a result of the jump is the average frequency (taken in a definite but very simple way) of the classical vibrations in a hypothetical classical motion from the n' to the n'' orbits. Also the intensity of the emitted radiation is correlated to the amplitude factor $A_{p_1 p_2 \ldots p_k}$ in the corresponding expression for the co-ordinate. In particular, if this is zero, that particular frequency and plane of polarisation will be absent in the emitted radiation. This is the essence of it, at any rate. For example, if it had been foreordained that the paths of the electrons in atoms were to be circles instead of Keplerian ellipses, we should have $x = A \cos(\omega t + a)$ simply. Since the only value of p which occurs here is 1, the only possible jump is $n' - n'' = 1$. Each spectral series would reduce to a single line. For example, Balmer's series would be replaced by $H\alpha(3 \rightarrow 2)$. Such an arrangement would have lightened the burdens of spectroscopists very considerably.

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* Davisson and Kunsman, Science, Vol. 54, p. 522 (1921); Phys. Rev., Vol. 21, p. 637; Vol. 22, p. 242 (1923).

I shall pass over a Paper by Bohr, Kramers and Slater,* which attempted to dispose of the contradiction between the quantum theory and the wave theory of light ((1) above), and another by Kramers† on the theory of dispersion, which helped to lead up to the further developments.

Whilst these remarks do not apply to these particular theories, it may be pointed out that some of the suggestions which have been made would introduce a new principle (or lack of principle) into physics by violating human free will. To account for the photoelectric effect, for example, it has been suggested that the emission of a quantum of radiation by an atom A and its subsequent absorption by an atom B involves a collusion between the atoms A and B of such a nature that the emission from B is predetermined by the event of ejection from A. Now suppose A is an atom on a star which the light left years ago, and B is an atom on a photographic plate of a telescope set up to photograph the star. It is obvious that there are a large number of things which I might do which would prevent that predetermination from being carried out. It is, of course, an indication of the seriousness with which the difficulties have been felt that such views should have received consideration.

The next step was by Heisenberg,‡ who made a kind of extension of the Correspondence Principle by pointing out that there was a very close analogy between the classical dynamics and the quantum dynamics of a peculiar kind. He pointed out that the things that we really know about atoms are the frequencies and amplitudes of the radiations they absorb and emit. We have no knowledge of such things as the position and time of revolution of an electron in an atom, and, in fact, these things may just be figments of our imagination. The key to what is real is probably to be found in the radiation. The properties of the radiation expressed in the corresponding classical and quantum theory forms are exhibited in the following table :—

Classical.	Quantum Theory.
$\nu(n, a) = a\nu(n) = a\frac{1}{h}\frac{dE}{dn}$	$\nu(n, n-a) = \frac{1}{h}\{E(n) - E(n-a)\}$
$\nu(n, a) + \nu(n, \beta) = \nu(n, a+\beta)$	$\nu(n, n-a) + \nu(n-a,\ n-a-\beta) = \nu(n, n-a-\beta)$ or $\nu(n-\beta)(n-a-\beta) + \nu(n, n-\beta) = \nu(n, n-a-\beta)$
$A_a(n)E^{\iota\omega(n)at}$	$A(n, n-a)e^{\iota\omega(n, n-a)t}$

n and a (or β) are two numbers which are integral in the quantum theory column, ν and ω indicate frequency, E energy and A amplitude. Any quantity $x(n, t)$ which is a function of n and t can be expressed classically by the Fourier expansions

$$x(n, t) = \sum_{a=-\infty}^{+\infty} A_a(n)e^{\iota\omega(n)at} \text{ if it is periodic, or}$$

$$x(n, t) = \int_{-\infty}^{+\infty} A_a(n)e^{\iota\omega(n)at}\, da \text{ if it is not.}$$

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* Bohr, Kramers and Slater, Phil. Mag., Vol. 47, p. 785 (1924).

† Kramers, Nature, Vol. 113, p. 379 (1924) ; also Kramers and Heisenberg, Zeits. für Physik, Vol. 31, p. 681 (1925).

‡ Heisenberg, Zeits. für Physik., Vol. 33, p. 879 (1925).

According to Heisenberg, in the quantum theory $x(t)$ is to be taken as *represented* by the *totality* of all the values $A(n, n-\alpha)e^{\iota\omega(n, n-\alpha)t}$ and $x(t)$ squared as the totality of all the values $B(n, n-\beta)e^{-\iota\omega(n, n-\beta)t}$

$$= \sum_{\alpha=-\infty}^{\alpha=+\infty} A(n, n-\alpha)A(n-\alpha,\ n-\beta)e^{\iota\omega(n, n-\beta)t}$$

or the corresponding integral between $\pm\infty$ in the non-periodic cases. Heisenberg then asserts that if the quantities in the classical dynamics are translated in this way, and the operations of the classical equations are carried out according to the rules appropriate to the new type of quantities, the results form the answer to the corresponding problem in the quantum dynamics. For example, this method when applied to the non-harmonic oscillator, gives the energy levels as $(n+\frac{1}{2})h\nu_0$, as required by the band spectra, instead of $nh\nu_0$, as given by the older theory. An alternative independent development which has some advantages has been given by Dirac.*

The quantities which operate in this new quantum dynamics are not ordinary numbers, and the operations to which they conform do not agree in form with the laws of algebra. It was pointed out by Born and Jordan† that the laws which govern them are the same as those which govern the operations in the theory of matrices. The rules are framed so that the addition and subtraction of matrices are formally equivalent to algebraic addition and subtraction. Multiplication is defined as following the rows and columns rule of determinants. It agrees with the associative law for multiplication, but not in general with the commutative law—i.e., $(pq)r = p(qr)$; but pq is not in general $= qp$. On the other hand, $p(q_1+q_2) = pq_1+pq_2$. This identification with matrices has enabled a great deal of existing mathematics to be brought to bear on these problems, but it is a kind of calculus which I do not think that most physicists will find attractive. Anyhow, the real identification of the physical quantities of the quantum theory as matrices is very doubtful. What is really essential in this aspect of the matter is that $pq - qp$ should not be zero. This is practically the position taken by Dirac.

It will be seen that this theory of Heisenberg, Born, Jordan and Dirac is essentially and fundamentally one of discontinuities. I shall now pass on to the second way in which it has been attempted to widen the foundation of the older quantum theory, the so-called wave-mechanics. This method starts from the basis that all mechanics, including the quantum mechanics, is ultimately dependent on a wave-motion, and, in fact, that the solution of a mechanical problem is the solution of the partial *differential* equations of an appropriate wave-motion. This is evidently a theory whose basis is essentially one of continuity, and it is a most remarkable thing that the results of the two theories are *mathematically identical.* The foundations of this method are about 100 years old, and due to Hamilton. The Hamiltonian dynamics originated as a development of Hamilton's theory of optics, a fact which most people have forgotten.

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* Dirac, Proc. Roy. Soc., A, Vol. 109, p. 642 (1925); Vol. 110, p. 561; Vol. 111, p. 405; Vol. 112, p. 661 (1926); Vol. 113, p. 621 (1927).

† Born and Jordan, Zeits. für Physik., Vol. 34, p. 858 (1925).

§ 3. Hamilton's Wave Mechanics.

According to Fermat's principle, the track of the rays of light between any two fixed points is determined by the equation

$$\delta\int\frac{ds}{v}=0 \text{ (The principle of least time.)} \quad . \quad . \quad (1)$$

In general, the time should have an extreme value which may not in all cases be a minimum. In non-homogeneous media v will be a function of the co-ordinates xyz, through the relation $c/v=\mu(x, y, z)$, the refractive index; so that the principle of least time may be written

$$\delta\int\mu(x, y, z)ds=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The variations δ have to be carried out in such a way that the terminal points are fixed.

For a conservative system in which the kinetic energy T is a homogeneous quadratic function of the generalised velocity components, the principle of least action takes the form

$$\delta\int 2T dt=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For a single material particle this is the same as

$$\delta\int\sqrt{E-V}ds=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where E is the total and V the potential energy, and the total energy is understood to be the same in the actual as in the varied motion.

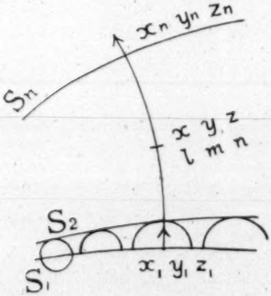

FIG. 1.

From (4) and (2) we see that the path of a moving particle in a space where the potential energy field is $V(x, y, z)$ is the same as that of a ray of light in the corresponding optical problem, provided

$$\mu(x, y, z)=\sqrt{E-V(x, y, z)} \quad . \quad . \quad (5)$$

Now we can work out geometrical optics either from Fermat's Principle or from Huyghen's Principle, and the results are the same. Turning to Fig. 1 we see that the successive wave fronts S_1, S_2, etc., are generated as the envelopes of infinitesimal spheres setting out simultaneously from every point of the preceding wave front and

$$t_n-t_1=\int_{x_1y_1z_1}^{x_ny_nz_n}\frac{ds}{v}=\frac{1}{c}\int_{x_1y_1z_1}^{x_ny_nz_n}\mu(ldx+mdy+ndz)$$

where $ds=ldx+mdy+ndz$ is an element of the ray path. l, m, n and μ are functions of x, y, z, but the integral on the right can only depend on the terminal values, so that

$$t_n-t_1=\varphi(x_ny_nz_nx_1v_1z_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This equation is the equation of the successive wave-surfaces. In isotropic media the ray s is always perpendicular to the wave front, and thus for every displacement $\delta x_1\, \delta y_1\, \delta z_1$

$$\frac{\partial\varphi}{\partial x_1}\delta x_1+\frac{\partial\varphi}{\partial y_1}\delta y_1+\frac{\partial\varphi}{\partial z_1}\delta z_1=0,$$

provided $l_1\delta x_1+m_1\delta y_1+n_1\delta z_1=0$, where l, m, n are the direction cosines of the wave normal s_1 to S_1 at x, y, z, and therefore subject to

$$l_1^2+m_1^2+n_1^2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It follows that

$$\frac{1}{l_1}\frac{\partial\varphi}{\partial x_1}=\frac{1}{m_1}\frac{\partial\varphi}{\partial y_1}=\frac{1}{n_1}\frac{\partial\varphi}{\partial z_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Similarly

$$\frac{1\,\partial\varphi}{l_n m_n}=\frac{1}{m_n}\frac{\partial\varphi}{\partial y_n}=\frac{1}{n_n}\frac{\partial\varphi}{\partial z_n} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and $$l_n^2+m_n^2+n_n^2=1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The 6 equations (6) and (8)–(10) enable $x_ny_nm_nl_nm_nn_n$ to be ascertained when $x_1y_1z_1l_1m_1n_1$ are given. Thus the optical paths in the medium are completely determined by a single function $\varphi(x_ny_nz_nx_1y_1z_1)$ (Hamilton's characteristic function).

But it is implicit in (5) that the same result must hold for the path of a particle in a corresponding dynamical system. If α and β are undetermined multipliers, we have from (8) and (9)

$$d\varphi=\alpha(l_1dx_1+m_1dy_1+n_1dz_1)+\beta(l_ndx_n+m_ndy_n+n_ndz_n)$$

In going out from S_n along ds_n φ increases by

$$d\varphi=\frac{\mu_n}{c}(l_ndx_n+m_ndy_n+n_ndz_n)$$

and in going out from S_1 along ds_1 it diminishes by

$$\frac{\mu_1}{c}(l_1dx_1+m_1dy_1+n_1dz_1).$$

Thus $$d\varphi=\frac{\mu_n}{c}(l_ndx_n+m_ndy_n+n_ndz_n)-\frac{\mu_1}{c}(l_1dx_1+m_1dy_1+u_1dz_1)$$

Putting $$X_nY_nZ_n=\frac{\mu_n}{c}(l_n,\ m_n,\ n_n),\quad X_1Y_1Z_1=\frac{\mu_1}{c}(l_1,\ m_1,\ n_1)$$

we get $$d\varphi=X_ndx_n+Y_ndy_n+Z_ndz_n-X_1dx_1-Y_1dy_1-Z_1dz_1$$

The quantities $X_nY_nZ_nX_1Y_1Z_1$ may be called the components of the normal *slowness* of the wave propagation.

We now proceed to the case in which S_n and S_1 are infinitely near wave fronts by putting $x_n=x_1+\lambda\delta t,\ y_n=y_1+\mu\delta t,\ z_n=z_1+\nu\delta t,\ X_n=X_1+p\delta t,\ Y_n=Y_1+q\delta t,\ Z_n=Z_1$

$+r\delta t$, $\varphi=W\delta t$ and $H=X\lambda+Y\mu+Z\nu-W$. We then find that if we regard H as a function of X, Y, Z, x, y and z,

$$dH=\frac{dx}{dt}dX+\frac{dy}{dt}\delta Y+\frac{dz}{dt}\delta Z-\frac{dX}{dt}\delta x-\frac{dY}{dt}\delta y-\frac{dZ}{dt}\delta z \quad . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

Thus

$$\left.\begin{array}{lll} \dfrac{dx}{dt}=\dfrac{\partial H}{\partial X} & \dfrac{\partial y}{\partial t}=\dfrac{\partial H}{\partial Y} & \dfrac{\partial z}{\partial t}=\dfrac{\partial H}{\partial Z} \\[2ex] \dfrac{\partial X}{\partial t}=-\dfrac{\partial H}{\partial x} & \dfrac{\partial Y}{\partial t}=-\dfrac{\partial H}{\partial y} & \dfrac{\partial Z}{\partial t}=-\dfrac{\partial H}{\partial z} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \quad (12)$$

If XYZ are the components of the momenta and x, y, z of the displacements, these are precisely Hamilton's canonical equations. They are clearly the differential equations which correspond to (6) and (8)—(10) when the optical parameters are translated according to (5). Conversely, (6)—(10) are the integrals of (12).

The integral in (3) or (4) may be written in the form $\int\Sigma p_s dq_s$, where the ps and qs are written instead of X and x, etc., as the components of generalised momentum and displacement, this being a more usual notation. Keeping, however, to the case of a single particle, let the variation of the integral be carried out in a rather different way, so that two infinitely near paths are compared which take the same time t_n-t_1, but do not have the same end points or the same total energy E. The result is

$$\delta\varphi=\delta\int_{t_1}^{t_n}(p_1\dot{q}_1+p_2\dot{q}_2+p_3\dot{q}_3)dt=\Big/ p_1\delta q_1+p_2\delta q_2+p_3 d\delta_3\Big/_{t_1}^{t_n}+(t_n-t_1)\delta E \quad . \; . \; (12.1)$$

whence

$$\left(\frac{\partial\varphi}{\partial q_1}\,\frac{\partial\varphi}{\partial q_2}\,\frac{\partial\varphi}{\partial q_3}\right)=+p_1p_2p_3 \text{ at } t_n \text{ and } =-p_1p_2p_3 \text{ at } t_1 \quad . \; . \; . \; . \; . \; . \quad (13)$$

and

$$\frac{\partial\varphi}{\partial E}=t_n-t_1 \text{ or } \varphi-E(t_n-t_1)=\text{const.} \quad . \; . \; . \; . \; . \; . \; . \quad (14)$$

Since φ is a function only of $x_n y_n z_n x_1 y_1 z_1$ and E, this equation is of exactly the same form as (6), which was got directly from Fermat's principle. If instead of taking the function φ we take another function φ', which is derived by subtracting the total energy from the integrand in (12.1), we have (Hamilton's Principle)

$$\delta\varphi'=\int_{t_1}^{t_n}(p_1\dot{q}_1+p_2\dot{q}_2+p_3\dot{q}_3-E)dt=\Big/ p_1\delta q_1+p_2\delta q_2+p_3\delta q_3\Big/_{t_1}^{t_n}-E\delta(t_n-t_1)\,. \quad (15)$$

since

$$\delta\int_{t_1}^{t_n}Edt=\int_{t_1}^{t_n}\delta Edt+E\delta(t_n-t_1)$$

if E is constant for the natural (unvaried) path. From (15) it follows that

$$-E=\frac{\delta\varphi}{\delta(t_n-t_1)} \text{ or } \varphi'+E(t_n-t_1)=\text{const.} \quad . \; . \; . \; . \; . \quad (16)$$

and

$$p=\text{grad }\varphi' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

By constant in (14) and (16) we mean independent of the time.

Since grad $\varphi' = p = \sqrt{2m(E-V)}$ (compare = "5), we can construct the surfaces φ', starting from any given value φ_0', by constructing the envelopes of the normals

$$\delta n = \delta\varphi_0'/\sqrt{2m(E-V)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or after the manner of Huyghens' principle. Now, if the time varies, it follows from (16) and (18) that the system of surfaces will not vary, but the values of φ' will travel from surface to surface along the normals with velocity, in the space of equation (18), u, given by

$$u = E/\sqrt{2m(E-V)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If, instead of supposing the surfaces to be fixed, we suppose them to wander so that each constantly takes the place of that which immediately preceded it, we get an equivalent picture which we may describe as a wave-motion in which the phase φ' travels with the velocity u. We may therefore call u the phase velocity. It is important to realise that this velocity is different from the velocity of the material particle. This is

$$v = (\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2)^{\frac{1}{2}} = [2(E-V)/m]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

All this is about a century old. The essence of it amounts to this: The motion of a particle in a field of force can be represented as a ray of a propagated wave surface. All classical dynamics is built up out of the motion of particles, and can therefore be equally well regarded as wave-motions with the energy travelling along certain rays, the paths of the particles. The correspondence holds on the optical side until the obstacles and apertures are diminished so that their dimensions become comparable with the wavelength. It does not include the diffraction group of phenomena.

§4. THE NEW WAVE MECHANICS.

A wave-theory of the motions of electrons in atoms has been developed in the last few years by L. de Broglie. The electron is regarded as a train of waves whose group velocity is shown to be v, the ordinary velocity of the electron, and whose phase velocity is C^2/v, where C is the velocity of light. I shall not follow de Broglie into the many interesting developments which he has made of these ideas. I shall, however, mention one of them which I believe to be of great importance. De Broglie makes this suggestion. Just as the corpuscular theory of light breaks down when the obstacles and apertures become comparable with the wavelength, so the particle theory of dynamics breaks down when the distances concerned are of atomic dimensions. And the reason is the same in both cases—namely, that both are wave phenomena, and in each case the dimensions concerned become camparable with the wavelength.

If we are to have a wave theory of mechanics valid down to atomic dimensions, one thing is clear—it will have to coincide with the Hamiltonian theory on the large scale. The way in which this extension is to be made is developed in a series of brilliant Papers by Schroedinger.* Let the wave-function be denoted by ψ. Then ψ will be of the form

$$\psi(x, y, z, t) = A(x, y, z)\sin\varphi'/K = A(x, y, z)\sin\left(-\frac{Et}{K} + \frac{S(x, y, z)}{K}\right) \quad . \quad . \quad . \quad . \quad (21)$$

This is required by (16), since $\varphi'(= -Et +$ a function of x, y, z only which we denote by $S(x, y, z)$) is a measure of the phase of the wave. Since Et and S have the dimensions of action, it is necessary to introduce the constant K in order to make the argument of the sine dimensionless. Now $A(x, y, z)$ represents the amplitude, and the frequency ν of the wave functions given by (21) is obviously

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* Ann. der Physik, Vol. 79, p. 361, p. 489, p. 734; Vol. 80, p. 437; Vol. 81, p. 109 (1926); Vol. 82, p. 257, p. 265 (1927). Phys. Rev., Vol. 28, p. 1049 (1926).

$$\nu = E/2\pi K$$

Schroedinger makes the very natural assumption that

$$K \text{ is a } \textit{universal constant} = h/2\pi, \quad \ldots\ldots\ldots \quad (22)$$

h being Planck's constant, which leads at once to the well-known universal relation between energy and frequency

$$E = h\nu \quad \ldots\ldots\ldots \quad (23)$$

The wavelength λ of the waves is

$$\lambda = \frac{u}{\nu} = h/[2m(E-V)]^{\frac{1}{2}} = h/mv \quad \ldots\ldots \quad (24)$$

Now for the nth Bohr circle of radius a_n we have $p = mva_n$ and $2\pi p = nh$, so that

$$a_n = nh/2\pi mv \quad \ldots\ldots\ldots \quad (25)$$

from which it is clear that h/mv is of the same order of dimensions as the atoms. This result is essential, because it is just at the atomic dimensions that the ordinary point-mechanics breaks down, and according to the ideas of de Broglie and Schroedinger it should break down when dealing with systems whose dimensions are comparable with the wavelength of the waves. This result also inspires confidence in the correctness of the assumption $K = h/2\pi$.

Equation (19) shows that the velocity of the waves depends on the frequency ν through the total energy E, so that the equation is an equation of dispersion. In the case of a single moving charged particle, the group velocity of the phase waves, which represent the motion, is equal to the actual velocity v of the moving particle.

On account of the breaking down of the ordinary mechanics, it is necessary to study the details of the wave-motion in dealing with atomic problems, and, as a preliminary to this, it is necessary to construct the equation of wave propagation. This process is in an experimental stage at present, the method adopted being to accept the simplest of any alternatives which offer themselves and to judge by the results. In the case of a single material point moving in an external field of force, Schroedinger takes the ordinary wave equation

$$\triangle^2\psi - \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad \ldots\ldots\ldots \quad (26)$$

and puts for u the value given by (19). This depends on the space co-ordinates through V and on the frequency ν through E. The connection between E and ν restricts the use of equation (26) to such functions ψ as depend on the time only through the factor $e^{\pm 2\pi i}\frac{E}{h}t$. Thus $\frac{\partial\psi}{\partial t^2} = \mp\frac{4\pi^2E^2}{h^2}\psi$. If we put

$$\psi = e^{2\pi i}\frac{E}{h}t\,X(x, y, z)$$

we get from (19) and (26)

$$\triangle^2\psi + 8\pi^2m(E-V)X/h^2 = 0 \quad \ldots\ldots\ldots \quad (27)$$

The wave function ψ is now to be derived by solving this partial differential equation, subject to such boundary conditions as may be appropriate to the problem.

Let us apply this to the hydrogen atom treated for simplicity as a single electron moving round a fixed inverse square centre of attractive force. Then $V=-e^2/r$, and in polar co-ordinates $r\theta\varphi$ (27) becomes

$$\frac{\partial^2 X}{\partial r^2}+\frac{2}{r}\frac{\partial X}{\partial r}+\frac{1}{r}\frac{\partial^2 X}{\partial \theta^2}+\frac{\cot\theta}{r^2}\frac{\partial X}{\partial \theta}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 X}{\partial \varphi^2}+\frac{8\pi^2 m}{h^2}\left(E+\frac{e^2}{r}\right)X=0 \quad . \quad (28)$$

The solution of this is sought in the usual way by putting $X=R(r)\times\theta(\theta)\times\Phi(\varphi)$, these functions each involving a single variable only, as indicated. With this substitution (28) can be broken up into three independent equations, and we find that Φ is the circular function $Ae^{in\varphi}+Be^{-in\varphi}$, θ is the spherical surface harmonic $P_m{}^n(\cos\theta)$, and $R(r)$ satisfies

$$\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}+\left(\frac{4\pi m_1 E}{h^2}+\frac{4\pi m_1 e^2}{h^2 r}-\frac{m(m+1)}{r^2}\right)R=0 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

($m=0$, 1, 2, 3, etc.). The limitation of n to whole numbers is necessary to make the dependence on the polar angles single valued. Now, Schroedinger has shown that the nature of the solutions of (29) depends very much on the value of the total energy E, which in this problem is quite definite, being equal to the sum of the kinetic energy of the moving electron and the potential energy $-e^2/r$. The only solutions of the equation (29) which are finite, continuous and single-valued throughout all space are those for which either

$$E>0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

or $$E=-2\pi m e^4/h^2 l^2,\ l=1,\ 2,\ 3,\ 4,\ \text{etc:} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Now the values (30) correspond to the hyperbolic orbits which are unquantized, and those given by equations (31) are the energy levels of Bohr's orbits. Actually; the functions which solve (29) are something like Bessel's functions, and are expressed by the following polynomial

$$X=x^m e^{-x}\sum_{k=0}^{l-m-1}\frac{(-2x)^k}{k!}\binom{l+n \text{ and}}{l-n-1-k},\ x=\frac{2\pi r}{h}\sqrt{-2m_1E} \quad . \quad . \quad . \quad . \quad (32)$$

so that the wave function which represents the whole motion will be

$$\psi=\Sigma\{A_n\cos n\varphi+B_n\sin n\varphi\}P_m{}^n(\cos\theta)x^m e^{-x}\sum_{k=0}^{l-m-1}\frac{(-2x)^k}{k!}\binom{l+n \text{ and}}{l-n-1-k} \quad . \quad (33)$$

where l, m, n are integers which are subject to the requirements $n<m+1$ and $l>m$ (there is no solution for $l<m$, the integral values are required to make the solutions single valued, and the solutions are only valid if $n<m+1$). These are all purely mathematical requirements well established in the theory of the harmonic functions which satisfy vibration problems; but they have a remarkable resemblance to the discontinuous requirements which we are familiar with in the theory of quanta. According to (31), l must be identified with the total quantum number. It takes the values 1, 2, 3 . . . ∞. The values admissible for m or n are 0, 1, 2, 3 . . . ∞. If we simply identify the azimuthal number k with $m+1$, the value zero for this quantum number is obviously excluded, as the facts require. It is a well-known*

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* Byerly's Fourier's Series and Spherical Harmonics, p. 197.

property of spherical surface harmonics (tesseral harmonics) that there are $2m+1$ of them of the mth degree. Since $2m+1=2k-1$, this seems to explain the mystery that there are $2k-1$ Bohr levels associated with the azimuthal quantum number k. In Schroedinger's theory these numbers m and n give the nodes of the postulated θ and φ vibrations. It appears that X regarded as a function of x (equation (32)) has exactly $l-m-1$ real positive roots, so that the number of spherical nodes is equal to the radial quantum number. It is interesting to inquire where these nodes are. They are all within a few Ångstrom units of the centre of the atom. After passing the last zero in going outwards from the centre of the atom, the function X soon reaches a maximum or minimum, and then rapidly tends towards zero permanently. Thus, although the theory presupposes the vibration phenomenon to extend from the centre of the atom to infinity, it is for all practical purposes confined within a region which we are accustomed to regard as of atomic magnitude. This is something like the theory of the electromagnetic mass of an electron which distributes the mass all over the universe except inside the electron in theory; but it nevertheless locates it at the electron for all practical purposes.

It is interesting to inquire what is to be expected of an atom of hydrogen when in its state of lowest azimuthal quantum number ($k=1$ or $m=0$), the so-called s state. Since $n<m+1$, $n=0$ and the tesseral harmonic reduces to $P_0(\cos\theta)=1$, so that

$$\psi_{m=0}=\sum_{k=0}^{l-1} Ae^{-x}\frac{(-2x)^k}{k!}\binom{l}{l-1-k},\quad x=2\pi r\frac{\sqrt{-2m_1E}}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

This expression is independent of the angles φ and θ, so that the atom in an s state possesses spherical symmetry. This result has been tested by R. G. J. Fraser, who orientated the hydrogen atoms in a beam of canal rays by a magnetic field (space quantisation). He then sought for a difference in the mean free path for the orientated, as compared with the non-orientated, atoms. The result was entirely negative, which is in accordance with the requirements of Schroedinger's theory, and entirely opposed to any other theory of atoms which works.

It is important to recognise that equation (27) is restricted to cases of not more than 3 dimensions in the space co-ordinates, such as that of the hydrogen atom treated as a one body problem. In general Schroedinger* has shown that for a system whose Hamiltonian function is

$$\tfrac{1}{2}\sum_{l=1}^{n}\sum_{k=1}^{n} a_{lk}p_lp_k+V_{(q_1q_2\ldots q_n)},\quad a_{lk}=a_{kl} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27.1)$$

the equation has to be replaced by

$$\Delta p^{\frac{1}{2}}\sum_{l}\frac{\partial}{\partial q_l}\left(\Delta_p{}^{-\frac{1}{2}}\sum_{k}a_{lk}\frac{\partial\psi}{\partial q_k}\right)+\frac{8\pi^2}{h^2}(E-V)\psi=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27.2)$$

where Δp is the determinant $|\Sigma\pm a_{ik}|$, when the frequency is restricted to E/h, or, if the frequency is unrestricted, it is to be replaced by

$$\Delta_p{}^{\frac{1}{2}}\sum_{l}\left(\Delta_p^{-\frac{1}{2}}\sum_{k}a_{lk}\frac{\partial\psi}{\partial q_k}\right)-\frac{8\pi^2V}{h^2}\psi\mp\frac{4\pi i}{h}\frac{\partial\psi}{\partial t}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27.3)$$

These more general equations are necessary for most of the problems mentioned in the sequel.

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* Schroedinger, Phys. Rev., Vol. 28, p. 1064 (1926).

I will now recount, without going into details, the results of the calculations of a number of problems by the new wave-mechanics.

(*a*) The Harmonic Oscillator.—The E levels are

$$(n+\tfrac{1}{2})h\nu_0 \qquad n=0,\ 1,\ 2,\ 3.\ \ldots \qquad (35)$$

instead of $nh\nu_0$.

(*b*) The Rigid Rotator with a Free Axis.—The E levels are

$$n(n+1)h^2/8\pi^2A \quad n=0,\ 1,\ 2,\ 3\ \ldots \qquad (36)$$

instead of $n^2h^2/8\pi^2A$. Since

$$n(n+1)=(n+\tfrac{1}{2})^2-\tfrac{1}{4} \qquad (37)$$

the new formula is equivalent to half quantum numbers where the differences of the levels alone are concerned.

(*c*) The Diatomic Molecule.*—The result is the same as the ordinary theory, except that half quantum numbers appear throughout.

(*d*) The Hydrogen Molecule Ion H_2+.—It appears† that the lowest azimuthal quantum number which is admissible is $k=m+1=2$, and that the ionisation potential of H_2+ is exactly equal to that of the neutral H atom—i.e., $Rh=13.5$ volts (R being Rydberg's constant). By imagining the neutral hydrogen molecule to be dissociated into hydrogen atom ions and electrons in two alternative ways it is easy to show that

$$I_{\mathrm{H2}}+I_{\mathrm{H2}+}=D+2I_{\mathrm{H}} \qquad (38)$$

Here I_{H2}, $I_{\mathrm{H2}+}$ and I_{H} are the ionisation potentials of H_2, H_{2+} and H respectively, and D is the heat of dissociation of H_2. If $I_{\mathrm{H2}+}=I_{\mathrm{H}}$ it appears that

$$I_{\mathrm{H2}}=D+I_{\mathrm{H}}=D+13.5 \text{ volts} \qquad (39)$$

The reliable experiments fix the value of I_{H2} at 15.9 volts almost with certainty to 0.1 volt, which gives $D=2.4$ volts. The most reliable estimate of D is about 4.3 volts.‡ Whether this discrepancy is in the experimental value of D or in the calculations is uncertain. But, at all events, the fact that the estimated value of I_{H2} is pulled down a lot from the value 23.7 volts, calculated by Pauli on the old mechanics, is hopeful.

(*e*) The Stark Effect.—The levels agree exactly with those calculated by Epstein, which are known to agree well with the experiments. The calculated intensities also agree well with the experimental estimates. In the calculation of the intensities of components of the hydrogen lines in the Stark effect it is necessary to make the hypothesis that the complex quantity ψ multiplied by its conjugate complex quantity represents the density of electric charge in the generalized space of the problem. In the simple hydrogen case this is the actual charge density, but in the more complex cases the interpretation is more involved, and perhaps still uncertain.

(*f*) Dispersion.—It is to be remembered that in the present theory the terms E_m are regarded as representing real vibrations of frequency E_m/h, and the line frequencies of the emission spectra are equal to the differences of their frequencies.

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* Fues Ann. der Physik, 80, 367 (1926).

† Alexandrow, Ann. der Physik, 81, 613 (1926).

‡ cf., O. W. Richardson, Proc. Roy. Soc., A, Vol. 113, p. 414 (1926).

It results from the theory that any atom with characteristic values E_m, etc., exposed to radiation of frequency ν has excited in it vibrations of the frequencies $\frac{1}{h}E_m \pm \nu$ (2 for each characteristic value), and these combine with the permanent vibration of frequency E_m/h to absorb or emit the observed scattered and refracted rays of the required frequency ν. The actual dispersion formula is very similar to one based on considerations of the Correspondence Principle, brought forward by Kramers to bring the refractory subject of dispersion within the quantum fold.

(*g*) The Compton Effect.—This has been treated in detail by W. Gordon,* who calculates both the intensity and frequency of the scattered radiation. This calculation is difficult, but the relations between the direction and frequency of the scattered radiation and the direction and velocity of the moving electron, which were deduced by Compton by treating the problem mechanically as that of the impact of a light dart on an electron, have been obtained by Schroedinger in a rather simple but very curious way. It has been shown by Brillouin† that the reflection of light, of wavelength λ, by a sound wave of wavelength L, can be deduced by representing the sound wave as a crystal whose atoms are in planes L apart. The light is found to be reflected at an angle θ given by Bragg's law, for the first order, $2L \sin \theta = \lambda$. If now the moving electron is represented by the equivalent "wave of electric density" according to Schroedinger's‡ ideas, and this is replaced by the equivalent crystal (in the method of Brillouin), then Bragg's reflection law, for the first order, gives the facts of the Compton effect as stated already, the incident light replacing the incident light in the crystal reflection case. In both these problems the crystal must, of course, be treated as a moving crystal, and the Doppler effect taken into account.

Schroedinger makes the following claim for the advantages of the wave-mechanics :—

(*a*) The laws of motion and the quantum conditions are deduced simultaneously from one simple Hamiltonian Principle.

(*b*) The discrepancy hitherto existing in quantum theory between the frequency of motion and the frequency of emission disappears in so far as the latter frequencies coincide with the differences of the former. A definite localization of the electric charge in space and time can be associated with the wave-system, and this, with the aid of ordinary electrodynamics, accounts for the frequencies, intensities and polarizations of the emitted light, and makes superfluous all sorts of correspondence and selection principles.

(*c*) It seems possible by the new theory to pursue in all detail the so-called transitions which up to date have been wholly mysterious.

(*d*) There are several instances of disagreement between the new theory and the older one as to the particular values of the energy or frequency levels. In these cases it is the new theory that is better supported by experiment.

If the present account does not establish these claims with conviction, it is not because the claims are overstated, but because this account is necessarily very brief and imperfect.

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* Zeits. für Physik., Vol. 40, p. 117 (1926).

† L. Brillouin, Annales de Physique, Vol. 17, p. 88 (1923).

‡ E. Schroedinger, Ann. der Physik., Vol. 82, p. 260 (1927).

To this I would add that the theory seems hopeful in some other ways. I will enumerate some of them.

(1) It offers, at any rate in principle, a way of escape from the first difficulty in our list. A co-ordination of phase between the exciting radiation and the material from which a photo-electric electron is ejected may be necessary. The requisite co-ordination may be a rather rare occurrence. However attenuated light is, we get interference (G. I. Taylor's experiment). This and other familiar phenomena show that light is not really a dart. How, then, shall we explain the photo-electric effect (or light absorption)? In the reverse process of recombination at the instant of interchange, the actual phase of the wave of the impinging electron will have to fit with the phase of the atom wave and with that of the emitted light in some appropriate way. In absorption or the photo-electric effect similar conditions will have to be worked backwards. The light may traverse many atoms before it comes to one in which the phases conspire in the right way. Owing to its high velocity of propagation, it does not give the atom much time to get into a suitable condition for interaction.

(2) The $\frac{1}{2}$ quantum numbers and the $j(j+1)$ factors referred to under (2) and (3) seem to come naturally out of this theory.

(3) I do not know that anyone has shown explicitly how the long free paths in argon come about, but there seems to be nothing in the theory which is abhorrent to them.

(4) The theory obviously gets rid of the difficulty about the orbital electrons intermingling, yet preserving their identities. What is supposed to be real in these consist of the modes of vibration of a pseudodynamical system.

(5) The calculated ionization potential of the hydrogen molecule seems still too high, although much lower than by the old methods. However, it is not certain that either the calculations or the experimental evidence are yet final in this matter. The neutral helium atom has not yet been calculated.

(6) It is too early yet to say whether it will account for the peculiar results found in the soft X-ray field and Davisson's results in electron reflection, but it seems in a general way to be the type of thing which is required for these phenomena.

Before concluding I should like to mention one or two difficulties in the way of the wave-mechanics. (1) In its present form it appears to offer no explanation of the atomicity of matter and electricity. (2) The way in which it uses the expression $-e^2/r$ for the potential energy in the hydrogen problem seems peculiar, since, according to the solution of the problem, the charge e is no longer at a distance r from the nucleus. In fact, in general, the theory seems to have to work with prescribed potentials given by the ordinary theory, and not with the potentials which it would itself deduce if it were a consistent theory. It is difficult to know what weight to assign to these objections at this stage. My own feeling is that, unless there is a substantial measure of truth in the theory, it will prove one of the most remarkable coincidences in the history of physics.

I wonder if I have removed that feeling of perplexity which I referred to at the beginning of my address. I do not expect I have. At any rate, I think you will probably agree with me that if the last thirty years has been the most wonderful in the history of physics, the next thirty promise to be equally interesting.

XIV.—ELECTRICAL POLARISATION IN SELENIUM CELLS AND THE EFFECTS OF DESICCATION.

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Received June 25, 1926.

ABSTRACT.

The Paper describes measurements of the secondary E.M.F.s displayed by selenium cells and the apparently abnormal effect of illumination on the corresponding secondary currents. Both are shown to be probably due to an invisible water film in parallel with the selenium. That this film is the principal seat of the polarisation seems to be proved by the fact that the polarisation disappears almost completely with prolonged drying. On this basis a satisfactory quantitative explanation of the various effects is given. An important practical consequence of the desiccation of the cell is the large increase obtained in its sensitivity to light.

INTRODUCTION.

THE investigation to be described originated in the observation that certain selenium cells, which had been used previously by one of the authors for photophonic purposes, exhibited secondary E.M.F.s after the passage of electric currents through them. This phenomenon was in itself not surprising; other types of selenium cells have probably frequently been observed to have a similar property. The peculiarity of behaviour which attracted attention was the fact that the polarisation voltage established with the selenium in darkness was very largely *diminished* by illumination, with partial recovery when the illumination was cut off. Even when the terminals of the selenium cell in the polarised condition were connected together by a constant very high resistance, so that it was in fact behaving as a secondary cell and giving out a small but finite current, the effect persisted—i.e., the potential difference between the terminals was *reduced* by illumination. The conclusion was inevitable that the secondary current given out by the polarised selenium cell also *became less* under the action of light, in contradistinction from the well-known enhancement of current which the same selenium cell, in common with others, displayed under illumination when used in the ordinary way. Apparently the cells, carrying a current driven by an externally applied voltage, possessed what is usually known as positive light sensitiveness, whereas they were *negatively* light sensitive in respect of their own polarisation currents.

But in the course of closer examination of this surprising phenomenon it was discovered, almost accidentally, that the polarisation itself was profoundly modified by long exposure of the cell to the effects of phosphorus pentoxide. In these circumstances the capacity of the selenium cells to be polarised almost completely disappeared, and the electrical resistance greatly increased. It has been considered reasonable to attribute these changes to thorough desiccation, and the behaviour of the cells can be accounted for satisfactorily, except in one respect of relatively small importance, on the assumption that the polarisation observed was not due to the selenium itself, but to something in the nature of a water film, removable by persistent drying, in parallel with the selenium between the electrodes of the cell.

An important practical result of the investigation is the experimental fact, which accords with the theory put forward, that the light sensitiveness of the selenium cells is greatly increased by desiccation. Drying reduces the absolute conductance, but the *ratio* of the conductance under light action to that in the dark is much enhanced.

DESCRIPTION OF THE SELENIUM CELLS.

The selenium cells used were made to the design of Prof. H. Thirring, and the construction has been described by him.* The electrodes are of copper foil, arranged in the form of a minute condenser, the sheets being separated by mica. The latter is slightly recessed, and the selenium layer, about 0·01 mm. thick, is spread over the copper edges so as to fill the shallow interspaces. A circular area about 6 mm. in diameter is so covered, and protected by a superimposed sheet of transparent celluloid. A sheet of ebonite with a circular hole in the appropriate position limits slightly the surface of selenium exposed, and forms the front of the cell, which, however, is not hermetically sealed. The whole unit is in the convenient shape of a rectangular parallelepiped 4 cm. long, 2 cm. broad and 1 cm. deep, the face containing the selenium layer being 4 cm. by 2 cm. The electrodes are connected to two brass blocks, insulated from one another by ebonite, which form the 4 cm. by 1 cm. faces, and the cell is conveniently held for experimental purposes by insulated metal spring clips bearing on these surfaces and connected to suitable terminals.

The cells, as supplied, although of like dimensions and nominally of the same construction, vary considerably in electrical resistance. In complete darkness this is of the order 10 to 20 megohms. In common with other selenium cells, those under consideration vary considerably in conducting power during use, for reasons which sometimes remain obscure. The light sensitiveness also is a variable factor, differing from cell to cell, and changing in one and the same cell with progress of time and use. On the average it has a high value; the current carried, for the relatively high voltages customarily applied (20 volts or so), is increased something like ten-fold by the illumination due to 100 candles at one metre.

PRELIMINARY EXPERIMENTS.

In order to observe the polarisation E.M.F. of the selenium cells it was necessary, because of their high resistance, to use an electrostatic method of measurement. A Dolezalek quadrant electrometer was therefore employed as a voltmeter in parallel with the terminals of the cell. The insulation was found to be sufficiently high for the purpose by measuring the rate of leakage of a mica condenser of known capacity. Under these conditions the insulation resistance proved to be approximately 15,000 megohms, and since the condenser used for this test was no doubt itself responsible for part of the leakage, it was safe to conclude that the insulation of the electrometer alone was in excess of the above value. Compared with this, even the high internal resistances of the selenium cells were negligible, and practically the full value of any polarisation E.M.F. was therefore recorded. As used, with a needle voltage of about 94, the sensitivity of the electrometer was such as to give approximately

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* Phys. Zeit., Vol. 21, p. 67 (1920), and Proc. Phys. Soc., Vol. 39, p. 97 (1926).

1 mm. scale deflection per millivolt. The motion of the needle was highly damped, so that practically no oscillation occurred, but the slowness of the motion prevented reliable readings being taken until one minute after a potential difference had been applied to the quadrants. Only slow variations of this difference could be properly observed.

The currents passing through the selenium cells due to applied voltages were measured by means of a 400 ohm moving-coil galvanometer originally giving a scale deflection of about 3·6 mm. for 10^{-8} ampere; the same galvanometer was sometimes used also to observe the secondary currents in the selenium cells, which in these circumstances were virtually short-circuited by the galvanometer, the internal resistance of the selenium being relatively extremely high.

Only small voltages (of the order of 1 volt) were applied to the selenium cells in these experiments. There were two main reasons for this; first, in order that the effects of possibly small polarisation on the current might not be obscured; and, second, to avoid, if possible, the well-known increase of the conducting power of selenium which arises from the continued application of the considerable voltages customarily used. From both these points of view the choice of voltage made appears to have been correct.

The selenium cell under investigation was held in a clip with ebonite insulation, red fibre having previously been rejected on account of its relatively high conducting properties and liability itself to polarise. The whole was enclosed in a wooden box which could be rendered light-tight or opened for purposes of irradiating the selenium. Having applied a measured voltage to the selenium for a measured time, the current carried being observed meanwhile, the applied voltage was removed by breaking the circuit, and the electrometer immediately connected to the terminals of the selenium cell. Readings of the electrometer were then commenced as soon as possible, and continued for considerable periods of time, thus recording the decay of the polarisation E.M.F. Usually the polarisation of the selenium cell and the early subsequent electrometer readings were carried out with the cell in complete darkness; then, at a chosen time, the enclosing box was opened, thus exposing the cell to illumination, for a period long enough to observe the accompanying fall of E.M.F. The box was then closed again and the subsequent growth of E.M.F. recorded.

All the selenium cells tested—some half-a-dozen—showed the effects to be described, although in markedly different degree. For a detailed investigation one was chosen for which the polarisation was large, and typical results for this are shown in Fig. 1. The upper curve in this diagram shows the decay of polarisation voltage, the selenium cell being all the time in darkness. The polarisation here is the result of the application of 1 volt to the selenium cell for one hour, during which the current in the selenium had fallen from a value in excess of $5{\cdot}9 \times 10^{-8}$ amp. to $4{\cdot}5 \times 10^{-8}$ amp. in the first 30 minutes, and had thereafter remained practically constant until the removal of the voltage. The first reliable reading of the electrometer, taken one minute afterwards, was equivalent to 0·269 volt—a value too high to insert conveniently in the diagram. This upper curve is included for purposes of comparison with the lower one, which relates to the effect of illumination on the polarisation E.M.F. of the same selenium cell on another occasion. The decay of the E.M.F. was here allowed to proceed until it was not too rapid, the cell having been previously polarised by the same voltage (1 volt) for the same length of time (one hour). At the time indicated by *A* the cell was exposed to Pointolite lamp illumination approximately equal to $400 \frac{\text{candles}}{\text{metre}^2}$. One minute afterwards the small electrometer reading *B* was taken and the illumination immediately cut off. The subsequent changes of the E.M.F. are shown—first a substantial rise and eventually a resumption of the decay. The dotted curve *AC* shows by comparison with the upper curve what probably would have been the course of events if the cell had not been irradiated; for, although the decay curves were not strictly reproducible, they were approximately so and of the same general shape. It is noticeable that the recovery of the E.M.F. after the cessation of illumination is slow, just as is the recovery in the normal photo-electric effect of the cells; but it seems likely that after a sufficient time the recovery curve would coalesce with the continuation of *AC*.

The remarkable effect displayed by the portion *AB* of the curve is that the illumination causes a large *reduction* of the difference of potential between the terminals of the cell. Since the insulation was not absolutely perfect this must be interpreted as corresponding to a *reduction*, due to illumination, of the small current flowing in the circuit. In other words, the current through the selenium cell was apparently *diminished* by illumination when it was due to its own polarisation E.M.F., while the same cell, actuated by an externally applied E.M.F. gave, under conditions otherwise the same, the normal enhancement of current.

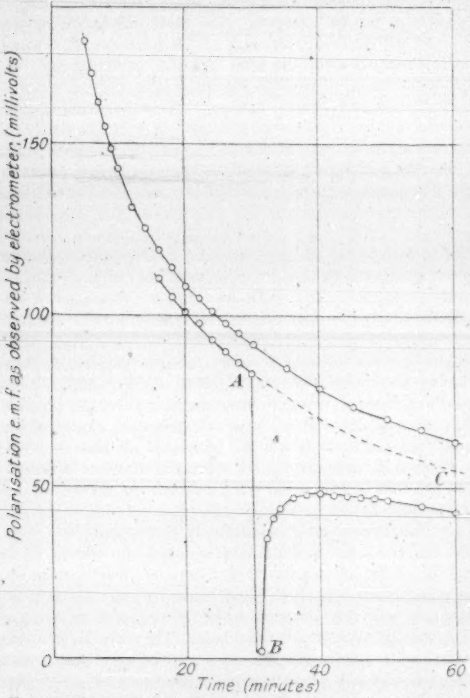


Fig. 1.—Upper Curve: Decay of Polarisation E.M.F. of Selenium Cell 466 in dark. Lower Curve: Effect of Illumination. $400 \frac{\text{Candles}}{\text{Metre}^2}$

Experiments were also carried out with high resistances, of the same order as that of the selenium cell itself, in circuit with the cell, so as to cause the polarisation

E.M.F. to produce larger and definitely measurable currents. The electrometer readings were thus, of course, reduced approximately to one half, but qualitatively the effect of illumination was the same as before—namely, a large reduction of the difference of potential between the selenium cell terminals, and, by inference, of the current flowing through the external high resistance.

It was found also that the cells could be polarised while exposed to light, although the effect was revealed to a marked extent only after the illumination had been cut off. During polarisation under these conditions the current carried was, of course, much larger, but after the removal of the applied voltage, the cell still being illuminated, the polarisation E.M.F. measured by the electrometer was found to be very low. On darkening the cell, however, this E.M.F. rose to a maximum and eventually decayed after the manner of the curve in Fig. 1 subsequent to the point *B*.

The next step was to make current observations directly with the cell virtually short-circuited through the galvanometer immediately after polarisation in the manner already described. At this stage of the experiments the accuracy of the direct readings of current thus obtainable was low, since the sensitivity of the galvanometer was only sufficient to give readings of the order of a few millimetres at most. By the time the observed polarisation current had ceased to decay too rapidly—a necessary condition for satisfactory comparison—the galvanometer deflection would be only one or two millimetres. Under these conditions it was found that illumination produced *no observable effect* on the polarisation current. Subsequent measurements of a more accurate character, recorded later, have shown this not to be strictly true, but as a first approximation it is sufficient to indicate the great difference of the behaviour of the observed polarisation current obtained from the selenium cell according to whether the resistance external to it is high or very low.

The above are the main observations which led to a closer and more accurate investigation of the phenomenon, with a view to establishing quantitative relationships in what appeared to be a new photo-electric property of selenium. This work, now to be described, discovered instead the origin of the effects not to be in the selenium itself, which, apparently, is responsible for little, if any, of the observed polarisation.

Experiments with Improved Arrangement of Apparatus.

The new arrangement of the electrical circuit was fundamentally the same as before, permitting either the observation of the polarisation E.M.F. of the selenium cell on " open circuit " by means of the electrometer, or the polarisation current by means of the galvanometer which then virtually short-circuited the cell. A potentiometer was added to enable calibrations of the electrometer and galvanometer to be made from time to time as necessary, and to provide a range of voltages for application to the selenium cell. It was also found possible to increase the sensitivity of the galvanometer, which had a mirror of sufficiently good optical quality, by the expedient of increasing to more than 4½ metres the distance from the mirror to the observing scale. The sensitivity was thus raised to 16·4 mm. per 10^{-8} ampere, as compared with 3·6 mm. previously. This enabled the direct observations of current to be made much more accurately than in the preliminary experiments. It was also arranged so that the zero could be checked without disturbing the current through the cell. The zero actually remained remarkably constant.

The period and damping of the galvanometer suspension were such that a reliable current reading could be taken with a lapse of only five seconds after large changes of current, such as those involved by making and breaking the circuit. Fig. 2 shows a typical polarisation curve, obtained by applying 1 volt for one hour to a selenium cell, and then short-circuiting the cell through the galvanometer. The general behaviour appeared to be similar, although on a very much smaller scale as regards current, to that of an ordinary voltameter. An attempt was therefore made to find a polarising voltage small enough to be opposed completely by the back E.M.F., so that the current would be reduced to zero. Back E.M.F.s of the order of $\frac{1}{3}$ volt had been obtained with the application of 1 volt; accordingly $\frac{1}{3}$ volt was applied to the same cell, and the diminution of current observed. It was found that the current fell from the value $1{\cdot}95 \times 10^{-8}$ amp. at 30 seconds after the application of the voltage to $1{\cdot}25 \times 10^{-8}$ amp. after half an hour, but that, although the voltage application was continued without intermission for approximately 45 hours, a current of $0{\cdot}92 \times 10^{-8}$ amp. still survived, and showed no tendency further to diminish. The back E.M.F. was thus found not to be a constant quantity, but to depend on and diminish with the applied voltage continuously. This was confirmed by a series of observations with a range of applied voltages between 0·33 and 1, in which electrometer measurements were made after polarisation until the current was steady. It was found that the voltage-time curves (of the type of the upper curve in Fig. 1) were all similar to one another—i.e., the ordinates at any time were in constant proportion, and that the polarisation voltages represented by these ordinates were roughly proportional to the voltage originally applied. This is seen in the following table :—

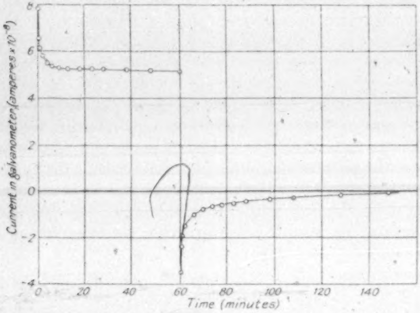


FIG. 2.—POLARISATION OF SELENIUM CELL 466 AS SHOWN BY GALVANOMETER VIRTUALLY ACTING AS SHORT CIRCUIT. ONE VOLT APPLIED FOR 1 HOUR; THEN REMOVED FROM CIRCUIT.

TABLE I.

Applied voltage.	$\frac{\text{Polarisation voltage.}}{\text{Polarisation voltage due to 1 volt.}}$
1·00	1·00
0·70	0·65
0·60	0·57
0·50	0·52
0·40	0·39
0·33	0·37

The increased sensitivity of the galvanometric arrangement made possible a more accurate investigation of the effect of illumination on the polarisation current in the short-circuited selenium cell. The cell, having been polarised by 1 volt for one hour, was connected to the galvanometer and illuminated and darkened successively for equal periods at regular intervals, at first one minute, and later with longer intervening periods. The results are shown in Fig. 3, from which it will be seen that although the effect of illumination is small—quite negligibly so in comparison with the normal effect in selenium—the current is definitely increased by light. The increase, for the illumination specified, is about 10 per cent., compared with about 2,000 per cent. given by the cell under normal conditions.

Comparisons of one selenium cell with another showed that polarisation for equal periods produced effects of the same kind but widely different in magnitude. The course of the decay of polarisation E.M.F. was, however, similar in all cases, the curves being related by proportionality of ordinates at corresponding times.

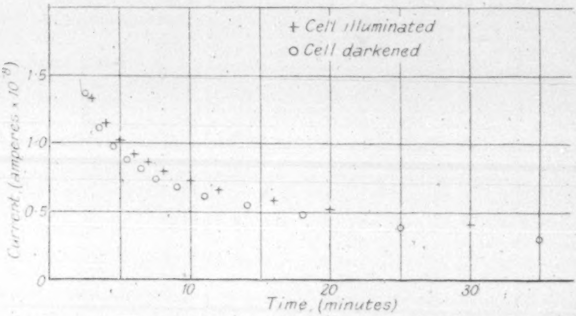


Fig. 3.—Effect of Illumination of 400 $\frac{\text{Candles}}{\text{Metre}^2}$ on Current through Galvanometer Short-circuiting Polarised Selenium Cell 466.

In obtaining the above results the importance of the effects of temperature had early been recognised as the possible cause of the observations not being strictly reproducible. Evidence was obtained that the current in the selenium cell with a constant applied voltage, and also the polarisation E.M.F. itself, increased rapidly with temperature. Steps were therefore taken to obtain more precise control over the temperature, so as to study the phenomena with more exactitude.

Effects of Desiccation.

The wooden box containing the selenium cell was itself placed in a metal enclosure surrounded by melting ice, 0°C. having been chosen as a suitable constant temperature for the continuation of the observations. In order to avoid the possible condensation of water on the cell and its connections, owing to the air becoming saturated, a tray of fused calcium chloride was introduced with the box. A constant temperature of 0·7°C. was reached in the locality of the selenium cell, and it was found that thepolarisation of the cell was much reduced, the back E.M.F. being little more than one-fifth of the previous value at atmospheric temperature (about 15°C.). This was naturally at first attributed to the temperature change, but even with the now constant temperature strict reproducibility of observations was still not possible. On allowing the cell to return to atmospheric temperature, while leaving it continuously in darkness and exposed to the drying agent, it was found that the cell had not recovered even approximately its former polarisation properties. The gradual reduction of current on applying 1 volt had almost disappeared; the actual current was diminished to about one-half; and the polarisation E.M.F., as indicated by the electrometer, had similarly diminished. Thus the effect of the drying agent was made evident.

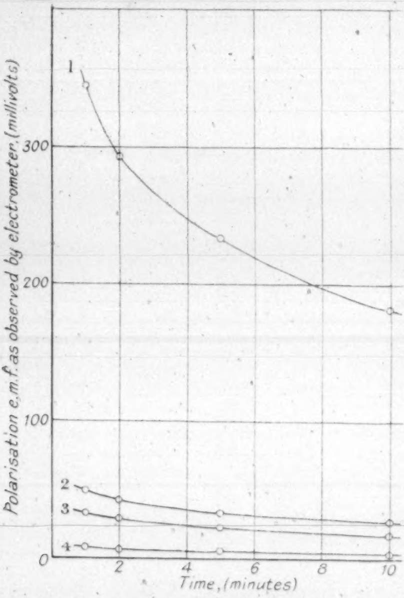


FIG. 4.—POLARISATION OF SELENIUM CELL 466—EFFECT OF DESICCATION.

Curve 1 Drying—none
,, 2 ,, 6 days
,, 3 ,, 10 ,,
,, 4 ,, 33 ,,

At this juncture a specially designed and newly made container was brought into use. It is unnecessary here to describe it in detail. The essential features were as follows: It consisted of an air-tight vertical brass cylinder with an ebonite top fitted with a double glass window and carrying suitable insulated terminals to lead to the enclosed selenium cell and the thermo-couple used to determine the temperature of the cell. The cylinder was actually immersed as deeply as possible in a large water bath thermally insulated, but was also suitable for jacketing with steam, etc. The selenium cell was supported inside with its sensitive surface facing upwards and its temperature was ascertained by a thermo-couple, one element of which was inserted in a small brass block soldered to one of the brass clips which made the electrical contacts with the cell, and the other in the water bath.

This couple showed any small difference in temperature between the cell and the water bath, the temperature of the bath being read with a mercury thermometer graduated in 1/10°C. A vessel containing phosphorus pentoxide was placed at the bottom of the container, so that prolonged drying of its contents might progress. With the arrangement described it was found possible to maintain the temperature of the selenium cell constant at 15°C. to within $\frac{1}{3}$° during a whole day.

With the selenium cell in darkness continuously and with the temperature adjusted very nearly to 15°C. during observations, readings were taken of the current due to the application of 1 volt and of the polarisation E.M.F. subsequent to one hour's application of this voltage. The readings extended over 33 days. Progressive drying was accompanied by a marked diminution of the actual current carried and the rapid disappearance of both the fall of current after the application of the voltage and the secondary E.M.F. of polarisation. The effects as regards the current are shown in Table II, and Fig. 4 shows the behaviour of the polarisation E.M.F. as measured by the electrometer.

TABLE II.—*Current Produced in Selenium Cell 466 by the Application of 1 volt During the Process of Desiccation.*

Duration of drying in days.	Current in 10^{-8} amp.			
	After 10 secs.	After 1 min.	After 10 min.	Steady value.
None	8·45	7·03	5·96	5·92
6	2·61	2·57	2·56	2·55
10	2·37	2·37	2·37	2·37
33	2·18	2·18	2·18	2·18

In the curves in Fig. 4 the ordinates are throughout in constant proportions to within the accuracy of the observations. These proportions, as percentages of the effects in the undried cell (Curve I) are given in Table III, which makes evident the rapid reduction of polarisation procured by drying, and points to the probability of its complete disappearance ultimately.

TABLE III.—*Variation of Polarisation E.M.F. in Selenium Cell 466 with the Period of Drying of Cell.*

Duration of drying in days.	$\frac{\text{Polarisation E.M.F.}}{\text{Polarisation E.M.F. in undried cell}} \times 100$
0	100·0
6	14·5
10	9·7
33	2·2

No simple relation has been found connecting these quantities; it is not, for example, an exponential one.

PROPERTIES OF THE DRIED CELL.

It was anticipated that the removal of the polarisation described, which was presumably attributable in some way to moisture of unknown, and perhaps varying amount, would result in the selenium cell becoming more constant in its electrical properties. This, to a certain extent, proved to be the case. Within certain limits of applied voltage the current was proportional to it, and did not vary from time to time for no apparent cause as had happened in undried cells. Provided the temperature was the same the current could be reproduced by applying the same voltage, and the cell displayed a definite resistance of about 46 megohms at 15°C. Above about 1·5 volts, however, Ohm's law ceased to be valid, the current becoming more and more in excess of that appropriate to this law as higher potential differences were used. At the same point, also, the well-known "enhanced conductance" effect began to appear—namely, that continued application of the voltage caused the current to grow with time. The apparent resistance of the cell became variable between wide limits, depending upon its previous history, and a long time was required for the cell to regain its primitive condition. Thus the property of the dependence of the conductance upon the duration of the current flow is assignable, in part at any rate, to the selenium itself, and is not wholly removed by desiccation. No constant resistance can be specified for the selenium cell except with a limitation as to the maximum current used. For the cell in question this maximum was

$$\frac{1\cdot 5}{46\times 10^{6}}=3\cdot 3\times 10^{-8} \text{ ampere approximately.}$$

Desiccation produced a striking enhancement of the light sensitivity of the selenium cell—that is, the ratio, with a fixed applied voltage, of the current in the cell under illumination to that in complete darkness was much increased. An exact figure for the ratio cannot, unfortunately, be given, since the need, now apparent, for precise measurements before desiccation was not foreseen. Moreover, a breakdown of the illuminating lamp during the progress of the work makes strict comparison still more difficult. The combined evidence of various pieces of experimental data points, however, to the conclusion that desiccation had increased the light sensitivity in the proportion of about 4 : 1, and that this increase was due almost entirely to the *reduction* of the "dark" conductance of the cell. This, it will be seen, is in accordance with the theoretical explanation of the general phenomena which it is now proposed to present.

Proposed Explanation of the Effects.

The effects which have been described can be accounted for satisfactorily on the assumption that something in the nature of a water film exists in the virgin selenium cell in such a way as to form a conducting path between the electrodes, in parallel with the selenium itself, and that the polarisation which is observed is due, in the main at least, to the development of a back E.M.F. in the water film during the passage of currents from extraneous sources. The exact localisation of the supposed water film, which, of course, is not visible, remains obscure; whether it is on the selenium itself, the celluloid cover or the ebonite insulating the electrodes, or, indeed, whether it is in the form of a film at all, has not been decided. But the almost complete disappearance of the polarisation with prolonged exposure to a drying agent, accompanied as it is by a reduction of the conductance of the cell, is in qualitative agreement with the previous existence of the film as the seat of the polarisation and the conductor of additional current. It is possible to go further, and, from available data, to obtain consistent quantitative agreement on the above assumptions.

For this purpose we may regard the virgin (i.e., undried) selenium cell as consisting of two conductors in parallel, first the selenium itself having a resistance S, and, second, a water film of resistance W, capable of developing a maximum back E.M.F. e_w as a consequence of the continued application of a definite voltage. In the thoroughly desiccated cell e_w may be regarded as having vanished, and W as having become infinite, leaving only S as the available conductor, in which there is little, if any, polarisation. On this basis, S has already been estimated to be 46 megohms.

In order to estimate e_w and W from the observations it is not possible to proceed directly, because the recording instruments—galvanometer and electrometer—were not capable of giving respectively the initial values of the total current upon application of the voltage and the potential differences between terminals of the cell upon the removal of the voltage. Further, the experimental curves are so steep in the regions of the first readings that extrapolations to time=zero would constitute little more than guesses. The curves only provide certain limiting values; for example, that W is less than 16 megohms and that e_w is greater than 344 millivolts. There is, however, a definite relation between e_w and W, so that if one is known the other is determinate. Referring to the current values in Table II, the steady value of the total current in the undried cell, $5{\cdot}92\times10^{-8}$ ampere, is the sum of the currents in the selenium and in the water film. The former is $2{\cdot}18\times10^{-8}$ ampere, and the difference between this and the total current—namely, $3{\cdot}74\times10^{-8}$ amp.—passes through the water film under the action of 1 volt less the back E.M.F., which at this stage has become fully established. Thus

$$1\text{ volt} - e_w = 3{\cdot}74\times10^{-8}\text{ amp.}\times W \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

From this point the procedure has been to choose values of W (and by consequence of e_w), and by trial and error find which gives the most consistent quantitative account of the various phenomena observed, while not being inconsistent with other checks such as those mentioned earlier. The value so obtained for W—namely, 12 megohms—and the corresponding value of e^w (550 millivolts) do in fact provide a quite satisfactory explanation of all the phenomena. Referring again to Table II, we may find the initial value of the total current by recognising that then there is no back E.M.F., and that the current in the water film is $\dfrac{1\text{ volt}}{12\text{ megohms}} = 8{\cdot}33\times10^{-8}$ amp. The total current is this plus that in the selenium ($2{\cdot}18\times10^{-8}$ amp.), and equals $10{\cdot}5\times10^{-8}$ ampere, a value which is smoothly connected with the subsequent values. With regard to the electrometer readings (uppermost curve in Fig. 4), equation 4, which is developed later, gives the unobserved initial value as

$$V = \frac{550}{1+\dfrac{12}{46}}\text{ millivolts} = 440\text{ millivolts.}$$

This again is in keeping with the general subsequent trend of the curve.

We have now to consider the effect of illumination on the system assumed when polarisation has been established and the current output from the system itself is being measured. In order on the one hand to preserve generality, and on the other to take account of a possible relatively small polarisation in the selenium itself, we shall suppose that a back E.M.F. e_s exists in the selenium. The system under consideration will thus be that represented in Fig. 5, where R is the resistance of the measuring instrument. Application of the current distribution laws gives the relation

$$C\left(1+\frac{R}{S}+\frac{R}{W}\right)=\frac{e_w}{W}+\frac{e_s}{S} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where C is the current in the resistance R in which measurements are made. The effect of illumination may be supposed to have the single effect of reducing the value of S to a value S', according to the recognised property of selenium. In the actual experiments with the desiccated cell the ratio S/S' was found to be about 90 for the illumination used—namely, the equivalent of 100 c.p. at 50 cm. For purposes of calculation the values taken will therefore be

$W=12$ megohms.
$S=46$ „
$S'=0{\cdot}5$ „

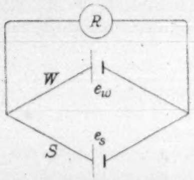


Fig. 5.—Schematic Representation of Undried Selenium Cell after Polarisation and under Observation with Polarisation Measuring Instrument of Resistance *R*.

(*a*) *Measurements with galvanometer.*

In this case the value of R is 400 ohms—a quantity negligible in comparison with W, S and S'. Equation (2) therefore reduces to

$$C=\frac{e_w}{W}+\frac{e_s}{S}$$

It follows that if the selenium itself displays no polarisation—i.e., if $e_s=0$,

$$C=\frac{e_w}{W}$$

and the current is thus independent of the changes of resistance of the selenium accompanying illumination. This we have seen (Fig. 3) to be nearly true, and the main fact is therefore accounted for without assuming that the selenium polarises. The slight increase of current on illumination is, however, most easily explained by attributing to e_s a small value, which can be roughly calculated from the observations. (It should be noted that the actual current variation is in the opposite sense to that which the finite resistance of the galvanometer would produce—an effect which, in fact, is negligible.) If we suppose that the change of S to S' under illumination is the only photo-electric change, the current C' will be

$$C'=\frac{e_w}{W}+\frac{e_s}{S'}$$

and the current difference is given by

$$C'-C=e_s\left(\frac{1}{S'}-\frac{1}{S}\right)$$

According to the data represented in Fig. 3, the value of $C'-C$ is of the order of 10^{-9} ampere throughout, with a slight tendency to diminish with progress of time. The accuracy of the measurements scarcely justifies this variation being taken into account, and we may be content with obtaining the order of magnitude of e_s. With

$C'-C=10^{-9}$ ampere, and with the values of S and S' already given—namely, $S=46$ megohms, $S'=\frac{1}{2}$ megohm—we obtain

$$e_s=0{\cdot}5 \text{ millivolt approximately.}$$

This E.M.F. in the selenium, so small in comparison with that removed by desiccation and having its seat in the supposed water film, cannot be regarded as definitely established. It is too small to have been capable of definite direct measurement with the electrometer, the sensitivity of which was 1·6 millivolts per millimetre. But the definite, though small, change of current with illumination in the undried short-circuited cell has somehow to be accounted for, and the assumption of a relatively small polarisation in the selenium does this without being contradictory to the remaining experimental evidence.

(*b*) *Measurements with electrometer.*

In this case R becomes the leakage resistance of the electrometer, and its value is certainly more than 15,000 megohms. In comparison with R/W, R/S and R/S' unity is therefore negligible, so that equation (2) becomes

$$V=RC=\frac{\frac{e_w}{W}+\frac{e_s}{S}}{\frac{1}{W}+\frac{1}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

V being the potential difference actuating the electrometer, now fulfilling the functions of a voltmeter. Neglecting for the moment the small quantity e_s, we have

$$V=\frac{e_w}{1+\frac{W}{S}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4$$

and V will evidently diminish with decrease of S, such as accompanies illumination. This is in accordance with the observations; moreover, the insertion of the data gives approximate quantitative agreement. For, clearly,

$$\frac{V'}{V}=\frac{1+\frac{W}{S}}{1+\frac{W}{S'}},$$

V' being the terminal potential difference with the cell illuminated. Thus

$$\frac{V'}{V}=\frac{1+\frac{12}{46}}{1+\frac{12}{0{\cdot}5}}=\frac{1}{20} \text{ approximately.}$$

Of several measurements of the effect for the cell in question the most accurately observed was that the standard illumination reduced the electrometer indication from 144 millivolts to 8 millivolts, corresponding to a reduction ratio 1 : 18, which is not far from the ratio 1 : 20 indicated above.

A still closer agreement is obtained by allowing for the effect of e_s in the equation (3). According to this equation, the value $V=144$ millivolts corresponds to $e_w=180$ millivolts, the effect of e_s being quite negligible in this particular calculation. It becomes appreciable, however, under conditions of illumination when S has changed to the much lower value S'. The numerical value of V' is given by

$$V'=\frac{\frac{180}{12}+\frac{0{\cdot}5}{0{\cdot}5}}{\frac{1}{12}+\frac{1}{0{\cdot}5}}=7{\cdot}7 \text{ millivolts},$$

and

$$\frac{V'}{V}=\frac{7{\cdot}7}{144}=\frac{1}{18{\cdot}6},$$

as compared with the observed ratio 1 : 18.

LIGHT SENSITIVITY.

The light sensitivity of a selenium cell is the ratio of its electrical conductance under exposure to standard illumination to that in complete darkness. The measurements of this sensitivity for the cell in question were made, both before and after desiccation, with applied voltages in comparison with which the polarisation E.M.F. was negligible. According to our theory, therefore, if we denote by L_1 the light sensitivity of the undried cell

$$L_1=K\frac{\frac{1}{W}+\frac{1}{S'}}{\frac{1}{W}+\frac{1}{S}}$$

where K depends on the standard illumination chosen. The effect of drying being to make the conductance $1/W$ of the water film zero, the light sensitivity L_2 of the dried cell is given by

$$L_2=K\frac{S}{S'}.$$

The ratio

$$\frac{L_2}{L_1}=\frac{\frac{S}{W}+1}{\frac{S'}{W}+1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

is clearly greater than unity, since S' is less than S. This simply means that the presence of the water film in the undried cell, giving rise as it does to an additional conductance which suffers no change with illumination, masks to some extent the photo-electric property of the selenium. Drying reduces the absolute conductance of the cell, but enhances the light sensitivity. Insertion of the available numerical data in equation (5) gives

$$\frac{L_2}{L_1}=\frac{3{\cdot}8+1}{0{\cdot}04+1}=4{\cdot}6$$

which is sufficiently close to the roughly measured value 4 for the relative light sensitivities of the cell dried and undried.

This improvement of light sensitivity by desiccation is the most important practical result of the investigation. It may well be that the same process would procure similar advantages in other types of selenium cells.

POLARISATION IN THE DRIED CELL.

The proof provided by these experiments that the polarisation observed is almost entirely attributable to moisture does not exclude the possibility of a much more transient type of polarisation in the selenium-electrode system itself. The measuring instruments, owing to their deliberation of response, were incapable of giving information concerning what happens in the very early stages of the operations.

Some later experiments with the dried cell, which have had of necessity to be discontinued for the present, seemed clearly to indicate the existence of polarisation rapidly established on the application, and rapidly decaying on the withdrawal, of the voltage applied. The proof of this had to be indirect, and the method chosen was the customary plan of demonstrating a difference of apparent resistance according to whether direct or alternating current was used. Two commutators in phase on the same shaft were rotated at speeds which could be controlled. One caused periodic reversals of the current driven by 1 volt through the cell, and the other rectified this current for passage through the galvanometer. The steady direct current was first measured; then the commutators were driven at a known speed and the corresponding alternating current noted. At 22 cycles per second the current in the cell was increased to $4\frac{1}{2}$ times its D.C. value. Similar procedure with a non-inductive wire resistance of 1 megohm in place of the selenium cell resulted in a 25 per cent. reduction of current on causing it to alternate, an effect evidently due to contact inefficiency of the commutation. This imperfection of the apparatus prevented quantitative conclusions from being drawn; but the experiments, nevertheless, seem to demonstrate quite definitely the existence in the dried cell of appreciable polarisation more short-lived than that dealt with in the main part of this Paper.

DISCUSSION.

Dr. A. B. **WOOD**, in congratulating the authors, remarked that they had used Thirring cells, which are made on an improved design and are comparatively non-hygroscopic. He had found that cells of the Fournier d'Albe type, in which a coating of graphite on unglazed porcelain is divided by scratches bridged by selenium, are very hygroscopic. They could be freed from moisture by immersion in melted paraffin at 100°C., and in this way quite useless cells could be restored to activity, or cells with a dark/light resistance ratio of 4 : 1, for instance, would have their sensitivity increased up to a resistance ratio of 10 : 1.

Dr. D. **OWEN** suggested that the cells should be hermetically sealed. Were they enclosed during the experiments, and, if so, in what material?

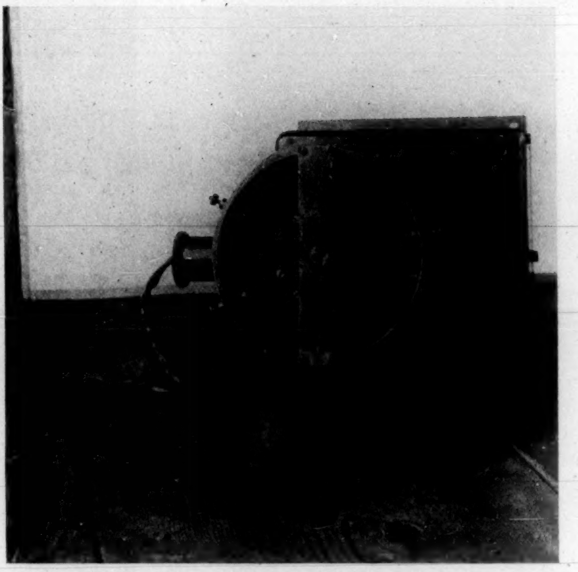
Dr. E. H. **RAYNER** disagreed with Dr. Wood as to the non-hygroscopic character of Thirring cells. Although the mica used in their construction is a good insulator in itself, moisture is able to penetrate into the interspaces between the laminæ, forming a surface layer of appreciable conductivity.

Prof. E. N. **DA C. ANDRADE** suggested that the cells might be sealed in glass vessels transparent to the illumination employed.

Prof. **RANKINE**, in reply to the discussion, said that Thirring cells vary considerably in quality, that used in the present experiments being a specially selected one. Prof. Thirring covers the front surface of the cells with collodion, which seals them against moisture on that side, but the back surface, on which there is no selenium, is unprotected. In the present experiments the cell was at first roughly sealed, together with phosphorus pentoxide, in a brass cylinder with a glass window. Subsequently the cylinder was hermetically sealed, being, however, in communication with an air reservoir consisting of a toy balloon to allow for changes of pressure. The difference between the times taken in the two cases to reach a steady hygrometric state was considerable. The experiments had not been extended to cells of the Fournier d'Albe type, but it was interesting to hear that they could be improved by drying. It was desirable for manufacturers to seal the cells before putting them on the market, and no doubt the paraffin used by Dr. Wood acted as a seal.

Dr. D. H. Black (communicated subsequently): I have read the authors' Paper with great interest and I should like to ask one question. Have any measurements been made on the effect obtained by reversing the direction of the applied potential through the cell after the galvanometer deflection has reached a steady value? If, as appears to be the case, the decrease of current with time is all due to the E.M.F. of polarisation which is set up, then on reversing the applied potential one would expect to obtain a steeper decay curve than the original, since the polarisation E.M.F. would then be in the same direction as the applied. If, however, there is a resistance change as well as a polarisation E.M.F., then it is quite possible for the reversal curve to lie below the original curve and to be of a different shape. I am at present engaged on some research on the electrical conductivity of oils, and I have studied, amongst other phenomena, the well-known decrease of current with time when a constant potential is applied across an oil film. Any attempt to account for all of this decrease by assuming a polarisation E.M.F. is rendered untenable by the shape of the current-time curve obtained by reversing the applied potential. The current on reversal starts at a low value, rises to a maximum, and then falls off to the same steady value as in the original case.

Authors' reply to Dr. D. H. Black: No measurements were made on the effect of reversing the direction of the applied potential difference across the undried cell after it had been polarised. This rather obvious variation would have been of interest, and would probably have occurred to the authors, but for the fact of the disappearance of the polarisation effect just after the arrangements had been completed for measuring it more precisely under controlled conditions. Attention was thus diverted to the importance of the complete elimination of the cause of polarisation.



PLAN OF MOTOR SHOWING POLEPIECES.

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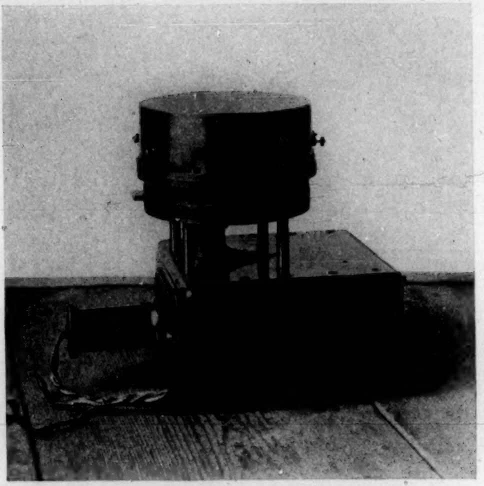
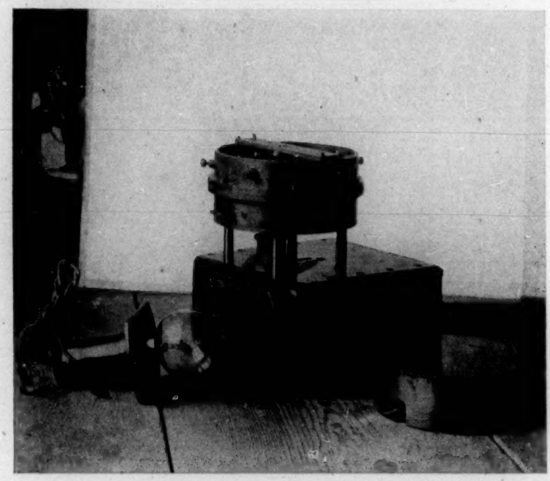


Motor with Lamp and Cap Fitted.



Motor with Lamp and Cap Removed.

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XV.—SYNCHRONOUS ALTERNATING CURRENT MOTOR AND MECHANICAL VIBRATING SYSTEM MAINTAINED BY A THERMIONIC VALVE AT THE FREQUENCY OF THE VIBRATING SYSTEM.

By T. G. Hodgkinson, *M.C., A.M.I.E.E.*

Received December 2, 1926.

ABSTRACT.

The design of a chronograph which will give a time trace subdivided into units and fractional units requires the association of a rotating system and a vibrating or time-keeping system.

If a low-frequency vibrating system be employed, the rotating system, although keeping good average time, may show exaggerated errors in unit subdivisions due to acceleration and deceleration between periods of control.

The Paper discusses the association of a high-frequency synchronous alternating-current motor with a high-frequency valve-maintained tuning-fork.

It deals with the design of valve-maintained motors and condenser-tuned valve-maintained tuning-forks and gives expressions for the frequency of the maintained system for motors coupled to tuning-forks both without and with tuning condensers.

IN 1921 the writer coupled a 1,000 cycle synchronous alternating-current motor designed for the purpose to a valve circuit maintaining a 1,000 cycle tuning fork, and showed that for determined load conditions the tuning fork determines the frequency of the system, while the motor is maintained in rotation by the valve. This motor was used to drive a chronograph, and the instrument was shown at the Royal Society Soirée in May, 1923; but outside Service reports the method of control has not been discussed.

In this Paper the method of control of such a system is investigated as a circuit problem, so that design conditions are laid down, and for these conditions the difference between the true fork note and the frequency maintained by the system is determined.

This difference is small, and compares with the differences in fork circuits without other load discussed by S. Butterworth* and by the writer,† provided, of course, that the motor output is small compared with the energy capable of being stored by the vibrating system while vibrating within a reasonable amplitude limit.

Tuning fork frequency control of synchronous alternating-current motors (notably the Rayleigh phonic motor, the Leeds Northrup converter and similar machines) have been employed for many years. These devices employ local circuits, including vibrating contacts operated by an electrically maintained fork, which interrupt a steady electromotive force applied to the machine, or, alternatively, control a governing load in the machine circuit; and the problem is of interest because the practicable use of vibrating contacts is limited to low-frequency schemes.

On the other hand, phase swinging of an alternating-current motor in a low-frequency scheme may cause serious time errors in a recording device driven by the motor, so that the use of higher frequencies becomes imperative for more accurate measurements.

The phase swing of a 1,000 cycle motor can cause only a relatively small time error, even if the swing is a large one; hence the reason of the writer's original effort, which was to produce a neat, accurate chronograph employing only one valve;

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* Proc. Phys. Soc., Vol. 32, page 345 (1920).

† Proc. Phys. Soc., Vol. 38, part I (1925).

however, the principles laid down in this Paper are not limited to tuning forks or one type of synchronous motors, and the subject has been dealt with as embracing the coupling of vibrating systems with rotating systems generally.

(1) THE MOTOR, VALVE MAINTAINED WITHOUT TUNING FORK (FIG. 1).

The particular motor mentioned earlier in this Paper as driving a chronograph has its winding distributed over eight poles, connected by a yoke, while the rotor is a laminated silicon steel disc, having twenty teeth ; but for the purpose of this work can be represented by a single winding surrounding a magnet, with a rotor having only one tooth.

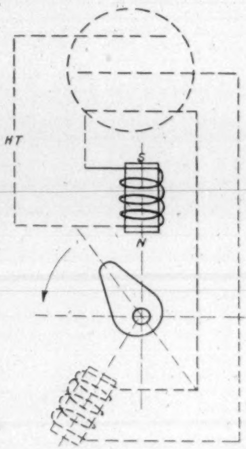


FIG. 1

The single winding has inductance l_1 and resistance r_1, and the amplitude of the E.M.F. produced in it when the machine is running is $V_M = V\frac{\omega}{\omega_F}$ where V is theE.M.F. when the frequency is that of the tuning fork we intend to couple the motor up with.

For the purpose of running the machine up to synchronism with the fork, or, if desired, for use when running with the fork, another magnet and winding can be employed.

This is shown dotted in Fig. 1, where the motor alone is shown coupled with a valve, and this method of maintenance of the motor alone is conveniently discussed before proceeding to the discussion of the motor coupled with the vibrating system.

The second motor winding has inductance l_2 and resistance r_2, while the E.M.F. produced in it when the machine is running at the frequency of the fork is $\frac{\mu}{m}V$, where m is the valve magnification factor and μ is a factor depending on coil dimensions.

This E.M.F. is arranged to lag behind the E.M.F. of the main motor winding by an angle θ, and is applied to the grid of the valve.

The main winding is included in the anode circuit of the valve, which has anode conductance s_1 and grid conductance s_2.

The anode current is i_1, the grid current is i_2, D is the time operator $\delta/\delta t$, and if we neglect all but the fundamental note the circuit equations are :

Grid Circuit.

$$(1/s_2 + r_2 + l_2 D)i_2 = \frac{\mu}{m}\frac{\omega}{\omega_F}V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

r_2 can easily be negligible compared with $1/s_2$, and it we neglect it here for the sake of tidiness, the E.M.F. in the valve grid is

$$V_g = 1/s\, i_2 = \frac{\frac{\mu}{m}\frac{\omega}{\omega_F}V}{1 + l_2 s_2 D}$$

Anode Circuit.

$$(1/s_1+r_1+l_1D)i_1=\left[\cos\theta+\sin\theta\frac{D}{\omega}+\frac{\mu}{1+l_2s_2D}\right]\frac{\omega}{\omega_F}V \quad . \ . \ . \ . \ . \ . \ (2)$$

If the reaction of the motor in the grid circuit is

$$\frac{\mu}{m}\frac{\omega}{\omega_F}V\sin\omega t$$

the resultant E.M.F. in the anode circuit is

$$V_R=[\sin\overline{\omega t+\theta}+\mu\cos\delta\theta\sin\overline{\omega t-\delta\theta}]\frac{\omega}{\omega_F}V \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (3)$$

and the current in the anode circuit

$$i_1=\cos\gamma[\sin\overline{\omega t+\theta-\gamma}+\mu\cos\delta\theta\sin\overline{\omega t-\delta\theta-\gamma}]\frac{\omega}{\omega_F}\frac{Vs_1}{1+r_1s_1} \quad . \ . \ . \ . \ . \ (4)$$

where $\tan\delta\theta=l_2s_2\omega$ and $\tan\gamma=\dfrac{l_1\omega}{1/s_1+r_1}$. If now $\dfrac{\sin\overline{\theta+\delta\theta}}{\mu\cos\delta\theta+\cos\overline{\theta+\delta\theta}}=\tan\varepsilon$, the power supplied by the valve

$$P_A=\frac{\mu\cos\delta\theta\dfrac{\omega}{\omega_F}Vi_1}{2}\cos\overline{\varepsilon-\gamma} \quad . \ . \ . \ . \ . \ . \ . \ . \ (5$$

and the power supplied to the motor

$$P_M=-\frac{\dfrac{\omega}{\omega_F}Vi_1}{2}\cos\overline{\theta+\delta\theta-\varepsilon+\gamma} \quad . \ . \ . \ . \ . \ . \ . \ (6)$$

Part of the power P_M is used to accelerate the rotor. Say that $P_M=P_\omega+$the accelerating power, and that $P_\omega=P\dfrac{\omega}{\omega_F}$

This assumes that the power P_ω is proportional to the speed ω—i.e., that the load is solid friction, and that P is the running power at the fork note. Substituting in (6) and making the accelerating power zero,

$$2P\tan\gamma_F(1+\tan^2\gamma)=\frac{V^2s_1}{1+r_1s_1}[\mu\cos\delta\theta\sin\overline{\theta+\delta\theta}\tan^2\gamma-(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})\tan\gamma]$$

where $\qquad \tan\gamma_F=\dfrac{l_1s_1\omega_F}{1+r_1s_1}$

Write $\qquad \dfrac{4P}{V^2s_1}(1+r_1s_1)=K$

and

$$\frac{\omega}{\omega_F}=\frac{K}{-(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})+\sqrt{(1+\mu\cos\delta\theta\cos\overline{\theta+\delta\theta})^2+K\tan\gamma_F(2\mu\cos\delta\theta\sin\overline{\theta+\delta\theta}-K\tan\gamma_F)}} \quad . \ . \ . \ (7)$$

As a matter of fact, this equation is a biquadratic, since $\delta\theta$ is variable; but for our purpose is conveniently discussed in this form.

There are two positive roots giving values for $\frac{\omega}{\omega_F}$, the lower of which is unstable that is, the lower root gives the speed at which the motor will start, and the upper the speed to which it will accelerate and remain stable.

The anode E.M.F., $\mu \cos \delta\theta \frac{\omega}{\omega_F} V = \frac{\omega}{\omega_F} V_A$, is often a determined quantity in design, limited by the H.T. supply, and if we require to do the maximum work in the motor for this E.M.F. the maximum value of P_M requires $\theta + \delta\theta = \pi - \gamma$. Substituting in equations (5) and (6) above

$$P_A = \frac{V_A{}^2 s_1}{2(1+r_1 s_1)} \cos \gamma \left[\cos \gamma - \frac{\cos 2\gamma}{\mu \cos \delta\theta} \right]$$

$$P_M = \frac{V_A{}^2 s_1}{2(1+r_1 s_1)} \cdot \frac{\cos \gamma}{\mu \cos \delta\theta} \left[1 - \frac{\cos \gamma}{\mu \cos \delta\theta} \right]$$

when the speed $\omega = \omega_F$ and the efficiency is

$$E = \frac{\mu \cos \delta\theta - \cos \gamma}{\mu \cos \delta\theta \left[\mu \cos \delta\theta \cos \gamma - \cos^2 \gamma \right]}$$

This efficiency is a maximum when $\mu \cos \delta\theta = \sin \gamma + \cos \gamma$, and is unity when $\gamma = 0$ and $\pi/2$ and 0·5 when $\gamma = \pi/4$.

As with all valve problems, maximum power output is not compatible with maximum efficiency. Maximum power output from the valve and motor requires $\mu \cos \delta\theta = 2 \cos \gamma$, and on this basis the power output is constant, $P_M = \frac{V_A{}^2 s_1}{8(1+r_1 s_1)} = \frac{P_A}{2}$, and the efficiency is 0·5 throughout the range of γ.

This elasticity of design possibilities is due to the fact that the value of θ, or the phase relationship between the back E.M.F. of the machine and the anode E.M.F., is a determined quantity, independent of load. However, when the motor is coupled to the fork this condition is nullified by the additional fork anode E.M.F., so that γ can safely be made equal to $\pi/4$, or adjusted to suit the coupling conditions.

With a non-conducting grid $\delta\theta = 0$, and if we require the maximum value of P_M when $\omega = \omega_F$, and make $\gamma = \pi/4$, V_M will be made equal to $V_A/\sqrt{2}$, $l_1 \omega$ equal to $1/s_1 + r_1$, and θ adjusted to $3\pi/4$.

Substituting these values in equation (7)

$$\frac{\omega}{\omega_F} = \frac{K}{\sqrt{K(2-K)}}$$

This is the starting speed. The higher speed is determined entirely by the power limit of the valve system.

Notice that with the conditions determined above $\frac{\omega}{\omega_F} < 1$ when $K < 1$.

The maximum power is $\frac{V^2 s_1}{4(1+r_1 s_1)} = \frac{V_A{}^2 s}{8(1+r_1 s_1)}$

and the motor will accelerate if this power is greater than P; in other words, if $K < 1$.

Guess $K=2/3$, then $\omega/\omega_F=1/\sqrt{2}$ for the starting speed. The upper limit is determined by the power supply, and of course by the fact that the grid can only be relatively non-conducting within well defined limits, and that the value of m (and so μ) is constant only over a limited portion of the valve characteristic.

For a conducting grid case, guess $\tan\delta\theta=0{\cdot}5\tan\gamma$. This requires $\tan\theta=-3$ for the maximum value of $P_M=P$ if $\gamma=\pi/4$.

The power supplied to the motor at the speed ω_F is

$$\frac{V^2 s_1}{4(1+r_1 s_1)}=\frac{V_A{}^2 s_1}{8(1+r_1 s_1)}$$

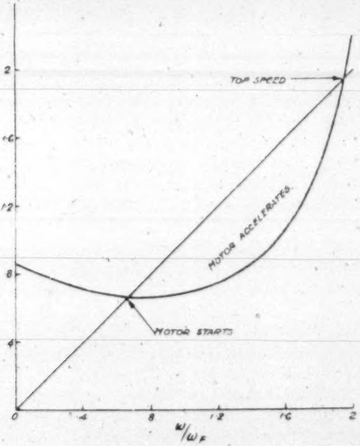
and the motor will accelerate if

$$\frac{V^2 s_1}{4P(1+r_1 s_1)}>1, \text{ or if } K<1 \text{ as before.}$$

In the Graph I, ω/ω_F is plotted against

$$\frac{K}{\sqrt{2}\sin\delta\theta\cos\delta\theta-1+\sqrt{(\sqrt{2}\sin\delta\theta\cos\delta\theta-1)^2+2K(2\cos^2\delta\theta-K)}}$$

assuming the above value for $\tan\delta\theta$ and making $K=0{\cdot}5$.



GRAPH 1.—SPEED SOLUTION FOR MOTOR AND VALVE (CONDUCTING GRID).

$$\frac{\omega}{\omega_F}=\frac{1/2}{\sqrt{2}\sin\delta\theta\cos\delta\theta-1+\sqrt{(\sqrt{2}\sin\delta\theta\cos\delta\theta-1)^2+(2\cos^2\delta\theta-1/2)}}$$

The curve represents values of right-hand side of the above expression for different values of θ, and the points of intersection with the straight line are the two solutions.

The motor will start when $\omega/\omega_F=0{\cdot}668$, and will accelerate until it is $1{\cdot}94$—i.e., between these values the power supplied to the motor exceed its demand at the speed.

If the motor is the determining factor, and we can do what we please with the valve and anode E.M.F. V_M, l_1, r_1, ω_1, and the power output required are fixed

$$P_M=\frac{V_M{}^2}{2l\omega}\sin\gamma\,(\mu\cos\delta\theta-\cos\gamma)$$

and the values of γ and $\mu\cos\delta\theta$ we have so far discussed may not meet the case if the value of $\frac{V_M{}^2}{2l\omega}$ is too small. For example, if $\mu\cos\delta\theta=2\cos\gamma$, the maximum power output requires $\gamma=\pi/4$, and is $V_M/4l\omega$, or if

$$\mu\cos\delta\theta=\overline{\sin\gamma+\cos\gamma},\ P=\frac{V_M{}^2}{2l\omega}\sin^2\gamma \text{ and is } \frac{V_M{}^2}{2l\omega},$$

when the valve conductance is very small and $\gamma=\pi/2$.

If we make

$$P_M = \frac{V_A{}^2 s_1}{2(1+r_1 s_1)} \frac{B \sin\gamma \cos\gamma}{(B \sin\gamma + \cos\gamma)} = \frac{V_M{}^2}{2l\omega} B \sin^2\gamma$$

the efficiency is $$\frac{B}{B+\sqrt{1+B^2} \sin\gamma \cos\gamma}.$$

This efficiency is unity when γ is zero and $\pi/2$, and the minimum value (when $\gamma=\pi/4$) is less than 0·5, if B is greater than or less than unity.

If $B \sin^2\gamma$ is required greater than, say, 0·5, it will be better to make γ greater than $\pi/4$ by selecting a suitable valve conductance and then to adjust B to the required value.

Example: Say $B \sin^2\gamma$ is required $=1{\cdot}5$. If $\gamma=\pi/4$, $B=3$, and the efficiency is 0·375; on the other hand, if $\gamma=\pi/3$, $B=2$, and the efficiency is 0·48.

We have discussed how to design motors to suit valves, or valves to suit motors, and proceed to discuss the problems connected with the coupling of the motor and the fork.

There are various ways of doing this, and a typical method, with the fork windings in series with the motor windings, has been selected as embracing all of these if they are considered on an energy distribution basis. The method of investigation is to get out a circuit equation for the anode circuit, very large, although systematic for both the untuned and condenser tuned fork cases, and then to evaluate the difference between the true fork note and the maintained note, and the ratio between the work impedance (the A^2/β of Mr. Butterworth's Paper) and the inductive impedance of the fork windings, required to maintain the whole system, for a few special practical cases.

This conforms with the method of Mr. Butterworth's Paper and my own Paper quoted, and the solutions can thus be compared with solutions of the fork circuit above. The condenser tuned fork is also discussed here as a special case.

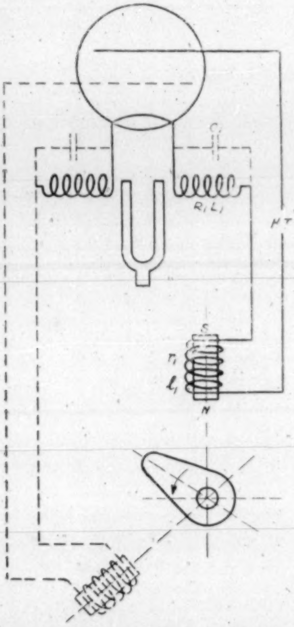


FIG. 2.—MOTOR WINDINGS IN SERIES WITH FORK WINDINGS WITHOUT CONDENSERS.

MOTOR AND TUNING FORK VALVE MAINTAINED AT THE FREQUENCY OF THE TUNING FORK.

The motor anode winding is connected in series with the fork anode winding in the anode circuit of a valve, while the motor grid winding is connected in series with the fork grid winding in the valve grid circuit.

Notation—

Mass of fork supposed collected at end of prong	α
Frictional force per unit velocity	β
Control force per unit displacement	γ
Force in magnet gap per unit current in anode coil	A_1
Force in magnet gap per unit current in grid coil	A_2
Fork anode coil resistance and inductance and shunt capacity	$R_1\ L_1\ C_1$
Grid anode coil resistance and inductance and shunt capacity	$R_2\ L_2\ C_2$
Anode and grid conductances	$s_1\ s_2$
Anode and grid currents	$i_1\ i_2$
Valve magnification factor	m
Amplitude of back E.M.F. of the motor in the anode circuit when the machine is running at the fork frequency	V
Resistance and inductance of motor anode winding	$r_1\ l_1$
Resistance and inductance of motor grid winding	$r_2\ l_2$
Amplitude of E.M.F. of the motor in the grid circuit (if a grid winding is used) when the machine is running at the fork frequency	$\frac{\mu}{m}V$

Back E.M.F. of the motor in anode circuit

$$V_{\mathrm{M}}=\left(\cos\theta+\sin\theta\frac{D}{\omega}\right)\frac{\omega}{\omega_{\mathrm{F}}}V, \text{ where } D=\delta/\delta t$$

Back E.M.F. of the motor in grid circuit (if used)

$$V_0=\frac{\mu}{m}\frac{\omega}{\omega_{\mathrm{F}}}V$$

In this work I have continued to use the notation of my Fork Paper (quoted earlier, the essential fork notation of which was acquired from Mr. S. Butterworth's Paper (also quoted), so that the results of this Paper may be compared with those of the earlier Papers.

The *Grid Circuit* equation, ignoring the resistance of the grid motor winding, which is supposed negligible compared with the resistance of the grid electrode, is

$$(1/s_2+\overline{L_2+i_2}D)i_2+\frac{A_2D(A_2i_2-A_1i_1)}{\alpha D^2+\beta D+\gamma}=V_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$V_g=\frac{i_2}{s_2}=\frac{V_0+\dfrac{A_1A_2Di_1}{\alpha D^2+\beta D+\gamma}}{1+\overline{L_2+l_2}s_2D+\dfrac{A_2{}^2s_2D}{\alpha D^2+\beta D+\gamma}}$$

The Anode Circuit.

$$(1/s_1+r_1+R_1+\overline{l_1+L_1}D)i_1+\frac{A_1D[(1+\overline{L_2+l_2}s_2D)A_1i_1-A_2s_2V_0]}{(1+\overline{L_2+l_2}s_2D)(\alpha D^2+\beta D+\gamma)+A_2{}^2s_2D} \quad . \quad . \quad (9)$$

$$=V_{\mathrm{M}}+m\frac{V_0+\dfrac{A_1A_2Di_1}{\alpha D^2+\beta D+\gamma}}{1+\overline{L_2+l_2}s_2D+\dfrac{A_2{}^2s_2D}{\alpha D^2+\beta D+\gamma}}$$

The power of the motor running steadily is $-\frac{\omega}{\omega_F}\frac{Vi}{2}\cos\varkappa$, where $\varkappa$ is the phase difference between V_M and i—i.e.,

$$P=-\frac{Vi\cos\varkappa}{2}\text{, or } V_M=-\frac{V^2}{2P}\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}i_1$$

$$\frac{V_M s_1}{1+r_1 s_1}=-\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1$$

$$\frac{V_0 s_1}{1+r_1 s_1}=-\frac{\mu}{m}\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1$$

since V_M is $\hat{\theta}$ in advance of V_0.

Substituting in equation (2) write

$$\frac{\overline{L_1+l_1\omega s_1}}{1+\overline{r_1+R_1}s_1}=\tan\gamma_1\text{, and } L_2+l_2s_2\omega=\tan\delta\theta_1\text{, and}$$

$$\left[\begin{array}{l}
\left(1-\frac{\omega^2}{\omega_F^2}+\frac{\beta}{\alpha\omega_F}\frac{D}{\omega_F}\right)\left\{\begin{array}{l}\left(1+\tan\gamma_1\frac{D}{\omega}\right)\left(\cos\delta\theta_1+\sin\delta\theta_1\frac{D}{\omega}\right)\\ +\frac{2}{K}\frac{\omega}{\omega_F}\cos\varkappa\left\{\begin{array}{l}\cos\overline{\varkappa+\delta\theta}+\sin\overline{\varkappa+\delta\theta}\frac{D}{\omega}\\ +\mu\cos\delta\theta\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\end{array}\right\}\end{array}\right\}\\
+\frac{A_1^2}{L_1+l_1}\frac{\tan\gamma_1}{\alpha\omega_F^2}\frac{D}{\omega}\left\{\cos\delta\theta_1\left(1-m\frac{A_2}{A_1}\right)+\sin\delta\theta_1\frac{D}{\omega}\right\}\\
+\frac{A_2^2}{L_2+l_2}\frac{\sin\delta\theta_1}{\alpha\omega_F^2}\frac{D}{\omega}\left[1+\tan\gamma_1\frac{D}{\omega}+\frac{2}{K}\frac{\omega}{\omega_F}\cos\varkappa\left\{\begin{array}{l}\cos\varkappa+\sin\varkappa\frac{D}{\omega}\\ +\frac{A_1\mu}{A_2 m}\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\end{array}\right\}\right]
\end{array}\right]$$

$$i_1=0$$

This expression of the motor E.M.F.s (rigid quantities), in terms of the varying current, phase and power, which have to adjust themselves to running conditions, is justified by the reservation that it requires that the system be working—i.e., that the motor is running in synchronism with and is supported by the fork.

Under these conditions the power supply to the valve is made such that the anode current and the fork amplitude are capable of growing to provide whatever power is required by the motor. The ratio K between the power required by the motor and $\frac{V^2 s_1}{4(1+r_1 s_1)}$ can have values greater than unity provided that the equations are satisfied, and that the fork amplitude is not allowed to grow excessive for its dimensions, and in subsequent examples values of K both above and below unity have been included.

Case (1).—Without Grid Reaction from the Motor.

If the grid conductance is negligible and the motor has no grid winding—i.e., there is no reaction from the motor, the solution we are concerned with if $\beta/\alpha\omega_F$ is very small—makes

$$1-\frac{\omega^2}{\omega_F^{\ 2}}=\frac{\beta}{\alpha\omega_F}\,\frac{\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}}{1+\dfrac{1+\cos 2\varkappa}{K}} \quad . \ . \ . \ . \ . \ . \ . \quad (10)$$

and the condition equation is

$$\left(m\frac{A_2}{A_1}-1\right)\frac{A_1^{\ 2}}{\beta(1/s_1+r_1+R_1)} \gtreqless 1+\frac{1+\cos 2\varkappa}{K}+\frac{\left(\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}\right)^2}{1+\dfrac{1+\cos 2\varkappa}{K}} \quad . \quad (11)$$

Writing

$$\frac{\tan\gamma_{1_F}+\dfrac{\sin 2\varkappa}{K}}{1+\dfrac{1+\cos 2\varkappa}{K}}=F(x)$$

and

$$\left(m\frac{A_2}{A_1}-1\right)\frac{A_1^{\ 2}}{\beta(1/s_1+r_1+R_1)}=N$$

we can rearrange our equation (9) for the anode circuit when the machine is running

$$\left[F(x)-\tan\gamma_{1_F}+(1-N+\tan\gamma_{1_F}F(x)\frac{D}{\omega}\right]i_1=\frac{\left(F(x)+\dfrac{D}{\omega}\right)V_M}{1/s_1+r_1+R_1} \quad . \ . \ . \ . \ . \quad (12)$$

The anode E.M.F. available for driving the motor is mV_g less the back E.M.F. of the fork in the anode circuit, and if the fork is large enough to store the energy, this E.M.F. can only change slowly, and is sensibly constant for any rapid change of load on the machine. If the current has reached a certain average value i_F, the anode E.M.F. is

$$\frac{(1/s_1+r_1+R_1)N\dfrac{D}{\omega_F}i_F}{F(x)+D/\omega_F}=\frac{N\dfrac{D}{\omega_F}V_M}{F(x)-\tan\gamma_{1F}+(1-N+\tan\gamma_{1F}F(x))\dfrac{D}{\omega_F}}$$

and the advance of V_M on this anode E.M.F. is

$$\theta_F=\tan^{-1}\frac{1-N+\tan\gamma_{1F}F(x)}{F(x)-\tan\gamma_{1F}}-\pi/2$$

Stable values of θ_F must make it greater than $\pi-\gamma_{1F}$—i.e., for stable running

$$\frac{N-(1+\tan\gamma_{1F}F(x))}{\tan\gamma_{1F}-F(x)}\gtreqless \operatorname{cotan}\gamma_{1F}$$

We have now two condition equations to be satisfied.

$$N \geqq \left(1+\frac{1+\cos 2\varkappa}{K}\right)(1+F^2(x)) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and

$$N \geqq 2+(\tan\gamma_{1F}-\operatorname{cotan}\gamma_{1F})\,F(x)$$

and from these

$$\tan\varkappa = \frac{2}{-K(\tan\gamma_{1F}+\operatorname{cotan}\gamma_{1F})} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This case is sufficiently discussed. We know that in order to maintain the motor in a stable state

$$\varkappa > \tan^{-1}\frac{2}{-K(\tan\gamma_{1F}+\operatorname{cotan}\gamma_{1F})}$$

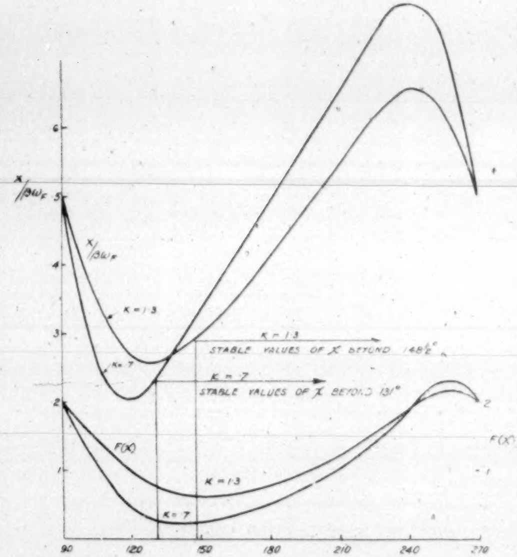
and substituting values of $\varkappa$ between this and $3\pi/2$ in the condition equation we can determine values of N, and so of $\frac{X}{\beta\omega_F}=A_1^2/\beta L_1\omega_F$ to maintain this condition.

Again, we can calculate the change in $1-\omega^2/\omega_F^2$ as $\varkappa$ varies in value between the above limits.

Further, from the relation $i_1\cos\varkappa=\frac{2P}{V}$, i_1 is determined, and the amplitude of the fork is determinate from the relation

$$(aD^2+\beta D+\gamma)y=A_1i_1$$

This decides the dimensions of the fork, since the amplitude of vibration must be kept within reasonable limits.



GRAPH II.—MOTOR AND FORK (UNTUNED, WITHOUT REACTION).

In Graph II, $X/\beta\omega_F$ is plotted against values of $\varkappa$ between $\pi/2$ and $3\pi/2$, and the point beyond which the system is stable when

$$\varkappa > \tan^{-1}\frac{2}{-K(\tan\gamma_{1F}+\operatorname{cotan}\gamma_{1F})}$$

is indicated. Notice that at this point $X/\beta\omega_F$ is increasing in value.

For this graph K is taken as 0·7 and 1·3, and the minimum value of $X/\beta\omega_F = 0{\cdot}295$. Guess the resistance r_1 of the motor and the resistance R_1 of the fork, small compared with $1/s_1$ and $\dfrac{L_1 s_1 \omega_F}{1+r_1+R_1 s_1}=1$. The minimum value of $X/\beta\omega_F$ is $\dfrac{2{\cdot}95}{m-1}$ if $K=1{\cdot}3$ and $A_2=A_1$, but a very much greater value of $X/\beta\omega_F$ will be required to give a stable system, since it increases in value at this point.

Compare this figure with $X/\beta\omega_F=\dfrac{2}{m-1}$ for the fork alone, given in the author's Paper (quoted).

Case (2).—With Grid Reaction from the Motor.

Grid conductance negligible and reaction from the grid winding of the motor.

In our treatment of the motor alone we decided that for a non-conducting grid we would make $\mu=\sqrt{2}$, $\tan\gamma=1$ and $\theta=3\pi/4$.

Substituting these values in the frequency and condition equations

$$1-\frac{\omega^2}{\omega_F{}^2}=\frac{\beta}{a\omega_F}\left[\frac{K\tan\gamma_{1_F}-\overline{1+\cos 2\varkappa}}{K+\sin 2\varkappa}\right. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$N \geqq \frac{(K\tan\gamma_{1_F}-\overline{1+\cos 2\varkappa})^2+(K+\sin 2\varkappa)^2}{K(K+\sin 2\varkappa)} \quad . \quad . \quad . \quad . \quad (16)$$

In this case the anode E.M.F. which is required to lag behind the back E.M.F. of the motor by an angle greater than $\pi-\gamma$ is

$$\frac{NV_M}{F(x)-\tan\gamma_{1_F}+(1-N+\tan\gamma_{1_F}F(x))\dfrac{D}{\omega}}$$

so that $\dfrac{1-N+\tan\gamma_{1_F}F(x)}{\tan\gamma_{1_F}-F(x)}$ requires to be $<\tan\gamma_{1_F}$—i.e.,

$$N \geqq 1-\tan^2\gamma_{1_F}+2\tan\gamma_{1_F}F(x)$$

Combining this with

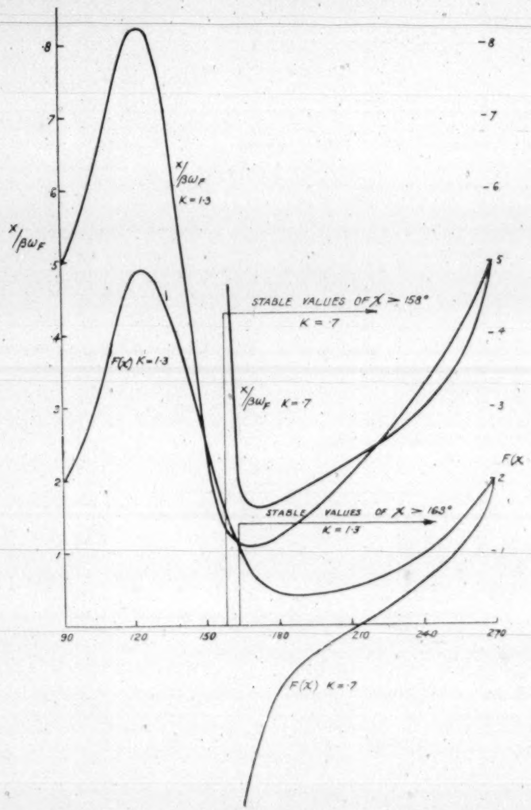
$$N \geqq \left(1+\frac{\sin 2\varkappa}{K}\right)(1+F^2(x)) \text{ above} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

$$\varkappa \geqq \tan^{-1}\frac{2}{-K(1+\tan^2\gamma_{1_F})} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

In the Graph III, $X/\beta\omega_F$ is plotted against values of $\varkappa$ between $\pi/2$ and $3/2\pi$, and the point beyond which the system is stable when $\varkappa>\tan^{-1}\dfrac{2}{-K(1+\tan^2\gamma_{1_F})}$ is shown.

K for this case is taken as 1·3 and 0·7, $\tan\gamma_{1_F}=2$, and $m=11$. The minimum value of $X/\beta\omega_F=0{\cdot}1185$ when $K=1{\cdot}3$ and $A_2=A_1$.

Notice that now at this point $X/\beta\omega_F$ is increasing very slowly in value, so that for a value, say, 2, the system is stable over quite a large range of $\varkappa$; also that values of $F(x)$ reach lower figures than in the case without reaction.



GRAPH III.—MOTOR AND FORK (UNTUNED, WITH REACTION).

Similarly to the preceding case, values of $\varkappa < 163°$ can be put into the frequency and condition equations, and the fork dimensions can be determined from the necessary value of i to maintain the load.

MOTOR WINDINGS IN SERIES WITH CONDENSER TUNED FORK WINDINGS (FIG. 2).

If this circuit is systematically analysed the circuit equation resulting is

$$\left[\begin{array}{l}(G_1H_1+s_1\overline{R_1+L_1D})(G_2H_2+s_2\overline{R_2+L_2D})\left(1+\dfrac{D^2}{\omega_F{}^2}+\dfrac{\beta}{a\omega_F}\cdot\dfrac{D}{\omega_F}\right)\\ +(G_2H_2+s_2R_2+L_2D)(s_1+c_1DG_1)\dfrac{A_1{}^2}{a\omega_F}D\\ +(G_1H_1+s_1\overline{R_1+L_1D})(s_2+c_2DG_2)\dfrac{A^{\ 2}}{a\omega_F{}^2}D-\dfrac{ms_1A_1A_2}{a\omega_F{}^2}D\end{array}\right]i_1=$$

$$\left[\begin{array}{l}(G_2H_2+s_2\overline{R_2+L_2D})\left\{H_1\left(1+\dfrac{D^2}{\omega_F{}^2}+\dfrac{\beta}{a\omega_F}\dfrac{D}{\omega_F}\right)+\dfrac{A_1{}^2}{a\omega_F{}^2}c_1D^2\right\}\\ \qquad\qquad +H_1(s_2+c_2DG_2)\dfrac{A_2{}^2}{a\omega_F{}^2}D\end{array}\right]s_1V_M+$$

$$\left[\begin{array}{l}m\overline{1+r_2s_2}\left\{H_1H_2\left(1+\dfrac{D^2}{\omega_F{}^2}+\dfrac{\beta}{a\omega_F}\cdot\dfrac{D}{\omega_F}\right)+\dfrac{A_1{}^2c_1D^2H_2+A_2{}^2c_2D^2H_1}{a\omega_F{}^2}\right\}\\ \qquad\qquad +\dfrac{A_1A_2Ds_2}{a\omega_F{}^2}\end{array}\right]s_1V_A \quad . \quad . \quad . \qquad (19)$$

where

$$G_1=1+r_1s_1+l_1s_1D \qquad\qquad H_1=1+R_1c_1D+L_1c_1D^2$$
$$G_2=1+r_2s_2+l_2s_2D \qquad\qquad H_2=1+R_2c_2D+L_2c_2D^2$$

TUNED FORK, WITHOUT MOTOR.

When we deal with the *Tuned Fork circuit alone*—i.e., when there are no motor windings in series with the fork windings in the electrode circuits, and we tune condensers so that

$$\frac{1+R_1s_1}{L_1c_1}=\frac{1+R_2s_2}{L_2c_2}=\omega^2{}_F$$

The frequency equation becomes (neglecting small terms if $\beta/a\omega_F$ is small):—

$$1-\omega^2/\omega_F{}^2=-\beta/a\omega_F\left[\frac{X_1}{\beta\omega_F}\ \frac{L_1s_1}{R_1c_1+L_1s_1}+\frac{X_2}{\beta\omega_F}\ \frac{L_2s_2}{R_2c_2+L_2s_2}\right]$$

where $A_1{}^2=X_1L_1$ and $A_2{}^2=X_2L_2$.

Notice that if $X/\beta\omega_F$ is small—and it can be very small with tuned systems—the difference between the fork note and the note of the maintained system can be very much smaller than these differences in untuned systems given in the author's Paper quoted.

The maintained note is slightly higher than the true fork note, and the plate and grid windings require to be oppositely wound.

If $A_2=-A_1$, $X_2=X_1=X$, and $R_2=R_1=R$, the condition equation becomes

$$X/\beta\omega_F \geqq \frac{\dfrac{R}{L\omega_F}\left(1+\dfrac{Ls_1}{Rc_1}\right)\left(1+\dfrac{Ls_2}{Rc_2}\right)}{\dfrac{m}{1+Rs_2}\,\dfrac{Ls_1}{Rc_1}-\left(2+\dfrac{Ls_1}{Rc_1}+\dfrac{Ls}{Rc_2}\right)} \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

Guess $\frac{s_2}{c_2}=0{\cdot}2\frac{s_1}{c_1}$, $\frac{R}{L\omega_F}=0{\cdot}3$ and $m=7{\cdot}2$, the optimum value of $s_1/c\omega_F$ is about $0{\cdot}8$ and

$$X/\beta\omega_F=0{\cdot}1285.$$

If the grid conductance is negligible

$$1-\omega^2/\omega_F{}^2=-\beta/\alpha\omega_F\frac{X/\beta\omega_F}{1+\frac{Rc_1}{Ls_1}}$$

and

$$X/\beta\omega_F\geqq\frac{Rc\omega_F\left(1+\frac{Rc_1}{Ls_1}\right)}{m-\left(1+2\frac{Rc_1}{Ls_1}\right)}$$

Compare this with the case in which we tune the condensers so that $Lc_1\omega^2{}_F=Lc_2\omega^2{}_F=1$. In this case

$$1-\frac{\omega^2}{\omega_F{}^2}=-\beta/\alpha\omega_F\frac{(Rc\omega_F+2X/\beta\omega_F)}{1+\frac{Rc}{Ls_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and

$$X/\beta\omega_F\geqq\frac{Rc\omega_F\left[\left(1+\frac{Rc}{Ls_1}\right)^2+R^2c^2\omega_F{}^2\right]}{\overline{m+1}\left(1+\frac{Rc}{Ls_1}\right)-2\left\{\left(1+\frac{Rc}{Ls_1}\right)^2+R^2c^2\omega_F{}^2\right\}} \quad . \quad . \quad . \quad . \quad (23)$$

Tuned in this way, the difference between the note of the system and the true fork note is greater, and the value of $X/\beta\omega_F$ is greater also. However, the case is of interest because I have dealt with the motor problem with the circuits tuned in this way, and the solutions can thus be compared.

There are other modes in which the system is maintained in vibration with the coils similarly wound, which depend to a much greater extent on the tuning of the condensers; but the solution given above is essentially that of a *fork oscillator* as opposed to a *circuit oscillator,* and requires the magnet coils *oppositely* wound.

With the coils *similarly* wound, for example, circuit notes may be maintained independently of any vibration of the fork, since the coupling is in the right direction to maintain such notes; whereas with the coils oppositely wound the only circuit note that can possibly be supported is a higher frequency difference note, and the coupling cannot easily be made sufficiently great to sustain this mode.

The principles outlined in this case as well as those of the Fork Papers mentioned are not limited to valve maintained forks, but must apply to self-maintained vibrating systems generally, such as the Campbell Hummer and various reed devices using carbon microphones to relay power. They do not exclude even contact driven devices if the phase of the make and break of the current supply is determinable.

TUNED FORK WITH MOTOR.

Including the motor in the problem the system is better studied when divided up into particular cases.

By way of simplifying expressions and dealing with the cases in which the grid conductance is negligible

$$\begin{array}{llcccc} R_1 & \text{is made} & = & R_2 & = & R \\ L_1 & ,, & = & L_2 & = & L \\ c_1 & ,, & = & c_2 & = & c \\ A_1 & ,, & = & A_2 & = & A \\ r_1 & ,, & = & r_2 & = & r \\ l_1 & ,, & = & l_2 & = & l \end{array}$$

$\dfrac{s_1}{1+r_1 s_1}$ is written s, $s_2=0$, and the condensers are adjusted so that $1-Lc\omega_F{}^2=\delta_1$—a small quantity.

Substituting

$$-\cos\varkappa\left(\cos\varkappa+\sin\varkappa\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{2}{K}i_1=V_M s$$

and

$$-\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\mu}{m}\frac{\omega}{\omega_F}\frac{2}{K}i_1=V_A s$$

in the circuit equation, this equation can be split up into :—

Frequency Equation

$$1-\frac{\omega^2}{\omega_F{}^2}=-\frac{\beta}{a\omega_F}\frac{[(Rc\omega_F+2\overline{1-\delta X}/\beta\omega_F)Y_x+\delta Z_x]}{Rc\omega_F Z_x-\delta Y_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and Condition Equation

$$X/\beta\omega_F\overset{=}{>}\frac{(R^2c^2\omega_F{}^2+\delta^2)(Y^2{}_x+Z^2{}_x)}{m+1Ls\omega_F(Rc\omega_F Z_x-\delta Y_x)-2\overline{1-\delta}Rc\omega_F(Y^2{}_x+Z^2{}_x)} \quad . \quad (25)$$

for the solution we are concerned with where

$$Y_x=Rs(1-lc\omega_F{}^2)+\delta+\frac{2\cos\varkappa}{K}[\delta(\cos\varkappa+\mu\cos\overline{\varkappa-\theta})-Rc\omega_F(\sin\varkappa+\mu\sin\overline{\varkappa-\theta})]$$

$$Z_x=\overline{L+\delta l}\,.\,s\omega_F+Rc\omega_F+\frac{2\cos\varkappa}{K}[\delta(\sin\varkappa+\mu\sin\overline{\varkappa-\theta})+Rc\omega_F(\cos\varkappa+\mu\cos\overline{\varkappa-\theta})]$$

Case I.

When there is no reaction by the motor in the grid circuit, V_A, r_2 and l_2 have no value, and the anode circuit equation can be re-written

$$\left[1/s+lD+\frac{R+LD}{\delta+RcD}\right]i_1$$

$$=\left[1-\frac{\overline{m+1}X/\beta\omega_F LD}{\left(F(\varkappa)+\dfrac{D}{\omega_F}\right)\left\{(1/s+lD)(\delta+RcD)^2+(R+LD)(\delta+RcD)\right\}+ \atop +X/\beta\omega_F LD\left\{m-1+2\left(1-lc\omega^2+\dfrac{cD}{s}\right)(\delta+RcD)\right\}}\right]V_M \quad . \quad (26)$$

where $1-\frac{\omega^2}{\omega_F{}^2}=\frac{\beta}{a\omega_F}F(\varkappa)$, and where the last term represents the portion of the anode E.M.F. for which the fork is responsible, and is intended to maintain steady in the case of a sudden load on the machine.

Clearly, if there is to be any lag of the current i_1 behind the resultant E.M.F. in the anode circuit, the inductance l of the motor winding must be greater than the inductance L of the fork anode winding, or δ must have positive value.

In order to provide stable running conditions this lag must have some value, and the power of the motor is a maximum when its value is $\pi/2$. This proportion of power taken by the motor is increased in a particular system by increasing the value of δ as well as by increasing the ratio l/L.

The condition that the anode E.M.F. for which the fork is responsible must lag behind the E.M.F. of the motor by an angle greater than π, less the lag of the current behind the resultant E.M.F. in the anode circuit, gives another relation—i.e.,

$$\begin{aligned}\overline{m+1}X/\beta\omega_F Ls\omega_F(\delta Z-Rc\omega_F Y) &\geqq \\ [(\delta^2+R^2c^2\omega_F{}^2)F(\varkappa)-\delta 2\overline{1-\delta}X/\beta\omega_F](Y^2+Z^2)& \\ -[\delta^2+R^2c^2\omega_F{}^2+Rc\omega_F 2\overline{1-\delta}X/\beta\omega_F]2YZ& \quad . \quad . \quad . \quad . \quad (27)\end{aligned}$$

where $Y=Rs(1-lc\omega_F{}^2)+\delta$ and $Z=\overline{L+\delta l}\,.\,s\omega_F+Rc\omega_F$.

This condition equation compared with the condition equation given earlier gives the minimum value of $\varkappa$ for which the system is stable—i.e.,

$$\varkappa_{\text{min.}}=\tan^{-1}\frac{2(Rc\omega_F Y-\delta Z)}{K(Y^2+Z^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

If the equations (24) and (25) are examined it will be appreciated that if $R/L\omega_F$ is small and Ls/Rc is greater than 2·5, the optimum value of δ is very nearly zero.

If Ls/Rc is small, negative values of δ will improve the value of both $X\beta/\omega_F$ and $F(\varkappa)$; but there is no point in making Ls/Rc small, and the examples shown in Graph IV are for a case in which $Lc\omega_F{}^2=1$—i.e., $\delta=0$, $Rc\omega_F=0{\cdot}2$, $Rs=0{\cdot}1$, $Ls\omega_F=0{\cdot}5$, $l/L=3$ and $m=11$.

$X/\beta\omega_F$ and $F(\varkappa)$ are shown for values of $K=0{\cdot}7$ and $1{\cdot}3$.

Case II.

When there is reaction by the motor in the grid circuit as well as in the anode circuit, say that

$$V_A=\frac{\frac{\mu}{m}V_M}{\cos\theta+\sin\theta\frac{D}{\omega}}$$

$$=-\frac{\mu}{m}\cos\varkappa\left(\cos\overline{\varkappa-\theta}+\sin\overline{\varkappa-\theta}\frac{D}{\omega}\right)\frac{\omega}{\omega_F}\frac{V^2}{2P}i_1$$

The circuit now becomes

$$\left[1/s+lD+\frac{R+LD}{\delta+RcD}\right]i_1=$$

$$\left[1-\frac{\overline{m+1}\,X/\beta\omega_F\,LD}{\left\{\frac{2X}{\beta\omega_F}LcD^2+(\delta+RcD)\left(F(x)+\frac{D}{\omega_F}\right)\right\}\left\{R+LD+(\delta+RcD)\,(1/s+lD)\right\}+\overline{m+1}\frac{X}{\beta\omega_F}\,LD}\right]\left(1+\frac{\mu}{\cos\theta+\sin\theta\frac{D}{\omega}}\right)V_M \qquad (29)$$

and what has already been said about the necessity for the current i to lag behind the resultant E.M.F. in the anode circuit for Case I applies equally in this case—that is, l must be greater than L or δ must have a positive value. The portion of the anode E.M.F. for which the fork is responsible and is intended to maintain steady for a period, in case of a sudden temporary load on the machine, is equal to the similar portion of the anode E.M.F. of Case I, multiplied by the operator.

$$\left(1+\frac{\mu}{\cos\theta+\sin\theta\frac{D}{\omega}}\right)$$

The effect of introducing reaction by the motor in the grid circuit of the valve is to increase the lead of the back E.M.F. of the motor on the fork portion of the anode E.M.F. by an amount

$$\tan^{-1}\frac{\mu\sin\theta}{1+\mu\cos\theta}$$

The condition that the anode E.M.F. for which the fork is responsible must lag behind the E.M.F. of the motor by an angle greater than π, less the lag of the anode current behind the resultant E.M.F. in the anode circuit gives the relation

$$\begin{aligned}&\overline{m+1}Ls\omega_F\,(\delta Y+Rc\omega_F\,Z)X/\beta\omega_F \geqq\\ &\qquad[\delta^2+Rc\omega_F(Rc\omega_F+2\overline{1-\delta X}/\beta\omega_F)](Z^2-Y^2)-\\ &\qquad\qquad[(\delta^2+R^2c^2\omega^2{}_F)F(x)-\delta 2\overline{1-\delta X}/\beta\omega_F]2YZ \quad .\ .\ .\ .\ .\ .\ .\ . \quad (30)\end{aligned}$$

and the minimum value of $\varkappa$ for stable running conditions is

$$\tan^{-1}\frac{0{\cdot}2(Rc\omega_F Z+\delta Y)}{-K(Y^2+Z^2)} \quad .\ .\ .\ .\ .\ .\ .\ .\ . \quad (31)$$

Two examples of this case are shown in Graphs IV and V. The values of μ and θ selected are those decided upon when we considered the motor and valve without the fork, i.e., $\mu=2\cos\gamma$ and $\theta=\pi-\gamma$.

$Lc\omega_F{}^2=1$, $l/L=3$, $Rs=0{\cdot}1$, $Rc\omega_F=0{\cdot}2$, $Ls\omega_F=0{\cdot}5$ and $m=11$. $X/\beta\omega_F$ and $F(\varkappa)$ are shown for values of $K=0{\cdot}7$ and $1{\cdot}3$.

Notice that in this case the useful portion of the $F(\varkappa)$ curve is negative—i.e., the note of the maintained system is slightly higher than the true fork note—also that the values of $X/\beta\omega_F$ for stable running are less than any of the earlier cases, and compare with the tuned fork case without the motor.

For the tuned systems the amplitude of the fork is determined from the relation

$$\left[\delta F(x) - Rc\omega_F - 2\overline{1-\delta}X/\beta\omega_F + (Rc\omega_F F(x)+\delta)\frac{D}{\omega}\right]y = -\frac{A}{\beta\omega_F}i_1 \quad . \quad . \quad . \quad (32)$$

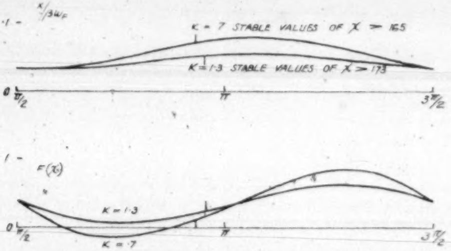
and since i is $-\frac{2P}{V\cos\varkappa}$ the design conditions are as estimable as they are for the untuned cases.

As an example, say

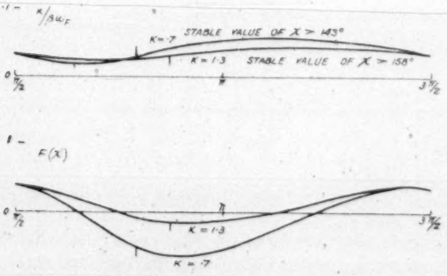
$P = 0{\cdot}045$ watts. $V = 50$ volts.
$\varkappa = 195°$. $\delta = 0$. $Rs = 0{\cdot}1$. $Rc\omega_F = 0{\cdot}2$.
$\omega_F = 2{,}000\pi$. $F(x) = -0{\cdot}2$. $X/\beta\omega_F = 0{\cdot}05$.

$$y = \frac{-\frac{A_1}{\beta\omega_F}i_1}{\sqrt{(\delta F(x) - Rc\omega_F - 2\overline{1-\delta}X/\beta\omega_F)^2 + (Rc\omega_F F(x)+\delta)^2}}$$

Guess $\beta = 20$ and $L = 2\times10^9$, $\frac{A}{\beta\omega_F} = \sqrt{X/\beta\omega_F \,.\, L/\beta\omega_F}$ and $y = 0{\cdot}174m/m$.



GRAPH IV.—MOTOR AND FORK. CONDENSER TUNED $1-LC\omega^2=0$: WITHOUT REACTION ON GRID.



GRAPH V.—MOTOR AND FORK. CONDENSER TUNED $1-LC\omega^2=0$: WITH REACTION ON GRID.

The results given above indicate the influence of the phase angle $\varkappa$ or the lag of the anode current behind the back E.M.F. of the motor in the anode circuit, on the maintained note of the system and the value of driving term $X/\beta\omega_F$, and so determine the design conditions.

All of these results depend entirely, for the approximation to truth, upon the value of $\beta/\alpha\omega_F$ being a very small quantity, and of the possibility of being able to ignore all but fundamental frequencies. The grid conductance has been ignored in the cases which include the motor, for the reason that the expressions are sufficiently cumbersome without it; however, consideration of the problem on the fork alone, in which it has been included, will give a clue to its effect in the motor problem.

In the foregoing work an effort has been made to determine design conditions, and to evaluate the differences between maintained notes of the systems and true fork notes for various combinations of fork systems with synchronous alternating current motors.

Both untuned and tuned systems are dealt with (as well as a special case dealing with a tuned fork without a motor), and the effect of introducing reaction from the motor in the grid circuit of the valve has been included.

Various other arrangements for tapping energy from the anode circuit may be made, and other arrangements for introducing reaction from the motor in the grid circuit are possible; but for similar powers and similar energy distribution the results must be similar to those given above.

In practice the design of efficient systems without reaction from the motor on the grid is difficult, and it is useful in any case to have a method of running the machine up to synchronism with the fork.

Where larger powers are concerned, or where cost of material and efficiency are not considered, it is expedient to amplify an E.M.F. from the anode circuit through succeeding stages of a power amplifier to the motor, and in this case the difference between the maintained note and the true fork note can be made independent of $\varkappa$ entirely, although in the examples shown the variation in $\varkappa$ is so small, in useful cases, that the maintained note is practically constant.

I wish to express my indebtedness to Mr. A. Hinchliffe, who has made up material and has collected a large amount of experimental evidence from the work; also to the Director of Artillery for his permission to offer the Paper for publication.

DISCUSSION.

Prof. O. W. RICHARDSON congratulated the author on the thoroughness of his investigation and on the precision attained by means of his design. When he saw a monumental Paper like the present devoted to engineering problems arising out of the application of thermionic systems, his feelings resembled those of a hen which has laid an egg and hatched an elephant.

Mr. R. S. WHIPPLE said that the instrument was a chronograph of high accuracy, but the actual error must depend ultimately on the accuracy of the fork. Presumably for an error of 1 part in 10,000 an "Elinvar" fork must be used, or a fork kept at constant temperature. He congratulated the author not only on the thoroughness of his theory, but on the excellence of his mechanical design; his drawings had been such that they could be used as shop drawings with very little preparation.

Dr. D. OWEN enquired whether the author had measured the mechanical amplitude of the fork directly as distinct from the anode current? In a case like the present, where the fork is coupled to an otherwise independent system, the amplitude was not independent of the frequency, and it would be interesting to learn what was the functional relation between frequency and amplitude.

AUTHOR'S reply: I wish to thank Professor Richardson for his eulogy of the work and at the same time express contrition for having contributed to the Presidential hen's little accident.

The reason for the design was not so much to measure time accurately, as to draw an accurate scale in which the finer sub-divisions of time occupy their proper places, and are uniform parts of the unit. For this purpose the ordinary steel forks supplied by the Cambridge Instrument Co., have been sufficiently accurate time keepers, but I have contemplated the use of an "Elinvar" fork and hope to secure one eventually. The fact that the small motors work with a power consumption, electrical and mechanical of a few milliwatts only, is proof of the excellence of the Cambridge Company's workmanship in their construction.

Dr. D. Owen asks a very important question. The frequency equation in the Paper for motors coupled to condenser tuned forks, gives a solution depending on the phase lag of the anode current behind the back electro-motive force of the motor, and if the machine hunts the frequency varies to an extent I have endeavoured to predetermine. With the motors we have made, and the small powers we deal with, although phase variation would be measurable, the accurate measurement of phase angles would be difficult. On the other hand, we could measure the variation of frequency with amplitude, varying the anode supply voltage, and although such a measurement would leave a number of quantities indeterminate, it would be of practical value.

We always keep the amplitude of the fork small, and calibrate and use the instrument under identical running conditions.

I have made isolated measurements of amplitude with 1,000 cycle and 50 cycle valve maintained forks, and have found the frequency variation of the latter (a fork without condenser tuning) with amplitude greater than I expected ; however, these measurements were associated with varying methods of control, and are not worth quoting in this connection.

The damping coefficient, 20 dyne c.m. seconds, given as a numerical example in the Paper for a 1,000 cycle fork is small for forks I have had experience with, and values three times this figure are nearer the mark.

XVI.—MEASUREMENTS OF ABSORPTION COEFFICIENTS OF LIGHT FILTERS.

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Received January 3, 1927.

Abstract.

A new spectro-photographic method of measuring absorption coefficients is described. A portion of the slit of a spectrograph is covered by the absorbing medium, and in front of the photographic plate or of the slit a neutral wedge is placed. The resulting spectogram consists of two parts, one due to light which has passed through the filter and the wedge, the other to light which has traversed the wedge only. It is shown how, from a knowledge of the distance between two points, one in each part of the spectogram, which are of the same density, the absorption coefficient of the filter at any wavelength may be determined. The source of light need not be constant.

We have had occasion recently to measure the absorption coefficients of various filters for radiation, both in the visible and ultra-violet parts of the spectrum, and in the course of our work we have devised a simple and accurate method of determining these coefficients.

Ordinarily the intensity of a beam of light is measured by a spectro-photometer fitted with a photo-electric cell or some other type of energy measuring instrument, and the absorption of the filter under test is balanced at various wavelengths against that of a neutral optical wedge or some other standard of absorption. In order to allow for variations in the source of light, the beam is split into two parts, one beam traversing the filter under test, the other passing through the standard absorbing substance. Where this is not possible, the substance under test and the absorption standard are brought alternately into the path of the beam.*

This type of apparatus is necessarily elaborate, and, moreover, it is difficult to obtain a constant source of light and maintain it over an extended period, especially when working in the ultra-violet portion of the spectrum.

In the method which we have adopted, although only one beam of light is used, the constancy of the light source is not essential, and the exact time of exposure is immaterial, provided that sufficient light is transmitted.

The apparatus consists essentially of an ordinary spectograph with its accessories, and a neutral wedge whose density-gradient is known. The greater part of the slit of the spectograph is covered by the filter whose absorption coefficient is to be measured. Also the whole slit, or if more convenient the photographic plate, is covered by the neutral wedge. The resulting spectogram consists of two parts, in the one the light has suffered absorption by the wedge and filter, in the other the wedge alone has been operative. For any given wavelength there will be pairs of points, one in each part of the spectogram, at which the photographic density is the same. It will be shown later that the distance between any such pair of points, multiplied by the density-gradient of the wedge, gives the absorption coefficient of the filter for the wavelength under consideration.

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* Thorne Baker, Journ. Sci. Inst., p. 13, October (1925). W. D. Haigh, Journ. Sci. Inst., p. 211, April (1926).

DESCRIPTION.

In Fig. 1, *AB* represents the slit of the spectograph, *WW* a neutral wedge, *FF* the filter whose absorption coefficient is to be measured.

The wedge has a reference line at its thick end, and the filter is placed so that its edge comes to within about 2 mm. of the reference line at *O*. A spectrogram taken through this combination will appear somewhat as shown on the left of Fig. 1, where the source of light is a carbon arc and *FF* a blue filter. The lower part *YY'* corresponds to light which has passed through both wedge and filter. In the upper part *XX'* the image depends solely on the absorption by the wedge.

When the filter has a wide opaque strip at its edges, as is the case with liquids in glass cells, a rhomboid prism may be placed in front of the filter as shown in Fig. 2.

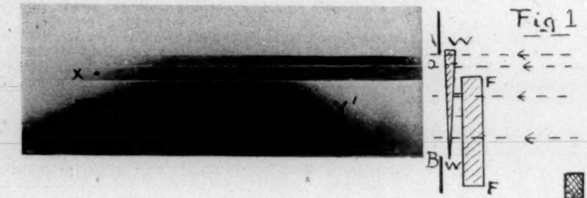

Fig 1

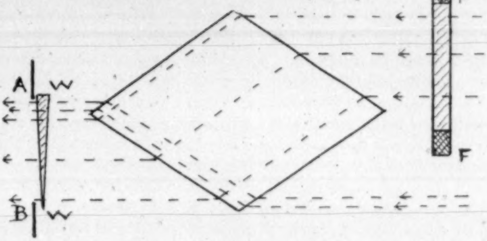

Fig 2

THEORY.

If I_0 is the intensity of the light incident on the instrument, and if x is the distance of a point X in the portion XX' from the image of the thin end of the wedge, the intensity of the light falling on the photographic plate at X is given by

$$I_x = I_0 \cdot 10^{-Kx}$$

K being the known wedge constant.

At a point Y in the lower part of the diagram, whose distance from the same reference line is y, the light intensity is I_Y where

$$I_y = I_0 \cdot 10^{-\mu\lambda} \cdot 10^{-Ky}$$

μ_λ being the absorption coefficient of the filter for the wavelength λ.

If the densities at X and Y are the same

$$I_x = I_y$$

or

$$I_0 \,.\, 10^{-Kx} = I_0 \,.\, 10^{-Ky-\mu_\lambda}$$

or

$$\mu_\lambda = K(y-x)$$

$$= Kz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where z is the distance between X and Y.

It will be noticed that no characteristic of the plate or its treatment enters into this equation. We make the one assumption that if the sensitivity and the development are uniform over a small portion of the plate then for any given wavelength equal intensities give equal densities.

METHOD.

The plate after exposure and development is placed on the stage of any good micro-photometer, and the density is noted at a point X, say 1 mm. below the reference line on the part of the image where light has not passed through the filter. Another point Y is found on the lower part of the image, at the same wavelength, and at which the density is the same as at X. The measurements may be repeated, using slightly different positions of X (all at the same wavelength) and correspondingly different positions of Y. The mean value of the values obtained for the distance between X and Y is substituted in equation (1), giving the required absorption coefficient.

It is of course necessary that the incident radiation should be of uniform intensity over the slit. In practice the radiation does not fall on the slit or the wedge as a parallel pencil. It is therefore desirable to calibrate the wedge *in situ*, and not to take values of K measured in parallel light.

It is obvious that any variation in the intensity of the source has no effect on the relative densities at X and Y. Line spectra, using a wide slit, can of course be employed instead of continuous spectra. The wedge or filter may be placed either in front of the slit or of the photographic plate.

By taking the precautions for uniform illumination of the slit and uniform development of the plate which we have described elsewhere, results can easily and readily be obtained with an accuracy of about 1 per cent.

NOTE ADDED MARCH 10TH, 1927.

Since the above was written we have slightly modified the arrangement in order to reduce the exposure necessary. The filter which is being measured is made to cover a smaller part of the slit than indicated above. The point X in the above diagram is then taken near the densest edge of the image where the light has passed through both wedge and filter, and a point Y of equal density is found on the upper part of the image where the light passes through the wedge only. The theory is identical with that given above, but since the thinner parts of the wedge are used, the exposures can be reduced.

Stray Light.—Where a very strongly absorbing filter is being measured errors are likely to arise from light scattered from a part of the spectrum which is freely transmitted, causing fogging over other parts of the spectrum, during the long exposures which are necessary to get a measurable image at the wavelengths which are strongly absorbed. This can be eliminated by using subsidiary colour filters covering the *whole* slit which reduce the light of wavelength not required. To get reliable measures in all parts of the spectrum it may be necessary to take two or three spectrograms with suitable exposures and suitable additional filters covering the whole slit; this, however, entails very little extra work.

It is also possible that as the light from the upper parts of the spectrum is very intense compared with the lower, scattered light from the former may fog the latter. When this is the case the points X and Y will necessarily be a long distance apart, and the major part of the light causing the scattering can be eliminated by raising the filter being measured until its edge approaches the point Y in the upper part of the image. Again, it may be necessary to take two or three spectra, a part of the spectrum being measured on each.

XVII.—A COMPARISON OF THE BEHAVIOUR IN THERMAL DIFFUSION OF NITROGEN AND CARBON MONOXIDE, AND OF NITROUS OXIDE AND CARBON DIOXIDE.

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Received December 17, 1926.

(*Communicated by* Prof. S. W. J. Smith, *F.R.S.*)

Abstract.

The thermal separation obtained in gas mixtures containing nitrogen is compared with that obtained in mixtures containing carbon monoxide. A similar comparison is made of mixtures containing carbon dioxide or nitrous oxide. The gas analysis required in the measurement of the effect is made by means of the Shakespear katharometer. The behaviour of nitrogen is found to be similar to that of carbon monoxide. The effect given by carbon dioxide is generally a little greater than that given by nitrous oxide.

The pairs of gases examined provide a special case for the application of the Enskog-Chapman theory, as in each pair the molecular weights and mean collision areas are the same. It can thus be deduced that the molecular field of nitrogen is similar to that of carbon monoxide, and that the field of carbon dioxide differs little from that of nitrous oxide.

THE Enskog-Chapman theory* of thermal diffusion has shown how this effect in a mixture of two gases will depend on the masses and diameters of the gas molecules, and on the proportions of the gases in the mixture. The effect has also been shown to depend on the nature of the molecules: it will be greatest for molecules which behave like rigid elastic spheres, and will disappear entirely for Maxwellian molecules obeying an inverse fifth power law. A comparison of the observed effect and the maximum calculated effect will give information on the nature of the molecular fields of force. The theory has already been confirmed experimentally† in a number of important points.

The present series of experiments originated in a suggestion made by Prof. S. W. J. Smith as to the possibility of investigating whether any thermal separation can be obtained in a mixture of gases such as nitrogen and carbon monoxide, where the molecular weights and sizes are practically the same. For this special case the theory predicts that the effect will disappear, whatever may be the nature of the fields of force. As the thermal conductivities of nitrogen and carbon monoxide are also nearly the same, this pair does not lend itself to direct examination by the method employed by one of the present writers in previous experiments, in which the Shakespear katharometer was used for gas analysis. Any small amount of separation which might be obtained would be difficult to measure in this case. The suggestion, however, led to the idea of mixing the gases separately with a suitable subsidiary gas of considerably different thermal conductivity and comparing the effects which can be observed. For example, using hydrogen as a subsidiary gas, the effect in a mixture of carbon monoxide and hydrogen can be compared with that in a mixture of nitrogen and hydrogen. As the gases forming the two mixtures are practically the same in molecular size and weight, any difference in the thermal diffusion effect observed will be due to a difference in the forces arising during collision. If the effect is the same for the two pairs, it will indicate a similarity of the molecular fields of force of the two gases carbon monoxide and nitrogen. A similar system of comparison can be applied to carbon dioxide and nitrous oxide, as these two gases have also very nearly the same molecular weight and size.

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* Enskog, D., Phys. Zeit., Vol. 12, p. 538 (1911); Ann. d. Phys. (4), Vol. 38, pp. 742, 750 (1912); Chapman, S., Proc. Roy. Soc., A, Vol. 93, p. 1 (1916); Phil. Trans., A, Vol. 217, p. 157 (1917); Phil. Mag., Vol. 34, p. 146 (1917); Phil. Mag., Vol. 38, p. 182 (1919); Enskog, D., Arkiv. f. Nat. Astron.-o-Fysik., Stockholm XVI (1921); Chapman, S., and Hainsworth, W., Phil. Mag., Vol. 48, p. 602 (1924).

† Chapman, S., and Dootson, F. W., Phil. Mag., Vol. 33, p. 268 (1917); Ibbs, T. L., with a note by Chapman, S., Proc. Roy. Soc., A, Vol. 99, p. 385 (1921); Ibbs, T. L., Proc. Roy. Soc., A, Vol. 107, p. 470 (1925); Elliott, G. A., and Masson, I., Proc. Roy. Soc., A, Vol. 108, p. 378 (1925).

Eleven pairs of gases were examined in the course of the experiments. These can be grouped as follows for purposes of comparison: Carbon monoxide and nitrogen were each examined with hydrogen, carbon dioxide and nitrous oxide as subsidiary gases. Carbon dioxide and nitrous oxide were each examined with hydrogen, oxygen, nitrogen and carbon monoxide as subsidiary gases. Four of the pairs can thus be used for the purposes of each comparison. Mixtures of ethylene and hydrogen were also examined in order to compare the behaviour of ethylene with that of carbon monoxide and nitrogen. This comparison is of interest, as the molecules C_2H_4, CO and N_2 have practically the same masses, but the C_2H_4 molecule differs from the others in size and in its number of atoms. The somewhat related problem of obtaining a partial separation by thermal diffusion in mixtures of nitrogen and ethylene has been discussed* by Professor Chapman.

METHOD OF EXPERIMENT.

The method of experiment was essentially the same as that employed in a previous series† of experiments, but a few modifications were introduced to meet the special needs of this work. A uniform mixture of two gases is introduced into the apparatus, which can be regarded as two vessels maintained at different temperatures joined by a connecting tube. A glass bulb *A* (Fig. 1), of volume 99·8 c.c., surrounded by a steam jacket *B*, served as the hot side. This was joined by a connecting tube *C*, of length 2·5 cm., and internal diameter 4 mm., to a brass cap *S* screwed to a katharometer block *K*. The screw cap and katharometer block were kept at constant temperature by means of a water jacket supplied from the mains. The total volume of the cold side, which includes the inside of the screw cap and the katharometer cell, was 1·39 c.c. The length of the connecting tube is reduced as far as possible, so that the temperature gradient is steep, and the time for the effect to take place is short. For mixtures containing hydrogen the effect could be regarded as complete in about four minutes, for other mixtures about ten minutes was allowed.

As all the gases used in these experiments could be obtained in considerable quantities, it was possible to make the hot side larger than before, with the result that nearly all the effect will show itself as a change on the cold side, where it is measured. The corresponding small effect on the hot side can be calculated. The thermal separation is expressed throughout as the difference in percentage composition by volume, between the gas on the two sides when equilibrium is reached. By making the hot side large we reduce the possibility of error due to uncertainty of the exact volumes of the connecting tube or cold side. For the purpose of making a comparison it was convenient to work with the hot side at steam temperature throughout. This facilitates the examination of a large number of mixtures requiring many observations. Except for the mixtures containing hydrogen, the mass ratio of the molecules is small, so that the maximum separation obtained with the hot side at about 100°C. and the cold side at about 10°C. will not be great. For these mixtures it was found to be less than 0·5 per cent. There is no advantage, however, in using a greater temperature range, as this tends to make the conditions of experiment less steady. The method adopted gave consistent results, and it was found that satisfactory measurements could thus be made on mixtures of small mass ratio.

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* Chapman, S., Phil. Mag., Vol. 34, p. 146 (1917).

† Ibbs, T. L., Proc. Roy. Soc., A, Vol. 107, p. 470 (1925).

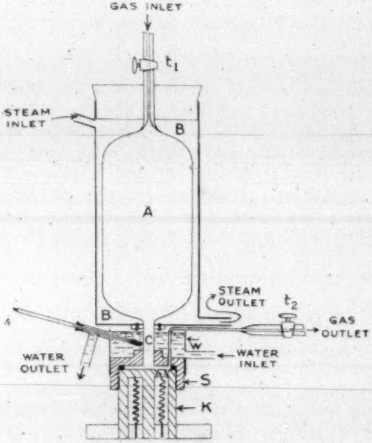

FIG. 1.—DIAGRAM OF APPARATUS.

In some of the experiments the gases were mixed in a cylinder under pressure. Steam was passed through the jacket, and the cold water turned on. The gas mixture was then allowed to flow through the apparatus, entering at tap t_1 and leaving at tap t_2. In this way the flow of gas through the connecting tube overcomes the tendency for thermal separation, and gas of very nearly uniform composition fills the apparatus. The flow is then stopped, taps t_1 and t_2 are closed, and thermal diffusion takes place, the change of composition of the gas on the cold side being measured by the katharometer. The temperature of the steam gives the hot side temperature T_1, and the temperature of the cold water gives the cold side temperature T_2.

In other cases, where it was undesirable to mix the gases under pressure in a cylinder, the following rather different procedure was adopted. The gases were first passed separately into the apparatus, and allowed to mix by diffusion at atmospheric pressure. Steam was then passed through the heater. If the taps were kept closed this would result in a change of pressure, which would itself affect the katharometer readings. To avoid any difficulty in making a correction for this, tap t_2 is opened when the heater is applied. Some gas, therefore, is driven out from the hot side through the connecting tube and atmospheric pressure is maintained. By applying the steam suddenly to the heater the gas escapes before much thermal separation has taken place. The loss of this gas, therefore, does not make any serious change in the mean composition of the gas in the apparatus. The glass bulb *A* is at the temperature of the room before the steam is applied, and this may differ by a few degrees from the temperature of the cold water. This will result in a small initial difference in composition between the gas on the two sides due to the thermal diffusion effect. What is actually measured when the steam is applied is the further change in composition which takes place on the cold side. In this case T_2 can be regarded as the temperature of the glass bulb before the steam is applied, and T_1 the temperature after the steam is applied. Special experiments were made which showed that this method and the flow method previously mentioned gave concordant results.

With the hot side at 100°C. and the cold side at 10°C., the value of $\log_{10}\frac{T_1}{T_2}$ is nearly 0·12, T_1 and T_2 being expressed in absolute measure. For the purposes of comparison it has been convenient to work out all results for $\log_{10}\frac{T_1}{T_2}=0{\cdot}12$. No serious error can be introduced by assuming for all mixtures that the separation is proportional to $\log_{10}\frac{T_1}{T_2}$ for the temperature ranges actually used.

The bridge arrangement of the katharometer was similar to that previously adopted.* The instrument is used for two distinct purposes : (*a*) for the analysis of gas mixtures, accurate to about 0·5 per cent. ; (*b*) for the measurement of the small changes in composition produced by thermal diffusion, with a probable accuracy of about 0·005 per cent. Eight new calibration curves were required. These were made by means of apparatus designed for making experiments on small quantities of gases, which it is hoped to describe later. A number of mixtures of known composition were made for each pair of gases by using a constant volume gas burette of the type described by Travers.†

We are indebted to Dr. G. A. Shakespear for his kindness in lending the katharometer used in this work.

PURITY OF GASES.

As this method of experiment requires considerable quantities of gases, it was convenient to use them as supplied commercially in cylinders under pressure. These gases could generally be regarded as sufficiently pure for the purpose. A satisfactory way of examining the purity is by testing the gas from the supply cylinder without admixture for thermal diffusion in the usual way. A small amount of impurity having about the same molecular weight as the gas will have little effect, whereas the same proportion by volume of an impurity of considerably different molecular weight may produce a greater effect which cannot be neglected. The nitrogen, carbon monoxide and hydrogen were practically pure, and the carbon dioxide and nitrous oxide contained only small amounts of impurity. We are indebted to Prof. F. W. Burstall, the Vice-Principal of the University, for providing a quantity of carbon monoxide of a high degree of purity. This was made in the plant in his department, and the gift was particularly welcome as the gas cannot be obtained commercially in a pure state. The oxygen contained about 3·0 per cent. of argon as an impurity. In mixtures of this gas with carbon dioxide or nitrous oxide the effect of the impurity would not be serious, previous experiments having shown that argon and carbon dioxide behave somewhat similarly. The ethylene contained about 1·0 to 1·5 per cent. of hydrogen. This impurity is serious, except in mixtures of the gas with hydrogen, and only measurements on this pair are recorded.

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* l.c., p. 475.

† Travers, M. W., "Study of Gases," p. 69.

RESULTS.

The results for the eleven pairs of gases are shown in Tables I, II and III. All are expressed as the percentage separation obtained when $\log_{10}\frac{T_1}{T_2}=0{\cdot}12$. The percentage of the lighter gas in the original mixture is stated throughout. The results

TABLE I.

Nitrogen and Carbon dioxide.		Carbon monoxide and Carbon dioxide.		Nitrogen and Nitrous oxide.		Carbon monoxide and Nitrous oxide.	
Per cent. N_2 in mixture.	Separation per cent.	Per cent. CO in mixture.	Separation per cent.	Per cent. N_2 in mixture.	Separation per cent.	Per cent. CO in mixture.	Separation per cent.
3·75	0·044	11·8	0·132	16·3	0·160	11·4	0·114
12·5	0·133	26·4	0·269	24·4	0·218	23·0	0·205
17·6	0·185	32·1	0·334	34·7	0·309	33·3	0·318
26·9	0·282	39·9	0·403	39·3	0·342	40·8	0·363
35·3	0·366	44·6	0·402	41·0	0·352	47·8	0·388
40·6	0·394	50·25	0·415	41·0	0·362	51·7	0·397
44·9	0·407	51·2	0·428	45·6	0·387	55·3	0·398
45·1	0·401	59·8	0·401	49·0	0·394	62·0	0·352
49·8	0·421	67·8	0·341	54·8	0·384	68·0	0·312
57·0	0·409	77·0	0·261	56·6	0·381	80·7	0·199
67·2	0·355	...	...	59·6	0·365	...	...
80·8	0·199	...	...	72·0	0·269	...	...
92·9	0·086	...	...	84·1	0·156	...	...

TABLE II.

Carbon dioxide and oxygen.		Nitrous oxide and oxygen.		Carbon dioxide and hydrogen.		Nitrous oxide and hydrogen.	
Per cent. O_2 in mixture.	Separation per cent.	Per cent. O_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.
20·0	0·203	17·1	0·157	3·5	0·158	2·0	0·119
32·9	0·321	23·1	0·244	8·4	0·433	9·4	0·445
39·8	0·364	27·0	0·281	13·6	0·683	11·1	0·573
43·3	0·383	33·3	0·345	15·7	0·831	12·5	0·605
45·5	0·396	35·0	0·377	19·2	0·95	18·7	0·930
47·0	0·383	38·6	0·376	22·2	1·15	25·6	1·24
49·4	0·400	42·4	0·386	23·0	1·18	31·0	1·44
53·8	0·392	46·1	0·393	24·4	1·21	36·4	1·58
55·8	0·391	47·9	0·391	28·3	1·43	38·1	1·63
57·3	0·385	50·0	0·407	32·8	1·56	40·8	1·71
58·5	0·388	52·9	0·397	34·0	1·54	43·9	1·83
61·1	0·372	57·9	0·395	37·1	1·74	50·1	1·93
68·7	0·323	64·5	0·369	42·1	1·94	53·2	1·99
76·6	0·282	80·0	0·249	46·7	1·99	61·0	2·07
79·6	0·292	85·7	0·203	63·5	2·18	69·0	2·02
87·5	0·204	93·8	0·128	71·4	2·10	72·7	1·97
90·4	0·170	...	...	78·0	1·95	78·6	1·88
...	...	...	...	80·0	1·83	83·5	1·52

TABLE III.

Carbon monoxide and hydrogen.		Nitrogen and hydrogen.		Ethylene and hydrogen.	
Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.	Per cent. H_2 in mixture.	Separation per cent.
5·5	0·282	5·2	0·264	5·4	0·242
8·0	0·405	9·6	0·451	10·5	0·466
15·7	0·841	18·9	0·967	17·7	0·787
24·4	1·21	25·4	1·34	25·1	1·105
30·1	1·49	36·9	1·80	29·0	1·28
35·7	1·69	46·6	2·25	39·6	1·60
42·4	1·95	54·7	2·54	48·2	1·73
48·5	2·18	55·8	2·60	58·3	2·16
55·0	2·54	61·0	2·63	64·4	2·40
61·7	2·53	62·1	2·58	74·5	2·34
63·3	2·64	66·4	2·62	74·8	2·28
65·3	2·62	68·2	2·54	82·0	2·23
70·4	2·46	76·4	1·98	84·6	1·69
77·8	2·19	81·6	2·03	85·1	1·52
81·9	1·65	85·0	1·45	89·6	1·37
83·4	1·57	87·2	1·21	...	...

are also shown graphically in Figs. 2, 3, 4 and 5. These composition-separation curves show clearly how the amount of separation depends on the proportions of the gases in the mixtures. More than three hundred measurements of thermal diffusion were made. Many of the values given represent the mean of a number of observations on the same mixture. In addition, numerous experiments were made which served to check the accuracy of different portions of the work.

DISCUSSION OF RESULTS.

All the mixtures examined showed the thermal diffusion effect of the sign predicted by theory: in all cases there was a tendency for the heavier molecules to move to the cold side, and the lighter molecules to the hot side. Previous results showing the influence of the mass ratio of the molecules in a mixture on the shape of the composition-separation curves are confirmed. When the mass ratio is nearly unity the curves are nearly symmetrical, the maximum effect being given in a mixture containing about 50 per cent. of the two gases (Figs. 2 and 3). For mixtures containing hydrogen, in which the mass ratio of the molecules is greater, the curves are unsymmetrical, and the maximum occurs in a mixture considerably richer in hydrogen than in the other gas (Figs. 4 and 5).

NITROGEN COMPARED WITH CARBON MONOXIDE.

Examination of the results shows that the effect is very nearly the same in mixtures containing either nitrogen or carbon monoxide and a subsidiary gas. The graphs drawn in Fig. 2 illustrate this clearly, the upper curve representing the effect for mixtures of either nitrogen with carbon dioxide or carbon monoxide with carbon dioxide; and the lower curve for either mixtures of nitrogen with nitrous oxide or carbon monoxide with nitrous oxide. In Fig. 5 it will be seen that the effect is very nearly the same for mixtures of nitrogen with hydrogen as for

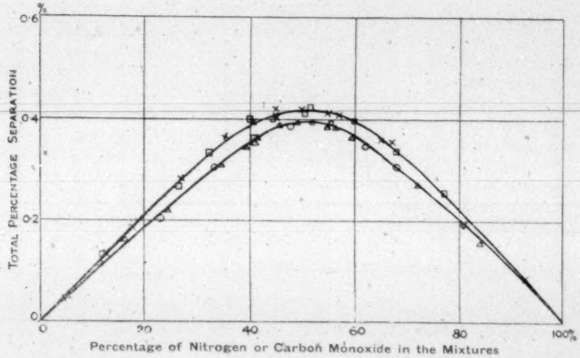

FIG. 2.

Composition—Separation Curves.

$\left[\log_{10}\frac{T_1}{T_2}=0{\cdot}12\right]$

Upper Curve { 1. Nitrogen and Carbon Dioxide . . ×
Upper Curve { 2. Carbon Monoxide and Carbon Dioxide ⊡
Lower Curve { 3. Nitrogen and Nitrous Oxide . . . △
Lower Curve { 4. Carbon Monoxide and Nitrous Oxide ⊙

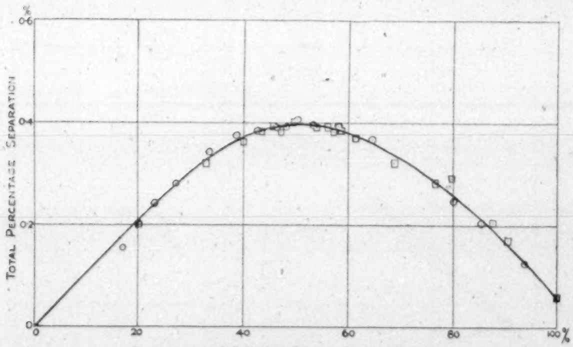

FIG. 3.

Composition—Separation Curves.

$\left[\log_{10}\frac{T_1}{T_2}=0{\cdot}12\right]$

1. Oxygen and Carbon Dioxide . . . . . . ⊡
2. Oxygen and Nitrous Oxide . . . . . . ⊙

NOTE.—The effects shown by the two mixtures are practically the same. The separation shown at 100 per cent. is due to impurity in the Oxygen used.

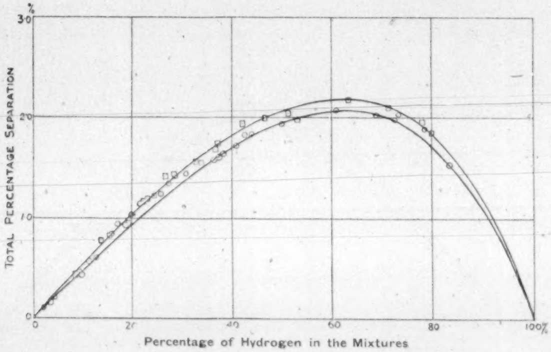

FIG. 4.

Composition—Separation Curves.

$$\left[\operatorname{Log}_{10}\frac{T_1}{T_2}=0\cdot12\right]$$

1. Hydrogen and Carbon Dioxide . . . ⊡
2. Hydrogen and Nitrous Oxide . . . . ⊙

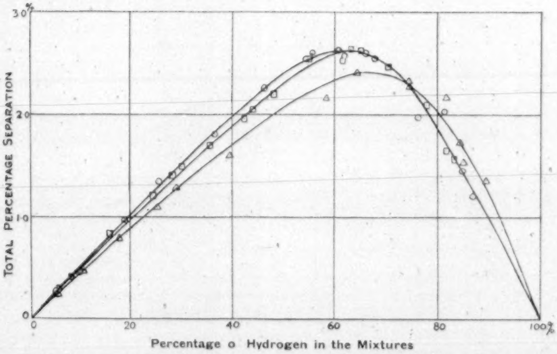

FIG. 5.

Composition—Separation Curves.

$$\left[\operatorname{Log}_{10}\frac{T_1}{T_2}=0\cdot12\right]$$

1. Hydrogen and Carbon Monoxide . . . . . ⊙
2. Hydrogen and Nitrogen . . . . ⊡
3. Hydrogen and Ethylene . . . . △

mixtures of carbon monoxide with hydrogen, although these two curves cannot quite be superposed completely. It may be considered that the effects produced by nitrogen or carbon monoxide in any of the mixtures do not differ by more than the possible errors of these experiments. C. J. Smith* has made an accurate comparison of the viscosities of these two gases, using Rankine's method. Measurements made at 15°C. and 100°C. indicate that the viscous properties of the two gases are identical over this range of temperature. The mean collision area, deduced by using Chapman's formula, is given as $0{\cdot}767\times10^{-15}$ sq. cm. for both gases, so that the molecules can be regarded as having the same size. The similarity of the behaviour in thermal diffusion therefore indicates that the molecular fields must be closely alike. This is of particular interest, as there is known to be a similarity of structure in the two gases, both having the same number (14) of extra-nuclear electrons. In the Lewis-Langmuir theory of molecular structure the two gases are considered to have the same electronic arrangement, and Langmuir† attaches importance to the idea that these gases can be regarded as examples of a special case where the whole molecule lies inside one cube formed by eight electrons. The fact that the two gases have very similar physical properties is in keeping with the idea of the similarity of their structure and external fields. The work of Bone‡ and his collaborators on the "activation" of nitrogen appears to indicate a further relationship between the two gases. Information given by viscosity measurements§ on nitrogen and carbon monoxide had previously indicated that their molecular fields are alike. Viscosity in a gas is, however, concerned with the forces arising between molecules of the same kind, while thermal diffusion is due to collisions between molecules of different kinds. The consistency of the behaviour of these two gases, of very different chemical properties, when in collisions of the latter kind appears to be of interest.

ETHYLENE.

The separation obtained in mixtures of ethylene and hydrogen is less than that in mixtures of nitrogen or carbon monoxide and hydrogen, as shown by the curve in Fig. 5. The mass ratio of the molecules in these mixtures is the same, but the size of the ethylene molecule is greater than that of carbon monoxide or nitrogen. The increased size of the heavier molecule which we have in ethylene-hydrogen mixtures would itself tend to increase the separation. The C_2H_4 molecule must therefore be regarded as softer than the molecule N_2 or CO. The curve for ethylene-hydrogen shown in Fig. 5 differs from the other two in shape, being more unsymmetrical. As the ethylene used in these experiments was impure, too much importance must not be attached to this difference in shape at present.

NITROUS OXIDE COMPARED WITH CARBON DIOXIDE.

A comparison of the behaviour of nitrous oxide and carbon dioxide in mixtures with subsidiary gases shows that the effects produced by the two gases are nearly the same. Fig. 3 shows that the effect for nitrous oxide and oxygen differs little from that for carbon dioxide and oxygen. In all other cases the effect given by nitrous oxide and a subsidiary gas is rather less than that given by carbon dioxide. Thus, in Fig. 2, the curve for carbon dioxide with either nitrogen or carbon monoxide lies above the curve for nitrous oxide with either nitrogen or carbon monoxide, and in Fig. 4 the curve for carbon dioxide and hydrogen lies above the curve for nitrous oxide and hydrogen. The work of C. J. Smith[*] has shown that the viscous properties of carbon dioxide and nitrous oxide are very nearly the same at 15°C. and 100°C., and he deduces that the mean collision area of the molecules of each of these gases is $0{\cdot}834 \times 10^{-15}$ sq. cm. These two gases of equal molecular weight and size are also similar in their physical properties, although in this respect they are not as closely alike as nitrogen and carbon monoxide. Again there is a similarity of structure, both gases having the same number (22) of extra-nuclear electrons. Langmuir considers that both have the same electronic arrangement: the outer electrons forming three cubes in line, the middle cube sharing two of its faces with the outer ones. An atomic nucleus lies in each cube. Rankine[†] has discussed the similarity of these two kinds of molecules. The structure suggested for the two gases is, however, not as compact as that suggested for nitrogen and carbon monoxide, where both nuclei are supposed to lie inside one electronic cube. The gases will differ in properties which depend on the distribution of the atomic nuclei in the molecules. We may expect a difference in the molecular moments of inertia; and although the total charge on the three nuclei is the same in both cases, we may expect some difference in the mean external field. These measurements show that there is a small difference in the fields, and that the CO_2 molecule must be a little harder than the N_2O molecule. Viscosity measurements[‡] have previously shown that the fields do not differ greatly, but indicated that the N_2O molecule was rather harder than the CO_2 molecule.

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* Smith, C. J., Proc. Phys. Soc., Vol. 34, Part 4, p. 155 (1922).

† Langmuir, I., Amer. Chem. Soc. Journal, Vol. 41, I, p. 868 (1919).

‡ Bone, W. A., and Haward, W. A., Proc. Roy. Soc., A, Vol. 100, p. 69 (1922); Bone, W. A., Newitt, D. M., and Townend, D. T. A., Proc. Roy. Soc., A., Vol. 103, p. 205 (1923).

§ cf. Jeans, J. H., Dynamical Theory of Gases, 3rd edit. p. 284.

The gases examined in these experiments provide a special and simple case for the application of the theory. A direct comparison of the thermal separation obtained in the mixtures gives information of a comparative nature on the molecular fields of certain gases. It also indicates a relationship between the external field and the molecular structure. The possibility of applying thermal diffusion to the investigation of such problems has been pointed out by Prof. Chapman. No attempt will be made here to compare the separation actually obtained with that predicted by theory. The results of the experiments have been communicated to Prof. Chapman, and it is hoped that he and Mr. Hainsworth will examine more fully their general bearing on the question of inter-molecular fields.

In the course of this work measurements were made on three pairs of gases which had previously been examined by one of the writers[§]—hydrogen and carbon dioxide, nitrogen and carbon dioxide, and hydrogen and nitrogen. For hydrogen and carbon dioxide the shape of the composition-separation curve and the amount of the effect are in agreement with previous results obtained by this method. The separation is, however, relatively less than that obtained by Elliott and Masson[||] when making experiments with the hot side at 490°C. The symmetry of the composition-separation curve for nitrogen and carbon dioxide, which was previously noted, is confirmed, many more observations being made in this case. The modified method employed gives steady conditions, which tend to give increased accuracy. The maximum separation obtained for hydrogen and nitrogen is relatively rather greater than that previously recorded.

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[*] l.c., p. 159.

[†] Rankine, A. O., Proc. Roy. Soc., A, Vol. 98, p. 369 (1920).

[‡] cf., Jeans, J. H., l.c.

[§] cf., l.c.

[||] Proc. Roy. Soc., A, Vol. 108, p. 378 (1925).

In conclusion, we desire to express our thanks to Prof. S. W. J. Smith for the help he has given by granting full facilities for the work.

DISCUSSION.

Prof. S. **Chapman** said that the molecular similarity of the gases discussed, which had already been shown by Rankine and Smith, was still more directly proved by the present method. Thermal diffusion is a phenomenon discovered by means of mathematical analysis, and it is very difficult to form any physical picture of the way in which it arises: wherever a temperature gradient exists in a mixture of gases, a separation of the components takes place until equilibrium with the mixing effect of ordinary diffusion is established. The phenomenon disappears when the attraction between molecules varies as r^{-5}, and its magnitude affords a measure of $(n-5)$ when the attraction varies as r^{-n}. Every point on the curves obtained by the authors gives a measure of n, whereas in the viscosity experiments n has to be inferred from the viscosity at two temperatures, from the formula $\log v = \{(n+3)/2(n-1)\} \log T$, where v is the viscosity and T the temperature. Further, the coefficient of $\log T$ is not very sensitive to variations of n, so that the present method affords a far more delicate measure. On the assumption that n is 2 in stars where ionisation is complete, the quantity $(n-5)$ would be negative, and the effect of thermal diffusion would be reversed in sign, the heavier molecules moving towards the higher temperature.

Prof. A. O. **Rankine** said that he had seen the authors' work while it was in progress, and was struck by the amount of labour involved, which was much greater than appeared from the Paper, since it necessitated precise variations in the percentages of the mixtures used.

Dr. **Ezer Griffiths**: There is just one question I should like to ask the authors. It concerns the arrangement of their apparatus. From the point of view of accuracy in the observations, would it not have been more satisfactory if the hot wire katharometer had been arranged so that one of the gaseous mixtures had access to one of the platinum spirals, and the other gaseous mixture to the other spiral? The observed quantity would then be the "difference" in the properties of the two mixtures, which is what the authors are seeking. Such an arrangement would have the added convenience that the two mixtures could be subjected to identical temperature conditions and interchanged from one side to the other to eliminate instrumental error.

Authors' reply: A differential method as suggested by Dr. Ezer Griffiths, where gas mixtures have access to both platinum spirals, has already proved to be of value for a number of purposes, in which the measurement of small differences is involved. Its use however in the type of thermal diffusion measurement described in the Paper, appears to present certain additional difficulties in the arrangement of the apparatus, and in making the measurements. In addition to observations of a comparative nature on the different pairs of gases, measurements of the actual amounts of separation produced in the gas mixtures were required.

XVIII.—RELIGHTING OF A NEON LAMP WHEN MOMENTARILY EXTINGUISHED AT VOLTAGES BELOW THE STRIKING POTENTIAL.

By ROBERT R. NIMMO, *M.Sc.*, Assistant to the Professor of Physics, University of Otago, Dunedin, New Zealand.

Received November 27, 1926.

(*Communicated* by Prof. R. JACK.)

SUMMARY.

The Paper gives an account of an investigation on the time for which the continuous discharge of a neon lamp may be interrupted without putting out the lamp. This time, which has been found to be of the order of 50 micro-seconds, depends on the voltage across the lamp and also on the current passing through it.

INTRODUCTION.

A NEON lamp discharge starts spontaneously if the striking voltage is applied to the lamp, but it would conceivably be possible to start the discharge at a lower voltage than this if a large supply of ions were initially present in the lamp. If a discharge be made to pass through the lamp and then interrupted for a short time, the supply of ions which was present when the discharge was passing will quickly diminish. If at the end of this short interruption a voltage smaller than the striking voltage be applied to the lamp, the discharge may recommence. It is of interest, then, to investigate the maximum time for which a lamp discharge may be interrupted so that sufficient ions are left in the lamp to allow the discharge to recommence, when the voltage applied to the lamp is always below the striking voltage.

This problem has been investigated by Oschwald and Tarrant.* Although they interrupted the lamp discharge for periods as short as 50 micro-seconds, the lamp never relit even when voltages very little below the striking voltage were used.

EXPERIMENTAL.

The arrangement used in interrupting the lamp discharge is illustrated in Fig. 1. The lamp used was an *I* neon lamp not previously overrun.† In series with the lamp was a resistance box *R* and a galvanometer shunt *G*. The switch *K* was of the rotating type, and was employed to connect an uncharged condenser in parallel with the lamp. Later in the rotation of the switch the condenser was discharged through the resistance *m*. When the condenser charges through the resistance *R* it can readily be shown that the voltage across the neon lamp is below the extinction voltage E_1 for a time‡ $RC \log_e \frac{E_0}{E_0 - E_1}$, where E_0 is the battery voltage and *R*§ and *C* the values of the resistance and capacity. The rest of the experimental arrangements were as follows. When the switch *Z* was open the condenser was already charged when connected across the lamp, and so opening *Z* really put the rotating switch *K* out of action; hence *K* could be started or stopped without the trouble of starting or stopping the motor driving it. One section (30 volts) of the accumulator battery was put on charge during the experiments, and by varying the charge rate the voltage across the lamp could be adjusted. The voltage used had to be below the striking voltage for the lamp, and so the switch *S* and the top part of the battery were used to light the lamp. The 10 mfd. condenser shown was important, for its charge kept the lamp alight while the switch *S* was put from "*a*" to "*b*," and also, owing to its large value compared with that of the condenser *C* it greatly minimised the effect that would be produced by the resistance of the battery. The condenser *C*, which in charging interrupted the lamp discharge, was a $\frac{1}{3}$ mfd. standard mica condenser.

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* Oschwald and Tarrant, Proc. Phys. Soc., Vol. 36, p. 262.

† The ballast resistance was of course removed.

‡ Some assumptions made in deducing this time are examined below.

§ For reasons given below, *R* does not include the battery resistance if this is small.

In the actual experiments, *Z* was opened, the lamp was lit and *S* was put to "*b*." The switch *K* had previously been started, and when *Z* was closed the interruption commenced. The voltage applied to the lamp and resistance *R* was kept constant, and, as usually happens, the current through the lamp gradually fell with time. It was on this account that a rotating switch was used. It was found that when the current passing through the lamp was large the charging of the condenser did not extinguish the lamp when *R* was, say, 100 ohms; but later, when the current had dropped considerably, the lamp discharge was extinguished on charging the condenser through 90 ohms. By using a rotating switch the lamp discharge was interrupted several times while the lamp current was approximately constant, and so it was possible to give *R* a definite value and go on interrupting the discharge until the current fell to such a value that the interruption was sufficient to extinguish the lamp. Thus the galvanometer reading gradually fell with time, and then suddenly it dropped to zero. The reading of the galvanometer was taken just before the sudden drop to zero, and this gave the lamp current for which the extinction interval used was just large enough to cause interruption of the discharge. The galvanometer reading just before extinction, the applied voltage when the current was flowing and the resistance used were all noted. The galvanometer reading just before extinction was somewhat variable, and so about ten values were taken with the same values of *R*, *C* and the applied E.M.F. The means of these sets are given in the table on next page.

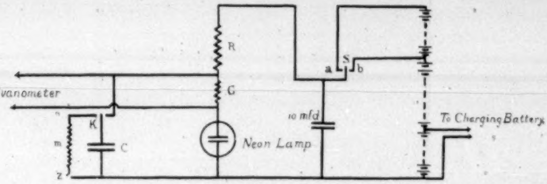

FIG. 1.

The table below shows the maximum time for which the lamp discharge could be interrupted without extinction occurring when the voltage[*] and current had the values stated. The time of interruption of the discharge was calculated on the assumption that interruption occurred when the lamp potential was below 142 volts, the extinction voltage for a continuous discharge. One lamp only was experimented upon.

RESULTS.

Voltage across lamp. Volts.	Current at extinction. Milliamps.	Resistance. Ohms.	Time of interruption. Microsecs.
170	6·5	110	66
	6·3	100	60
	5·9	90	54
	5·7	82	49
168	6·8	100	62
	6·3	90	56
	5·9	80	50
	5·5	70	44
166	6·5	80	52
	5·8	70	45
	5·3	65	42
164	6·2	70	47
	5·8	65	43
	5·6	60	40
	5·0	55	37
	4·9	53	35
162	6·0	63	44
	5·7	60	42
	5·2	55	38
	4·8	50	35
160	5·8	60	44
	5·4	55	40
	4·8	50	36
	4·5	45	33
158	4·6	45	37
	4·2	40	33
156	4·0	35	28
154	3·7	30	26
152	3·3	25	23
150	3·0	20	20

The first voltage used (170 volts) was very close to the striking voltage, possibly above it, for when this voltage was applied the lamp struck after about 30 seconds.

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[*] In giving the value of the voltage across the lamp no deduction has been made for the small potential drop across the resistance R in series with the lamp.

The lamp discharge during interruption was examined in a rotating mirror, and in general two very fine black lines were seen across the red image in the mirror. This was not the case when lamp currents less than 1 milliamp. were used, for then the mirror showed that the second interruption occurred before the lamp had recovered from the effect of the first. From the evidence afforded by the rotating mirror, it is considered that successive discharges had no cumulative effect in the cases for which values are given.

Discussion of Method and Results.

It was pointed out above that some assumptions were made in deriving the period of interruption. In the first place non-inductive resistances are assumed, but if properly wound resistance coils are used the inductance present will produce a negligible effect.

A considerable error might be caused by contact resistance at the switch. As care was taken to see that a good contact was made between the brush and the rotating brass segment at the switch, it is considered that errors due to contact resistance should be small.

One point of interest is the fact that the maximum interruption interval, which does not extinguish the lamp, depends to a considerable extent on the current passing. For a given voltage applied to the lamp this maximum interruption interval is much bigger for large currents than for small ones, as is to be expected. For a given voltage applied to the lamp, if below 166 volts, the ratio of lamp current to maximum interruption interval is approximately constant (within 4 per cent.) ; but for higher voltages applied to the lamp this ratio is found to be smaller for large currents than for small ones.

In conclusion, I wish to thank the British General Electric Company for the gift of a number of *I* neon lamps, and Dr. Jack and rest of the staff of the Physics Department for consideration which they have shown me and which has been of great value in enabling me to carry out this work.

DISCUSSION.

Mr. A. G. Tarrant said that he and Mr. Oschwald had recently measured not only the minimum time of interruption, but the actual voltage at relighting for interruptions down to 3 micro-seconds, and had found no case where lighting took place below the upper critical voltage. He suggested that in the author's apparatus surges of current may take place, owing to the presence of inductance in instruments and leads, and so raise the starting voltage above the applied D.C. voltage. This view was consistent with the effect of varying R as found by the author. Further, the author deduced the duration of the interruption from an expression involving the difference between two large voltages not accurately known. It would be of interest for the author and the speaker to experiment on one another's lamps, in order to eliminate divergences of result due to differences between the lamps.

Mr. R. P. Fuge said that he had measured the time taken to light up for various voltages lying between the upper and lower critical voltages, the lamps being allowed to rest in the dark for half an hour with their terminals short-circuited after each experiment. The time taken at a given voltage by a given lamp varied irregularly between astonishingly wide limits, but the averages for a number of trials lay on a smooth curve, part of which was accurately linear.

Dr. D. Owen considered that the principle of the author's method was sound. He had taken the opportunity to check the tests with a simplification of circuit, replacing the commutator by a simple mercury throw-in switch, and omitting the large condenser across the battery, the use of which by the author seemed open to question. In the case of the particular lamp used, the striking voltage was 145, the extinction voltage, 120 ; the E.M.F. of the battery after striking was reduced to 140 volts. Observations were made with a series of values of the capacity C

shunting the lamp, over the range 0.05 μF. to 1 μF., the corresponding values of the maximum series resistances R (from 20 ohms to 450 ohms) being found which just permitted of relighting of the lamp. On plotting the values of R against I/C a straight line was obtained, the intercept on the axis of R being -17 ohms, which agrees reasonably well with the ascertained value of 21 ohms found for the internal resistance of the battery. Using the formula of the Paper this gives the value 45×10^{-6} seconds for the period for which the lamp was below extinction voltage without failing to relight. This confirms the author's result ; and considering the wide range of capacities taken, it seems difficult to admit the suggestion of another speaker that the relighting might be assigned to voltage surges which lift the voltage across the lamp, above the striking voltage.

Author's reply (communicated) : I would point that the supply circuit in which inductance might be expected, was quite free from instruments, and so any inductance in the leads would produce only a negligible effect. In any case similar results were obtained when no charging circuit was used, and in such circumstances surges could not have occurred. The difference between the two sets of results was possibly caused by the difference in the methods of extinction in the two cases, and not by errors in the experimental methods. I must admit that the derivation of the time of extinction is open to objection, but I would point out that, as the lamp certainly would not relight below the extinction voltage, the times of extinction given might be too small, and so the correct times would disagree still further with the results of Mr. Tarrant. In conclusion I may mention that there is at present no satisfactory way of defining the exact point at which the lamp relights much less of measuring it.

XIX—ELECTRICITY OF DUST CLOUDS.—PART I.

By G. B. Deodhar, *M.Sc.*, Physics Department, University of Allahabad (India).

Received December 16, 1926.

(*Communicated by* D. B. Deodhar, *M.Sc.*)

Summary.

1. Observations on electrification of various dusts have been made by using a large volume of air at a pretty high speed. The climatic conditions are totally different from those under which other similar observations were made.

2. The number of substances tried is considerably increased. In some cases the sign of electrification is the same as that observed by Rudge.

3. The factors governing the phenomena of electricity of dust storms are stated as follows : (1) Material of the dust. (2) Its size. (3) The gas raising the cloud. (4) The velocity of the gas. (5) The temperature. Account of the first two factors is given in the present communication.

4. It appears that the electricity developed is of frictional nature, which itself is, of course, unknown.

5. Some quantitative estimates of electrification of chlorides and nitrates of sodium and potassium are made. It is seen that chlorides of sodium and potassium are equally efficacious, whilst sodium nitrate is about 4½ times as efficacious as potassium nitrate.

6. Dusts of various roughly uniform sizes were obtained with the help of a few graded sieves. The magnitudes of the various sizes were obtained after very laborious observations under microscope. It is shown graphically that, other things being the same, the number of volts developed by blowing increases very rapidly as the size grows less.

Introduction.

THE subject of the electricity developed when a cloud of dust is raised by wind is of immense theoretical and practical importance, especially in the tropical regions like India, where large volumes of dust are blown up into the atmosphere, causing what are called dust storms. Such storms are very common in the months of April, May and June in the northern tracts of this country, and cause a great increase in the potential gradient. For the wireless operator the dust storms are a great menace, and demand his closest attention. For a laboratory worker the dust is not less annoying, since it interferes to a large extent with his work on electrometers and other things where electric leakage is to be avoided. It is thus essential to collect as much information on the electricity of dust clouds as possible, and work inside as well as outside the laboratory must be vigorously prosecuted. The factors coming into play in the production of electric charges during the raising of dust clouds by winds may be stated as follows : (1) The material of the dust, (2) its size, (3) the gas raising the cloud, (4) the wind velocity, (5) the temperature.

Exhaustive work is necessary to find out the exact nature of the influence of each of these factors. Let us now see what knowledge we possess over these several factors. W. A. Douglas Rudge has done interesting experiments on the nature of the electricity developed by raising a cloud of dust.[1 and 2] He has examined several kinds of dust, and is of opinion that " Acidic " bodies, such as finely divided silica or molybdic acid, give to the dust a positive charge, whilst basic bodies, such as lead oxide, or organic dusts, such as flour or coal, give a negative charge. Of course, to establish such a generalisation it is necessary to examine a very large number of substances, and so enough work should be done in this direction. As regards factor (2) it has been pointed out that in general for a given material fine dust produces a very large electrification. But no quantitative work is done in this direction. It appears that the remaining factors have not been definitely studied, and so information must be gathered on each of them before a satisfactory theory of the electricity of dust clouds is formulated.

I propose to give in a series of Papers an account of the exact influence of the size of dust particles, of the gas raising the cloud, of the velocity of the gas, and of the temperature of the enclosure in which the dust is blown. As a preliminary I propose to give in the present communication an account of my observations on the electricity of dust clouds and dust storms.

APPARATUS.

The apparatus used is quite different from that of Rudge, and the experimental conditions are also new. Instead of concentrated air, large volume is used, and the wind velocity is also pretty high (about 8 metres per second). The experimental conditions thus partially approach those found in Nature. Fig. 1 shows the plan of the apparatus.

It has a close similarity with the apparatus used by Dr. G. C. Simpson in the study of electricity of thunderstorms.(3) *AA* is a big cylindrical vessel of iron, having the base as shown in the figure. On the top of the conical portion *TT* an ebonite ring is mounted, and on this is fixed a piece of fine wire gauze. On this gauze some quantity of dust is placed. The cylindrical vessel is covered by a wooden lid, *L*, the under surface of which is covered with tin-foil. At *G′* there is a hole in the lid. This is covered with wire gauze, having electrical contact with the tin-foil, which, in its turn, has contact with the vessel. The vessel *AA* is connected through a convenient key to a sensitive electrometer, which was worked at a sensibility of 420 mms./volt. The vessel is enclosed by a wire gauze screen, *SS*, which is earth-connected from several points. A strong current of air is obtained by a fan blower worked by a motor. The air current passes through the metallic pipe *B* into the conical portion *TT*, and thus the dust kept on the gauze *G* is blown up and collected in the vessel. The pipe *B* is, of course, kept free from the vessel. Any charge developed is given to the vessel, and is at once detected by the electrometer. The charge on the air can be obtained by suspending any suitable collector inside the vessel, and connecting it to a convenient electrometer. The present Paper gives an account of only the charge on the dust particles. The account of the charge on the air is reserved for a later Paper.

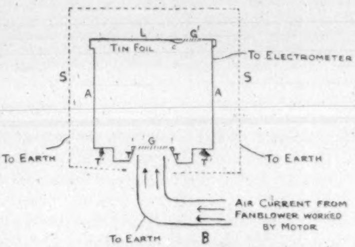

FIG. 1.—DUST APPARATUS.

ELECTRIFICATION OF VARIOUS DUSTS.

Rudge has tried many substances; but it was thought worth while to try many more kinds of dusts with an apparatus wholly different from that of Rudge, and under quite different climatic conditions. The temperature at the time of experiment was about 40°C. The electrometer was at first calibrated as usual. The sensibility was 420 mms./volt. The vessel was connected to the quadrants, and charged to a certain potential. Leakage was then observed without any air current passing into the vessel. Next, leakage with air current on was observed. The vessel with the connected quandrants was earthed, and then again kept insulated and uncharged. Air current was allowed to flow freely with full vigour. It was found that no electrical effect was produced. A small quantity of dust of a substance which was previously passed through two fine sieves was then carefully placed on the wire gauze *G*. The vessel and the quadrants earthed and then left insulated. Reading on the scale was noted. The dust was then blown by the air current and the reading of the electrometer again noted. After allowing for the leakage the difference between the two readings gave the electrification of the dust particles. For finding out simply the kind of electrification it was not necessary to find out the exact value in each case. The observations for various substances are collected in Table I below. For comparison Rudge's observations are also inserted.

TABLE I.

Substance.	Sign of electrification.		Remarks.
	Author.	Rudge.	
Ebonite dust	−	...	......
Rubber dust	+	...	When showered into the vessel (−).
Iron filings	−	−	Small.
Copper ,,	+	...	......
Brass ,,	+	...	......
Resin	+	...	......
Pulverised Lycopodium	+	...	......
Sulphur	−	...	......
Sealing wax	−	...	......
Precipitated chalk	−	−	......
Gulal (red)	...	...	......
Mica dust	(+), (−)	...	Sometimes (+), sometimes (−).
Road dust	+	...	......
Coal	+	...	......
Fine dust accumulating on things in laboratory	(+), (−)	...	Sometimes (+), sometimes (−).
Brick dust	+	...	Two pieces rubbed together both show feeble (+) charge.
Wheat flour	−	−	......
Gram	−	...	......
Coal ash	−	...	......
Wood coal	+	...	Two pieces rubbed against each other both gave (+) charge.
Glass	+	...	......
Rock salt	+	...	Two pieces rubbed against each other both showed (+) charge.
Zinc oxide	+	...	......
Turmeric	−	...	......
Pure $NaCl$	+	...	The salt must be pretty fine.
,, $NaNo_3$	+	...	......
,, KCl	+	...	......
,, KNO_3	+	...	......
,, NH_4Cl	+	...	......
,, $CaCl_2$	+	...	......
,, $BaCl_2$	+	...	......
,, Cobalt Chloride	+	...	Small. Difficult to determine, since the powder becomes moist very quickly.

SOME QUANTITATIVE DETERMINATIONS.

The observations made so far were all qualitative. It was next thought of making some quantitative determinations with a view to find out how the quantity of charge developed varies with the nature of the salt keeping other factors the same. The salts used were chemically pure. They were carefully pounded and passed through two fine sieves to obtain uniformity in size of the dust particles. Small quantity of each salt was measured by a sensitive balance, and carefully put on the gauze G, and the charge developed was measured as described above. Mean of several observations was taken as the charge developed by blowing a definite quantity of salt dust. A few observations are given in Table II :—

TABLE II.

Salt dust.	Mass.	Sign of charge.	Volts.
NaCl	5 grms.	+	1·13
NaNO$_3$	5 ,,	+	8·47
KCl	5 ,,	+	1·19
KNO$_3$	5 ,,	+	1·83

Of course, no claim is made for precision ; but it may be said with certainty that chlorides of potassium and sodium are equally efficacious, whilst potassium nitrate is very much less efficient than sodium nitrate. These observations are very few in number. Consequently no generalisation can be formulated. A more exhaustive study in this direction would be very interesting.

EFFECT OF SIZE.

Investigation on this point is beset with considerable amount of experimental difficulties. It is extremely difficult to secure particles of quite identical size. As a preliminary I have made some rough determinations on the electrification developed by raising a cloud of field dust, as this is expected to throw considerable light on the electricity of dust storms.

It is known for a long time that the presence of dust in the atmosphere has a very marked influence on the normal potential gradient. Rudge has made interesting observations in this connection. During a storm the magnitude of the gradient is considerably enhanced and the sign is either positive or negative, depending upon the nature of the dust.(4) I had no opportunity of determining quantitatively the potential gradient in a dust storm at Allahabad, which experiences heavy dust storms in summer (April, May and June) ; but a qualitative test of the high potential value is worth noting. During a dust storm a Geissler tube filled with helium was put in circuit with the aerial of our wireless set and the earth. It was found that the tube continued to glow quite brilliantly although intermittently. With the subsidence of the storm and settling of the dust the phenomenon vanished altogether, showing that during the storm the potential considerably exceeded the normal value. There is every reason to believe that the origin of the electricity of dust storms lies in the mutual friction between the dust particles blown up by wind. The dust storms at Allahabad are characterised by heavy winds and fine dust. I had no opportunity of measuring the wind velocity. A rough idea of the size of dust particles was obtained by allowing the dust to settle down at a sufficient height in a vessel, and observing the particles under microscope. The majority of the dust particles were below 0·01 mm. in size.

Next an attempt was made to get an idea of the way the electricity of dust particles varies with size. As remarked above the problem is pretty difficult. It is difficult to obtain a large number of dust particles of accurately uniform size. The following plan was therefore adopted to obtain particles, the size of which varied between certain limits which were not far between. Some quantity of dust was passed through various graded sieves, and thus the particles were classified in six different groups. Taking a weighed quantity for each group the charge developed due to blowing was determined as described in a previous section. The mean of several observations was taken in each case. A large number of particles belonging to a particular group was then observed under micrometer eyepiece, and by elaborate observations mean size of the particles deduced. The process was repeated for all the groups. The observations are given in Table III.

TABLE III.

Group.	Size.	Volts.
I.	3·15 mm.	1·40
II.	1·55 ,,	0·45
III.	0·86 ,,	0·35
IV	0·55 ,,	0·49
V	0·33 ,,	3·47
VI.	0·05 ,,	46·85

The whole process is, of course, very tedious, and gives only an approximate idea of the way in which the electrification of dust clouds varies with the size of the particles. Fig. 2 shows graphically how the charge varies with size.

It is seen at once from this figure that as the size decreases the volts produced mount up very high, thus explaining how the potential gradient mounts up considerably during storm or dust cloud raised by any means. Similar curve was obtained for barium chloride. It will be interesting to try many more substances in the same way.

As regards the remaining factors it appears very little is known. Work in this direction is in hand, and an account will be given in due course. During the course of the present observations it was incidentally observed that on halving the wind velocity almost the same electrification was obtained for the same quantity of dust.

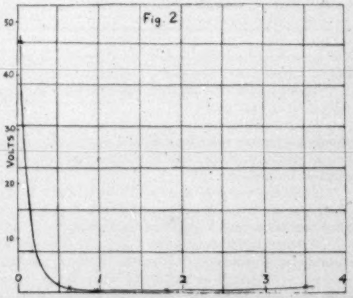
Fig. 2

DISCUSSION OF OBSERVATIONS.

It is easily seen from Table I that no definite rule can be given regarding the nature of electrification. Brass and copper, which are purely metallic dusts, differ from iron dust, but show similar electrification to various salts as well as some organic dusts. The electrification developed by friction between two pieces of a particular substance is also interesting, and an account is given in my note on anomalies in frictional electricity.(5) An examination of this note, as well as of Table I will bring out the fact that two pieces of the same substance can both be charged with the same sign. Obviously the dust abraded must be oppositely charged. This agrees with the observations of Hesecus,(6) and is in conflict with those of Rudge.

The quantitative work on salts is interesting, and it is contemplated to extend the observations on all possible chlorides. This will show how the electrification varies from group to group of the periodic table.

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DISCUSSION.

Dr. P. E. SHAW (communicated): These experiments by Mr. Deodhar, and the well-known earlier ones by Mr. Rudge, have special interest for me, as I have recently sent a Paper bearing on this topic to the Society. The effects, however, which I obtain are due to the impact of only two like bodies, and in this way the condition of the impacting surfaces can be ascertained and varied at will. The dust cloud experiments are interesting, as they are closely analogous to the natural phenomena found in the open air under dry, tropical conditions; but one may doubt if they are not altogether too complex to yield a solution of the problems involved. For instance, in these dust impacts the particles strike one another at all possible angles, and with a great range of speed. Such variables cannot be ignored. The curvatures of the striking surfaces must be very varied, and with the varied curvatures there will be different conditions of their adsorbed layers. Also, among the variables mentioned by Mr. Deodhar in the first clause of his Paper, the hygrometric state is not mentioned, though it is well known that moisture plays a very important rôle in tribo-electric effects.

Then, again, the charges observed in the apparatus shown are compounded, in unknown relations, of four parts:—

(1) Net charge, say $-ve$ left on the impacting particles; (2) the contrary, say $+ve$, and equal, charge on the ions emitted to the air; (3) charges of unknown sign and amount, due to impact of the particles on the walls and gauze of the chamber; (4) ions, corresponding to these, emitted to the air.

True, a net charge from all these remains in the chamber, and is recorded on the electrometer when the air has been blown through it; but how is it possible to disentangle the effects and eliminate those due to the chamber itself?

Without anticipating the results of my own Paper, I may say that the electrical nature of surfaces may change fundamentally when rubbed or struck, so that it is possible to obtain a $+ve$ or $-ve$ charge at will from like solids. In a dust cloud we may expect some impacts to yield net $-ve$, some net $+ve$.

When the particles are very small, then, for the same total mass in a cloud, there would be correspondingly more contacts and greater charges, as Mr. Deodhar finds. How far the charges on, and motions of, colloidal particles, as in the Brownian movements, can be attributed solely to electrical charge arising by their impact, as in these experiments, one cannot say; but there appear to be very interesting possibilities in this direction.

The molecular kinetic theory will account for two particles striking one another; but their rebound would, in part, be due to repulsion of like charges, caused by impact.

Mr. J. H. COSTE (communicated): This Paper deals with a subject of considerable interest, not only from the meteorological point of view, but in relation to industrial powders, and probably to explosions due to organic dusts.

The author might find that he could grade his particles more easily by a process of sedimentation after sieving. For pure substances, in which the particles are of uniform density, sedimentation in liquids which have no solvent action, and which are relatively dense or/and viscous should be fairly satisfactory. It would probably be as well not to rely entirely on the calculation of particle size by Stokes' equation, but to verify the dimensions by the microscopical method, as Mr. Deodhar has done. Sedimentation is likely to be more trustworthy than elutriation.

A method of precipitation from solution might be found to yield particles more uniform in size and form than grinding does—e.g., NaCl can be precipitated from very dilute alcoholic solutions by dry gaseous HCl, yielding fine particles and other salts by adding to their solutions suitable liquids in which they are sparingly soluble.

The curve, Fig. 2, showing that voltage produced is an inverse function of particle size, lends support to the view that the change is directly related to the total surface on which it is distributed, and that with the velocities used a maximum change is obtained.

It would be interesting to know what precautions were taken to ensure the freedom of the air from dust before it reached the powder, and whether its humidity was determined or regulated.

DEMONSTRATION ILLUSTRATING SOME PHYSICAL PROBLEMS CONNECTED WITH APPLE TRANSPORT.

By Ezer Griffiths, *D.Sc., F.R.S.*

A SKETCH was given of some of the problems which had been tackled in connexion with the refrigerated transport of fruit from overseas.

The first problem was to determine the most effective method of cooling down a compactly stowed mass of fruit when the ship had to perform the dual function of precooling and carrying. Fruit must not be frozen, a fact which sets a limit to the temperature gradients. An additional difficulty is the fact that fruit generates heat in the course of processes analogous to respiration. The author showed a number of graphs giving the results of investigations, and illustrated by a model a new system of vertical drainage for producing channels through the cargo.

The second problem was to provide means of removing the carbon dioxide produced by the fruit. An apple will generate its own volume of carbon dioxide in 40 hours if kept at a temperature of 70°F. and in 14 days if kept at 34°F. An experiment was shown to prove the generation of carbon dioxide by apples. The presence of carbon dioxide above 12 per cent. causes disease in the fruit. The suggested systems for supplying moisture-free air for ventilation were illustrated.

The third problem was that of instrument design. An improved form of resistance thermometer bulb was shown which was robust and had good heat dissipating capacity. One notable feature was the method of arranging the resistance coil on the internal surface of a brass tube. Junction boxes and auxiliary gear were also considered in detail. For the experiments on board ship a number of special instruments were constructed, such as hygrometers, air current measuring instruments, etc.

XX.—ACOUSTICAL EXPERIMENTS WITH A MECHANICAL VIBRATOR.

By E. Mallett, *D.Sc., M.I.E.E., A.M.I.C.E.*

Received December 15, 1926.

Abstract.

In this Paper some preliminary experiments are described with a mechanical device vibrating a piston at one end of a tube so that a sound wave is emitted at the other. The particle velocities in the sound wave are measured by a Rayleigh disc, and resonance curves are drawn. It is shown how the energy in the sound wave can be calculated from the results obtained. The experiments are directed towards obtaining a standard source of sound, and the results are encouraging, but a considerable amount of work remains to be done.

Contents.

I. Description of apparatus.
II. Sound amplitude measurements :—
  (i) Measurement of amplitude of piston with different rollers ;
  (ii) Curves of sound amplitude for different piston amplitudes ;
  (iii) Resonance curves.
III. Pipe theory on electrical analogy.
IV. Numerical results and conclusions.
Appendix : Critique of Paper on "The Degradation of Acoustic Energy."

I. Description of Apparatus.

A MECHANICAL vibrator was used to give known amplitudes of vibration of variable frequency to a piston at the end of a pipe, so as to produce the simplest condition possible from the mathematical point of view. The principle on which the vibrator works will be understood from Fig. 1. *R* is a roller, with an eccentric spindle at *B*, and resting on a large driving wheel *D*. The spindle of the roller is carried at the end of a rocking arm *AB*, pivoted at *A*, and there is a means provided of keeping the surfaces of *R* and *D* in contact. When *D* is rotated *R* will be rotated, because of the friction between *R* and *D*, and *B* will execute a vibratory movement along a vertical line, because it is not centrally placed in *R*. Further, since the spindle is rigidly fixed in the arm *BA*, the whole arm will be caused to rock about the pivot *A*, and the piston *P* fitting in the end of a pipe will also vibrate along the axis of the pipe.

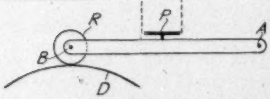

Fig. 1.

Naturally such an apparatus was not made without many difficulties arising. The means of holding the roller on to the driving wheel, for instance, was a difficulty finally solved by using a thick rubber block pressing upon the end of the arm. Lubrication of the roller was effected by means of a wick passing through a hole in the centre of the spindle with radial cuts to the bearing surface. Mechanical resonances in the bar and in the framework holding the whole contrivance occurred, and were investigated by fixing an iron armature to the arm and a telephone receiver to the frame with its poles close to the armature, so that as the arm vibrated an electromotive force was produced in the receiver coils.

The difficulties and the way in which they were overcome, with a complete description of the final apparatus are given in the writer's M.Sc. thesis in the University of London Library. This apparatus gives practically a sinusoidal vibration free from resonances, that is of constant amplitude, up to a frequency of about 500 cycles per second. The actual vibration of the spindle was shown to be expressed by

$$x=k\left(\cos \omega t-\frac{1}{2}\frac{k}{a}\cos 2\,\omega t\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

where k is the eccentricity and a the radius of the roller. Higher harmonics than the second are quite negligible.

To convert the vibrator to a mechanically-driven organ pipe the whole apparatus was turned through a right angle, so that the piston P became vertical and the pipe horizontal. The piston was made as good a fit in the pipe as possible, and the end of the pipe was made airtight by means of a rubber disc fixed as shown in Fig. 2. The pipe was rigidly clamped to the frame, and the far end was also supported.

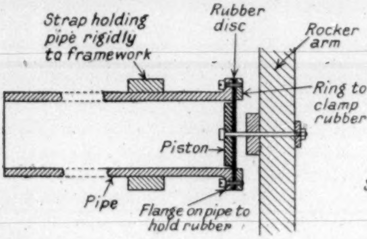
Strap holding pipe rigidly to framework
Rubber disc
Rocker arm
Ring to clamp rubber
Piston
Pipe
Flange on pipe to hold rubber

FIG. 2.

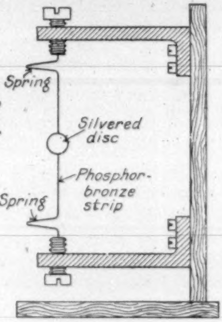

FIG. 3.

It was the original intention to obtain different amplitudes of vibration by fixing the apparatus to be vibrated at different positions along the rocker arm; but with the present pipe arrangement this proved to be impracticable, so a set of rollers was made having the same diameter but different eccentricities.

The driving wheel was driven by an electric motor, and changes of frequency were obtained by simply altering the motor speed.

Measurements of the particle velocities of the sound wave from the pipe were made by means of a Rayleigh disc arranged as shown in Fig. 3. This robust arrangement was possible, as the sound velocities to be measured were large.

II. SOUND AMPLITUDE MEASUREMENTS.

(i) *Amplitudes of Piston with Different Rollers.*

Four rollers were made each of ¼ in. diameter, but having different eccentricities, and so giving different amplitudes of vibration to the piston. The amplitudes were measured by a microscope while the machine was at rest by rotating the driving wheel and taking the difference of the readings with the vibrating bar in its top and bottom positions. The measurements were made at a distance of 12·15 in. from the pivot, the distance of the piston was 8 in. from the pivot, and that of the roller 16 in. from the pivot. The actual diameters of the rollers were measured by a micrometer gauge. Details of the rollers are given in the following table :—

Roller number.	Diameter of roller.	Measured lift of bar.	Lift of roller.	Throw of piston.	Amplitude of piston.
	Ins.	Ins.	Ins.	Ins.	Ins.
1	0·2496	0·00600	0·00790	0·00395	0·00197
2	0·2496	0·00380	0·00500	0·00250	0·00125
3	0·2494	0·00295	0·00388	0·00194	0·00097
4	0·2498	0·00074	0·000975	0·00048	0·00024

(ii) *Sound Amplitude with Different Rollers.*

The Rayleigh disc was now placed in front of the organ pipe and the machine speeded up. Two marked resonances were obtained within the allowable speed limit, one corresponding to the fundamental mode of the pipe, and the other to its first overtone, at three times the frequency. The maximum deflections of the disc were measured at these two frequencies with each of the four rollers in turn, and a figure proportional to the particle velocity obtained as the square root of the deflection.

The readings were as follows :—

Roller number.	Piston amplitude. Ins.	Fundamental.		First overtone.	
		Deflection of disc. δ	$\sqrt{\delta}$	Deflection of disc. δ	$\sqrt{\delta}$
1	0·00197	5·6	2·36	28·0	5·30
2	0·00125	2·0	1·41	10·3	3·21
3	0·00097	1·3	1·14	6·4	2·51
4	0·00024	—	—	0·4	0·63

The sound amplitudes ($\sqrt{\delta}$) are plotted against the piston amplitudes in Fig. 4. It is seen that two straight lines result, each passing through the origin. This quite satisfactorily indicates that the particle velocity in the sound wave outside the tube is directly proportional to the piston amplitude.

(iii) *Resonance Curves.*

Resonance curves taken with different rollers by varying the speed of the motor, and therefore the frequency of vibration of the piston, and measuring the particle velocity at the mouth of the tube by means of the Rayleigh disc are shown in Fig. 5 for the fundamental vibration, and in Fig. 6 for the first overtone.

The speed of the driving wheel was measured by a Moule tacheometer fixed to its spindle, and the frequency of vibration calculated on the assumption of no slip between wheel and roller, which has been shown to be at any rate very nearly true. No importance can, however, be attached to the apparent variations of resonant frequency with amplitude indicated by the curves. The frequency determinations were not sufficiently accurate for this, as the Moule tacheometer seemed to vary a little from day to day, and the temperature was not always quite the same. But it is thought that the differences of frequency are fairly accurate for any one curve.

Another difficulty was to keep the speed of the motor sufficiently constant for accurate readings both of the Rayleigh disc deflection and of the frequency to be taken. The method adopted was to brake the driving wheel judiciously with the finger to keep the Rayleigh disc deflection steady, and then read the tacheometer. Then, again, to avoid room resonances, such curves as these should be taken in a sound absorbing chamber, but none was available at the time. The presence of "coupled circuit" distortion is indicated in the fundamental resonance curves. These curves can therefore only be looked upon as preliminary. An efficient speed control, an accurate method of frequency measurement, and an absorbing chamber are necessary for more exact work.

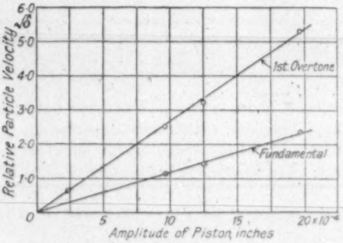

FIG. 4.

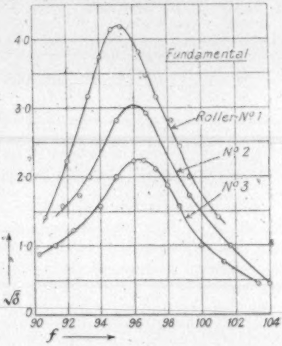

FIG. 5.

III. PIPE THEORY ON ELECTRICAL ANALOGY.

The theory of the acoustical vibrations in the pipe can be written down by analogy with the electrical oscillations in a transmission or telephone line, particle velocities in the acoustical case corresponding with electrical currents, and alternating air pressures with electrical potential differences. In the previous experiments the particle velocity at the "sending end" of the pipe is that of the piston, while the Rayleigh disc measurements give a figure which is proportional to the particle velocity at the "receiving end"—that is at the open end of the pipe. Thus the telephone equation likely to be of most value is

$$\frac{I_s}{I_r} = \cosh Pl + \frac{Z_r}{Z_0} \sinh Pl \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (2)$$

where I_s = sending end particle velocity = piston velocity

I_r = receiving end velocity

l = length of pipe

and $P = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$

is the propagation constant of the pipe line. Numerous experiments have shown that the loss of energy in a smooth tube such as the one used is very small. For instance, in an experiment in which resonance curves were obtained by altering the pipe length (*see* Acoustic Experiments with Telephone Receivers, J.I.E.E., 63, p. 512, Fig. 16) the resonance curve at the second overtone of the pipe was substantially the same as that of the fundamental, although the pipe length had been increased from about 7 cms. to about 42 cms. The R and G of the pipe can therefore be neglected, and the propagation constant is

$$P = j\omega\sqrt{LC} = j\beta.$$

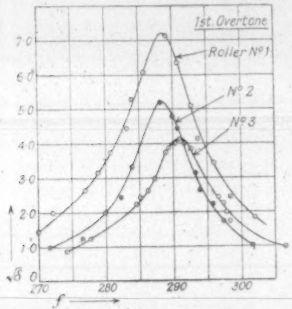

FIG. 6.

L, in the acoustical case, is m, the linear mass in grams per cm. run, which is equal to ρS, where ρ = density of air, and S = cross-sectional area of pipe.

$\frac{1}{C}$ is the "stiffness" of the air, $s = S\gamma\rho_0$ dynes, where ρ_0 is the normal pressure of the air, and γ the ratio of the specific heats.

The wave velocity (velocity of sound)

$$= \frac{1}{\sqrt{LC}} = \sqrt{\frac{S}{m}} = \sqrt{\frac{\gamma\rho_0}{\rho}} = c \text{ cms./sec.}, \; \beta = \omega\sqrt{LC} = \frac{\omega}{c}\ cm^{-1}$$

$$Z_0 \text{ the characteristic impedance} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} = mc \quad . \; . \; . \; . \quad \frac{\text{grams.}}{\text{sec.}}$$

Z_r is the load impedance, occasioned in the acoustical case by the pipe being open to the air at the end. Z_r may be written as $R+jX$, and both R and X are unknown. The dimensions of R and X must be gms./sec.

Returning the electrical symbols as they are more familiar, the ratio of the particle velocities at the two ends of the tube becomes

$$\frac{I_s}{I_r}=\cosh j\omega\sqrt{LC}l+(R+jX)\sqrt{\frac{C}{L}}\sinh j\omega\sqrt{LC}l$$

$$=\sqrt{1+X^2\frac{C}{L}}\cos\left\{\omega\sqrt{LC}l+\varphi\right\}+jR\sqrt{\frac{C}{L}}\sin(\omega\sqrt{LC}l),$$

where
$$\tan\varphi=X\sqrt{\frac{C}{L}}\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(3)$$

The locus of this expression as ω varies on the assumption that R and X remain constant, and neglecting φ for the moment, is an ellipse, with, in the present case, the horizontal major axis $\sqrt{1+X^2\frac{C}{L}}$ much greater than the vertical minor axis, $R\sqrt{\frac{C}{L}}$. The minimum value of the ratio—i.e., the maximum value of I_r—occurs then (still neglecting φ), when $\cos\omega\sqrt{LC}l=0$—i.e., when $\omega\sqrt{LC}l=\frac{2m+1}{2}\pi$, where m is any integer, including 0.

This is the same as
$$\frac{\omega}{c}l=\frac{2m+1}{2}\pi,$$

or,
$$f=\frac{2m+1}{4}\cdot\frac{c}{l},$$

the familiar expression for the resonant modes of a pipe open at one end and closed at the other.

Taking into account the modification introduced by the terminal impedance as φ, for the fundamental resonance occurs when

$$\omega\sqrt{LC}l+\varphi=\frac{\pi}{2},$$

$$\omega=\left(\frac{\pi}{2}-\varphi\right)\cdot\frac{1}{\sqrt{LC}l},$$

or,
$$f=\frac{C}{4l}-\frac{\varphi}{2\pi l}\cdot c.$$

Thus the resonant frequency is lowered. This is usually taken account of by assuming that the pipe is in effect a little longer—the well known " end correction."

φ is small, so that approximately

$$\varphi=\tan\varphi=X\sqrt{\frac{C}{L}},$$

and
$$f=\frac{c}{4l}-\frac{1}{2\pi}\cdot\frac{X}{Ll}.$$

Proceeding to the first overtone, resonance occurs when

$$\omega\sqrt{LCl}+\varphi^1=\frac{3\pi}{2}.$$

leading to

$$f=\frac{3}{4}\cdot\frac{c}{l}-\frac{\varphi'}{2\pi\sqrt{LCl}}.$$

In order that this may be three times the frequency of the fundamental, φ' must be three times φ—i.e.,

$$\varphi'\doteq\tan\varphi'=3X\sqrt{\frac{C}{L}}.$$

Similarly for the higher modes, the reactance of the load must increase, in order to fit in with the classical experimental observations. Writing $X=\omega L'$, this is the same as saying that L' remains constant.

For then resonance occurs when

$$\omega\sqrt{LCl}+\omega L'\sqrt{\frac{C}{L}}=\frac{2m+1}{2}\pi,$$

or,

$$f=\frac{2m+1}{4}\cdot\frac{c}{\left(l+\frac{L'}{L}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$\frac{L'}{L}$, therefore, is the end correction, and L' can be determined experimentally by accurate observations of the resonant frequencies.

The load resistance R is found from the resonance curves. In equation (3) $X^2\frac{C}{L}=\frac{\omega^2}{c^2}\left(\frac{L'}{L}\right)^2$. $\frac{L'}{L}$ will be something less than 1 cm., and $\frac{\omega^2}{c^2}$ is very much less than one, so that $\sqrt{1+X^2\frac{C}{L}}$ is very nearly one. Also, since φ is small, at or near the resonances, when $\cos(\omega\sqrt{LCl}+\varphi)=0$, $\sin(\omega\sqrt{LCl})$ will be very nearly one, positive or negative according to the overtone. Considering small departures of ω from the various resonant values ω_0.

$$\begin{aligned}\cos\{\omega\sqrt{LCl}+\varphi\}&=\cos\{(\omega-\omega_0)\sqrt{LCl}+\omega_0\sqrt{LCl}+\varphi\},\\&=\cos\left\{(\omega-\omega_0)\sqrt{LCl}+\frac{2m+1}{2}\pi\right\},\\&=(\omega-\omega_0)\sqrt{LCl}\times(-1)^m\end{aligned}$$

very nearly, and $\sin(\omega\sqrt{LCl})=(-1)^m$ very nearly.

So that equation (3) can be written

$$\frac{I_s}{I_r}=(-1)^m\left\{(\omega_0-\omega)\sqrt{LCl}+j\sqrt{\frac{C}{L}}R\right\} \quad . \quad . \quad . \quad . \quad (5)$$

The curves have been examined according to this expression in two ways (*a*) by considering only the sizes of the quantities, and (*b*) by the circle construction.

(*a*) Considering sizes only, equation (5) gives

$$\left(\frac{I_s}{I_r}\right)^2 = (\omega_0 - \omega)^2 LCl^2 + \frac{C}{L} R^2,$$

$$= 4\pi^2 LCl^2 (f_0 - f)^2 + \frac{C}{L} R^2.$$

Since I_s is very nearly constant over the resonance, and the readings δ of the Rayleigh disc are proportional to I_r^2, say, $I_r^2 = A\delta$

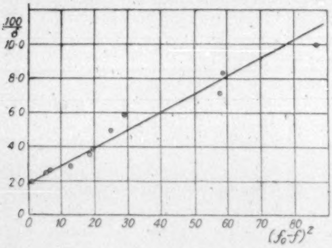

FIG. 7.

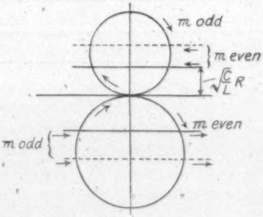

FIG. 8.

$$\frac{1}{\delta} = \frac{A}{I_s^2} \cdot 4\pi^2 LCl^2 (f_0 - f)^2 + \frac{A}{I_s^2} \cdot \frac{C}{L} R^2.$$

So that plotting $\frac{1}{\delta}$ against $(f_0 - f)^2$ a straight line should be obtained whose slope is $\frac{A}{I_s^2} \cdot 4\pi^2 LCl^2$, and whose intercept $= \frac{A}{I_s^2} \cdot \frac{C}{L} R^2$; and the ratio intercept/slope $= \frac{R^2}{4\pi^2 L^2 l^2}$.

and

$$\frac{R}{Ll} = 2\pi \sqrt{\frac{\text{Intercept}}{\text{Slope}}}$$

The curves of Figs. 5 and 6 were all examined in this way. As an example, the figures for the first overtone with No. 1 roller are given in the following table, and the curve $\frac{100}{\delta}$ plotted against $(f_0 - f)^2$ is drawn in Fig. 7.

No. 1 Roller.—Resonant frequency = 288·3 *cycle/sec.*

f	$f_0 - f$	δ	$(f_0 - f)^2$	$\frac{100}{\delta}$.
279	9·3	10	87	10
280·7	7·6	14	58	7·15
283·3	5·0	20	25	5·0
284	4·3	28	18·5	3·6
289·3	1·0	51	1·0	2·0
290·7	−2·4	40	5·7	2·5
292·7	−4·4	26	19·4	3·84
293·7	−5·4	17	29	5·9
296	−7·7	12	59	8·3
285·7	2·6	37	6·7	2·7
284·7	3·6	35	13	2·86

In this case the slope of the line is 0·105 and the intercept is 1·8. So that

$$\frac{R}{Ll}=2\pi\sqrt{\frac{1\cdot 8}{\cdot 105}}=26\cdot 1.$$

The results with the other rollers form similar curves, and are collected in the following table :—

Roller Number.	Fundamental.			First overtone.		
	Intercept.	Slope.	$\frac{R}{Ll}$	Intercept.	Slope.	$\frac{R}{Ll}$
1	5·3	1·17	13·4	1·8	0·105	26·0
2	11·0	1·7	16·0	3·5	0·295	21·6
3	19·5	3·1	15·7	6·0	0·35	26·0
			Mean = 15			Mean = 25

The differences of the values of $\frac{R}{Ll}$ found with different rollers can only be attributed to experimental error at this stage. There is no indication of any increase of load with amplitude.

(*b*) The examination of the curves by the circle and straight line method* is perhaps simpler and more illuminating. Equation (5) gives

$$\frac{I_r}{I_s}=\frac{1}{(-1)^m\left\{(\omega_0-\omega)\sqrt{LCl}+j\sqrt{\frac{C}{L}}R\right\}}$$

The locus of the denominator over the resonant ranges of frequency is a series of horizontal straight lines, above and below the origin in order of the harmonic. These inverted to give $\frac{I_r}{I_s}$ become a series of circles, as indicated in Fig. 8, below the origin with m even and above the origin with m odd, the circle described clockwise in each case as ω is increased. The phase angle α between I_r and I_s is evidently given by

$$\tan\alpha=\frac{(\omega_0-\omega)\sqrt{LCl}}{\sqrt{\frac{C}{L}}R}=(\omega_0-\omega)\frac{Ll}{R}$$

so that the circle construction carried out on the resonance curve gives for the resulting straight line, plotting ω,

$$\frac{d\omega}{d\tan\alpha}=\frac{R}{Ll}$$

or, plotting frequencies, as has been done,

$$\frac{R}{Ll}=2\pi\cdot\frac{df}{d\tan\alpha},$$

$$=2\pi\times\text{slope of line.}$$

The construction is carried out in Fig. 9 on the first overtone with No. 2 roller, and gives quite a satisfactory straight line with a slope of 3·39. This multiplied by $2\pi=21\cdot 4$ in close agreement with the value for $\frac{R}{Ll}$ found by the other method.

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* J.I.E.E., Vol. 62, p. 524.

IV. NUMERICAL RESULTS AND CONCLUSIONS.

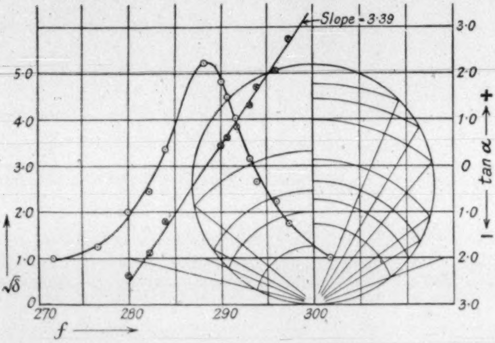


FIG. 9.

It is proposed in this section to make calculations of the acoustical energies involved and to indicate some directions in which the work may usefully be extended. The following details are required:—

Length of pipe = 89 cms. = l.

Internal diameter of pipe = 1·98 cms.

Area of pipe = S = 3·1 sq. cms.

Velocity of sound (from Kaye and Laby) at 22°C., the temperature at which the resonance curves were taken = 34,440 cms./sec. = c.

At 22°C., fundamental resonance frequency without end correction $= \frac{c}{4l} = 96{\cdot}8$ cycles per second.

The actual frequency from the resonance curves was about 96·0, corresponding to a pipe length of 89·8 cms., or an end correction of 0·8 cm. However, Kaye and Laby give end correction with no flange of 0·57 × radius = 0·57 cm. The frequency measurements were not sufficiently accurate for this determination, so the end correction will be taken as 0·6 cm. Thus,

$$\frac{L'}{L} = 0{\cdot}6 \text{ cm.}$$

$$L = m = \rho S = 0{\cdot}001196 \times 3{\cdot}1 \text{ gms./cm.}$$
$$= 0{\cdot}0037 \text{ gms./cm.} = m.$$
$$L' = 0{\cdot}0022 \text{ grams.}$$

——	Fundamental.	First overtone.
$\frac{R}{Ll}$	15	25 $\frac{1}{\text{sec.}}$
R	$= 15 \times 0{\cdot}0037 \times 89$ $= 5{\cdot}0$	$25 \times 0{\cdot}0037 \times 89$ $= 8{\cdot}3$ gms./sec.
$X = \omega L'$	$2\pi \times 96 \times 0{\cdot}0022$ $= 1{\cdot}33$	$2\pi \times 288 \times 0{\cdot}0022$ $= 4{\cdot}0$ gms./sec.
$Z_r = R + jX$	$5{\cdot}0 + j1{\cdot}33$	$8{\cdot}3 + j4{\cdot}0$ gms./sec.
Tan $\varphi = X\sqrt{\frac{C}{L}} = \frac{\omega}{c}\left(\frac{L'}{L}\right)$	$\frac{2\pi \times 96}{34{,}440} \times 0{\cdot}6 = 0{\cdot}0105$	$\frac{2\pi \times 288}{34{,}440} \times 0{\cdot}6 = 0{\cdot}0316$
φ	0·6°	1·8°

The acoustical energy radiated per second is I_r^2R (on the assumption, of course, that no energy is lost in the tube), where I_r is the root-mean-square value of the particle velocity in the mouth of the tube. At resonance this is from equation (5) given by

$$\frac{I_s}{I_r}=\sqrt{\frac{C}{L}}R=\frac{R}{mc},$$

so that

$$I_r=I_s\times\frac{mc}{R}.$$

Hence, power radiated

$$W=I_s^2\frac{m^2c^2}{R}\text{ ergs/sec.}=\frac{(\omega a)^2m^2c^2}{2R}\times 10^{-7}\text{ watts}$$

where a is the piston amplitude.

For the various rollers the particle velocities and energies are collected below : —

Roller number.	Amplitude a cms.	Fundamental.			First overtone.		
		$f=96$			$f=288$		
		I_s (max.) $\frac{\text{cms.}}{\text{sec.}}$	I (max.) $\frac{\text{cms.}}{\text{sec.}}$	W (milli-watts)	I_s (max.) $\frac{\text{cms.}}{\text{sec.}}$	I_r (max.) $\frac{\text{cms.}}{\text{sec.}}$	W (milli-watts)
1	0·0050	3·0	76	1·4	9·0	139	8·0
2	0·00318	1·9	48	0·58	5·7	88	3·2
3	0·00247	1·5	38	0·36	4·5	69	2·0

The Rayleigh disc was calibrated in a steady air stream, with a result that the max. value of the alternating particle velocity should be given by $10\sqrt{\delta}$. Thus the particle velocities measured at resonance with the No. 1 roller were 42 cms./sec. for the fundamental and 72 cms./sec. for the overtone, whereas the values of I_r were 76 and 139 cms./sec. respectively. Thus, while these figures are of the same order, it is evident that the Rayleigh disc is not measuring I_r. In fact, it would hardly be expected to do so, as the disc was not wholly in the mouth of the pipe, but projected from the end, and the sound intensities are falling off very rapidly as the wave spreads out and becomes spherical.

The figures obtained do, however, provide a demonstration that the calibration of the disc is the same at 96 cycles per second as at 288 cycles per second. The particle velocity V in a spherical wave at a distance r from the source is given by

$$V_r=\frac{A}{4\pi}\sqrt{\frac{1}{r^2}\cdot\frac{\omega^2}{c^2}+\frac{1}{r^4}},$$

and $\frac{\omega^2}{c^2}$ is very much less than one and r of the order 1 cm. So that very nearly $V_r=\frac{A}{4\pi}\cdot\frac{1}{r^2}$; that is, the measurements made will be independent of the frequency but depend only on the factor A, the amplitude of the source. At resonance for the fundamental

$$\frac{I_{r_0}}{I_{s_0}}=\frac{mc}{R_0}=\frac{0\cdot0037\times34{,}400}{5}=25\cdot4,$$

and for the first overtone

$$\frac{I_{r_1}}{I_{s_1}}=\frac{mc}{R_1}=15\cdot4.$$

The ratio of the velocities V_{r_0} and V_{r_1} for the fundamental and the first harmonic at resonance should, therefore, if the calibration of the Rayleigh disc is the same at each frequency, be given by

$$\frac{V_{r_1}}{V_{r_0}}=\frac{I_{r_1}}{I_{r_0}}=\left(\frac{I_{s_1}}{I_{s_0}}\right)\times\frac{15\cdot4}{25\cdot4}=3\times\frac{15\cdot4}{25\cdot4}=1\cdot8,$$

since the ratio of I_{s_1} to I_{s_0} is equal to the frequency ratio as the amplitude is the same.

The maximum values of $\sqrt{\delta}$ for the different rollers are given as follows from the resonance curves :—

Roller number.	Maximum $\sqrt{\delta}$		Ratio.
	First overtone.	Fundamental.	
1	7·2	4·4	1·64
2	5·2	3·05	1·7
3	4·15	2·25	1·85
			Mean = 1·73

The mean value of the ratio 1·73 is in quite good agreement with the value 1·80 found above.

The apparatus described has been shown to be capable of giving sound waves of variable frequency and amplitude which can be reproduced at will. It constitutes, in fact, a "standard source of sound." The actual frequencies obtained at present are low, but it is hoped by a modification of the design to obtain far higher frequencies. When a sound absorbing chamber is available it is hoped to extend the investigation to find, if possible, the actual loss in the tube, at the piston especially, so that the total power W radiated can be estimated with accuracy. With this knowledge the value A of the equivalent point source can be found from the expression

$$W=\frac{A^2\rho\omega^2}{8\pi c}.$$

Then by making velocity measurements with a Rayleigh disc in front of the tube at various distances when

$$V_r=k\sqrt{\delta}=\frac{A}{4\pi}\sqrt{\frac{1}{r^2}\cdot\frac{\omega^2}{c^2}+\frac{1}{r^4}}$$

the value of k can be found. In other words, the possibility arises of calibrating the Rayleigh disc absolutely at the frequency at which it is to be used.

The work described has been spread out over a number of years. The vibrator was made seven years ago in Prof. J. T. MacGregor-Morriss's laboratories at East London College, where Mr. Weaire and Mr. Niblett gave assistance in its construction. The adaption to the present acoustical work was carried out by Mr. Andrews at the City and Guilds (Eng.) College four years ago, and assistance in the actual measurements was given by Mr. F. S. Smith, a senior student. To Prof. MacGregor-Morris, in the early stages, and to Prof. C. L. Fortescue later the author has been indebted for ever-ready discussion and suggestion.

APPENDIX.

Critique of Mr. Hart's Paper on "The Degradation of Acoustical Energy" (Proc. Roy. Soc., A, Vol. 105, p. 80, 1924).

Mr. Hart in his Paper, which is reproduced in a book, "The Principles of Sound Signalling," by M. D. Hart and W. W. Smith, claims to show that acoustical energy propagated through air suffers a loss in propagation, the loss being greater the greater the flux of energy. This "degradation" is expressed mathematically by ${}_f\Omega_{\mathrm{E}}\frac{1}{cm}$, the figure by which E must be multiplied to obtain the loss per period from unit volume of the medium at sound intensity E (ergs per sq. cm. per second) and frequency f. The actual loss per second per c.c. is thus $f_f\Omega_{\mathrm{E}}E$. From experiments carried out at a frequency of 100 cycles per second values of ${}_f\Omega_{\mathrm{E}}$ are deduced, rising to 3·0 at $E=18\times10^{-4}$ ergs per sq. cm. per sec.

In the pipe experiments described above the sound intensities at 96 cycles reached 1·4 milliwatts=14,000 ergs per sec., and since the cross-sectional area of the pipe was 3·1 sq. cm., the flux of energy in the pipe was 4,500 ergs per sq. cm. per sec., a figure enormously greater than Mr. Hart's maximum at about the same frequencies, and which should involve, presumably, a far greater value of Ω. But taking only the value of 3·0 corresponding to $E=18\times10^{-4}$, the energy loss would be $3\times4{,}500$ ergs per c.c. per period, and the loss per second would be 96 times this, or $1{\cdot}3\times10^{6}$ ergs per c.c. This energy loss taking place in the air itself would, presumably, heat the air, the amount of heat produced being $\frac{1{\cdot}3\times10^{6}}{4{\cdot}2\times10^{7}}=0{\cdot}03$ calories per c.c. per sec.

Assuming no loss by radiation, the temperature rise every second would then be, taking the specific heat of air as 0·2417 calories per gram, or $0{\cdot}2417\times0{\cdot}0012=2{\cdot}9\times10^{-4}$ calories per c.c., $0{\cdot}03/2{\cdot}9\times10^{-4}=103^{\circ}$C—an absurdity. Actually, a sensitive thermo-couple was inserted in the tube, but no rise of temperature whatever could be detected.

It is clear also that if any appreciable loss were occurring in the tube, the loss increasing with amplitude, the resonance curves obtained would be distorted in consequence of a decay factor increasing with amplitude; but they appear to show no distortion of this kind, in spite of the very large amplitudes used.

It would seem, therefore, that if there is any degradation, it is of a different order of magnitude altogether from that found by Mr. Hart, and it seems worth while to examine how his figures were obtained. In the first place the measured intensities are very small. At 40 cms. distance from the siren the particle velocity is only about 0·01 cm./sec., whereas the velocities at that distance with the largest amplitude at 96 cycles/sec. given by the vibrator should be of the order 0·2 cm./sec. It seems possible, therefore, that the intensity measurements are in error.

But however this may be, the published results hardly seem to substantiate the conclusions drawn from them, according to the present writer's analysis. The calibration curve of the hot wire microphone used is not given in the Paper, but it may be derived from the theoretical " readings " given. Thus the following table gives the microphone calibration for E in arbitrary units for a " check " microammeter reading of 54 :—

Distance from source, d cms.	Theoretical " reading " of microphone.	$\left(\frac{50}{d}\right)^2 = E$ (theoretical) arbitrary units.
50	28·6	1·0
60	21·2	0·695
70	15·8	0·51
80	12·3	0·39
90	10·2	0·31
100	8·4	0·25

Next, from the curves in the Royal Society Paper, the actual experimental points are read off for a " check microphone reading " of 54 ; i.e., at a particular intensity at the source, for different distances, and these are expressed in intensities by means of a calibration curve drawn from the above table. The values are entered below :—

d cms.	Microphone reading.	Relative intensity E	Ed^2
50	24·4	0·82	2,050
60	17·2	0·555	2,000
70	13·6	0·43	2,100
80	10·2	0·31	1,980
90	8·4	0·25	2,020
100	7·0	0·21	2,100

The actual result appears to be a striking demonstration of the validity of the ordinary assumption, as judged by the constancy of the figures for Ed^2 entered in the last column.

DISCUSSION.

Major W. S. TUCKER : The author has described an entirely new sound-producing device of great interest, but I am not convinced as to the purity of the tone obtained. Would the air adjacent to the piston receive the same displacement as the piston itself ? Examination of the resonance curves quoted in the Paper suggests that the combination of source and resonator is not as efficient as might be expected. The resonance curves are not as sharp as those obtainable from a tube in which the radiation losses from the open end of the tube are the only losses to take into account. A simple calculation based on the figures of the Paper shows that the amount of sound in terms of amplitude reflected at the open end is between 90 and 95 per cent. of the incident sound. It can be calculated, however, that in a tube of these dimensions the loss in reflection must be less than 1 per cent. The resulting resonance curves would be much sharper than those depicted and it can only be assumed that there are losses within the tube which are not accounted for by radiation at the open end and that such losses must be considerable. I would suggest testing for the presence of harmonics by means of microphones suitably tuned.

Prof. E. N. da C. Andrade: The author gives the amplitudes of the motion of his piston to 1/100,000 of an inch, about half the wavelength of green light. Were measurements really made to this degree of accuracy? It also seems very risky to neglect elasticity and inertia, and to assume that a static measurement of the amplitude of the piston would give the actual amplitude for a frequency of oscillation corresponding to audible sounds. The curves in Fig. 5 are not all of the same shape, as they should be if the author's contentions were correct.

Dr. E. T. Paris said that the addition of a pipe to the vibrator seemed to introduce unnecessary complications. Why not dispense with the pipe?

Dr. E. G. Richardson said that Michotte in Holland has shown that the Rayleigh disc gives results which vary with frequency, and the difference in shape of the curves in Fig. 5 might be due to such variation. The speaker had studied the attenuation of sound in a rubber tube by using a hot wire for measuring intensity, resonance being thus avoided. The source of sound was a piston working in a cylinder and actuated by a cam driven by a motor, the speed being determined stroboscopically. He could recommend the stroboscope for determining the frequency, and a phonic motor for maintaining it constant, as a result of experience.

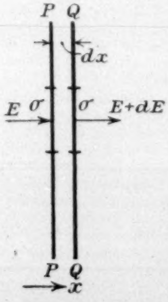
Mr. M. D. Hart: I am very glad that Dr. Mallett has published the figures given in the Appendix to his Paper as it has enabled me incidentally to discover and correct an unfortunate error that appears in the figures published in my Royal Society Paper. This error, which is in the values of Ω given in the Paper in question, is explained below. It has, however, no bearing on the results deduced by Dr. Mallett in his Appendix, such as, for example, that which results in the absurdity of a rise of 103°C. in the temperature of air through which a flux of 4,500 ergs per sec. is passing. Taking a value for Ω of 3 cm.^{-1}, he points out that with this energy flux the loss per second would be $1{\cdot}3\times10^6$ ergs per c.c., that is, that from a cubic centimetre heat energy would be produced in a given time at a rate of no less than 288 times the total energy input—an absurdity possibly even greater than that which he quotes! Such a result can, of course, only be obtained by reason of a complete misunderstanding of the meaning of the coefficient Ω. In the Royal Society Paper in question it is shown that for a spherical wave of which the energy flux at a radius r is E_r, the relation subsisting between E and r may be deduced from the definition of Ω to be

$$\frac{dE}{dr}+\frac{2E}{r}+n\,\Omega E=0$$

where n is the number of periods per second of the sound. The equivalent expression for a plane wave may be obtained from this by omitting the second term and substituting for r the quantity x, a length measured in the direction in which the plane wave is progressing. We thus obtain the equation

$$\frac{dE}{dx}+n\,\Omega E=0 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

a result that may also be deduced from first principles and from the definition of Ω as follows:



Let PP and QQ be two adjacent positions of the wave front separated by a distance dx. Let the energy flux at PP be E and at QQ be $E+dE$. Then the total energy passing through an area σ of the surface PP is σE and that emerging from the same area of QQ is $\sigma(E+dE)$. By definition of Ω the loss per period from the volume defined by this area and by the surfaces PP and QQ is $\Omega E\times\sigma dx$ and the loss per second n times this amount, so that

$$-\sigma dE=\sigma n\,\Omega E dx$$

or,

$$dE+n\,\Omega E dx=0$$

i.e.,

$$\frac{dE}{dx}=-n\,\Omega E$$

the same equation as (1) above. It is obvious that this rate of loss only continues constant if E and Ω are invariable with x, and it is equally obvious that this cannot occur since the loss must result in the decrease of E, and hence of Ω. The amount of such decrease of E over a definite distance x cannot be deduced unless we know the form of the relation subsisting between E and Ω. In the Royal Society Paper this relation was taken to be $\Omega = aE^p$, where p is about 1·6. Taking this value we obtain

$$\frac{dE}{dx} + anE^{2\cdot6} = 0$$

or

$$\frac{1}{a.n.} \frac{dE}{E^{2\cdot6}} + dx = 0$$

Integrating

$$\frac{1}{-1\cdot6an} E^{-1\cdot6} + x + C = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(ii)}$$

where C is a constant. This equation, also, could have been deduced from the equivalent expression for spherical waves given in my Paper. Inserting the value of 4,500 ergs per sq. cm. per second for E when $x=0$ and putting $n=100$, the value of C becomes equal to $2\cdot09 \times 10^{-3}$. We may then by substituting in (ii) a value for x of 1 cm. calculate what will be the corresponding value of E; actually 100 ergs per sq. cm. per second. Thus the amount of energy available per cubic centimeter for conversion into heat is 4,400 ergs for a length of pipe of 1 cm., so that over this length the average temperature rise would be no more than $\frac{4,400}{4,500 \times 288}$ or $\frac{1}{294}$ of that given by Dr. Mallett, resulting in a temperature rise of 0·35°C. instead of one of 103°C.

It may be emphasized that I have no reason for supposing that the value of Ω taken (i.e., 3 cm.$^{-1}$) is in fact appropriate to the energy flux in question of 4,500 ergs per sq. cm. per sec., and therefore that the temperature rise given above will accrue. The example has merely been worked out in order to call attention to the misconception arising in Dr. Mallett's Paper. Dr. Mallett further criticizes the interpretation of the experimental results which I obtained and has for the purpose worked out the results appearing in the two tables at the end of the Appendix to his Paper. The alleged discrepancy between his and my figures is in fact non-existent and the misconception appears to have arisen by reason of the fact that he has quoted only a particular set of experimental results and has neglected the treatment given in my Paper of the results as a whole. This treatment is given at some length and it is shown that the experimental results are satisfactorily consistent *inter se*. Without going into full details, which may be found by those interested by referring to the original Paper, it may be stated that the final result was given in the form of an equation connecting values of intensity at various different radii. From this equation, therefore, if any value of E be assumed to exist at a certain value of r, the value of E at any other value of r may be determined. If this be done for the intensity corresponding to the check microphone reading taken by Dr. Mallett it is found that the "transmission efficiency" from 50 to 100 cm. is about 98 per cent. This efficiency is defined as the ratio of the two relevant values of Er^2 and the figures in the last column of Dr. Mallett's second table correspond to these values. It is obvious from inspection that this column of figures is obtained from experimental values which have not been smoothed out and their mean error is so great as to obscure the small reduction of 2 per cent. referred to. It is therefore not surprising that this reduction is not apparent on inspection. I do not wish to labour the point unduly but I submit that the method of dealing with the results which is described in the Paper is such that its validity is self-evident, and that the final result is the best obtainable compromise between admittedly inaccurate data obtained under extremely difficult experimental conditions. If Dr. Mallett had applied his tests by means of the final equation given instead of to particular experimental points I think it would have been clear to him that the results had not been misinterpreted.

Apart from his critique of my work, there is only one point in Dr. Mallett's Paper which raises an issue on which I should like to remark. Measurement of acoustical energy flux by means of a Rayleigh disc is analogous to measurement of electrical alternating-current power with an ammeter. The dangers which beset the experimenter who makes such measurements may best be illustrated by an example. If a stationary wave be set up the Rayleigh disc may be made to give any reading we please between zero and a maximum by placing it at given points of the wave, but at all such points the energy flux is zero. The bearing of this is that it becomes almost essential to conduct all such measurements in free air with the concomitant host of experimental difficulties. Any measurements taken in a closed room are liable to be vitiated unless the walls of the room be completely absorbent, a Utopian state of affairs which, unfortunately, cannot be realized at present.

I may now revert to the error referred to in the first sentence of my reply. When the experiments in question were first performed it was intended to define the Coefficient of

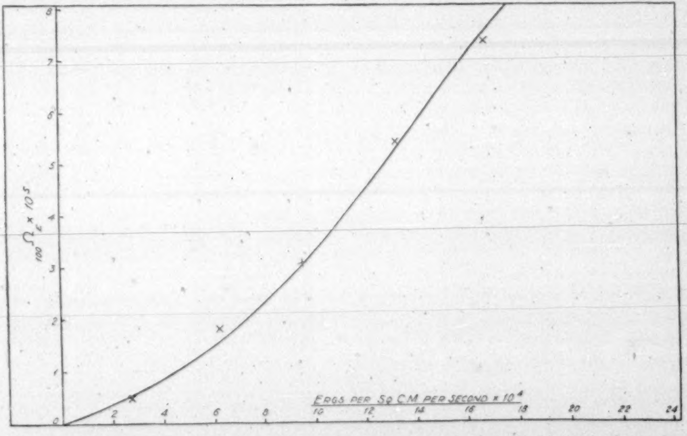
Degradation in terms of wavelength instead of period; this decision was subsequently altered, and I regret that the necessary alteration in some of the figures has been missed. The mathematics of the Paper are correct, as are the dimensions given of Ω, but the values of the coefficient have to be divided by the velocity of sound expressed in centimeters per second and a similar correction has to be made in the value of one of the constants in the expression connecting Ω and *E*. I append a figure showing a graph of the correct relation, the equation to which is

$$\Omega = aE^{1{\cdot}58}$$

in which a is equal to 1·67 grams$^{-1{\cdot}58}$ cm.$^{-1}$ seconds $^{4{\cdot}74}$ and also a table of corresponding values of Ω and *E*; the graph, equation and table superseding the incorrect ones given in my Paper.

E (ergs per sq. cm. per sec. $\times 10^4$)	Ω (cm.$^{-1} \times 10^5$)
4	0·8
8	2·31
12	4·47
16	7·15

Author's reply: The result of Major Tucker's calculations on the loss of energy in the tube is interesting. Since the resonance curve with a tube three times as long is substantially the same at the same frequency, it is clear that if there is this large loss in the tube it must be associated with the piston. It is clear also from the linearity of the curves of Fig. 4 that the proportionate loss does not increase with amplitude. I certainly imagine that the air in contact with the piston moves as the piston moves; the velocity of the latter is always far smaller than the velocity of sound. I have never suspected the presence of higher harmonics of considerable amplitudes; but I hope to make further experiments.



In reply to Prof. Andrade, it certainly was unduly optimistic to insert the last figure of the measurements on the roller lifts. Another experiment, not recorded in the Paper, showed that the electromotive force induced in the coils of the telephone receiver mentioned in Section I was directly proportional to the speed. This, I think, indicates that the amplitudes measured statically are reproduced dynamically.

The only reply to Dr. Paris is that if the pipe were omitted, the sound amplitudes would be very much smaller. It would however, be interesting to have the piston working in a large baffle plate.

I cannot think that over the small range of Fig. 5 the frequency differences can affect the behaviour of the Rayleigh disc.

Mr. Hart's criticism of my temperature rise calculation is not sound. In order to get 4,500 ergs /sec./sq. cms. into the air, with degradation such as he postulated, enormously greater power would have to be supplied at the piston end of the pipe. The *least* value of the flux in the pipe would be 4,500, and taking even this minimum value as holding over a centimeter the temperature rise would be as I calculated it. Of course it would be greater and greater the nearer one approached the piston. Mr. Hart has now, however, divided his degradation coefficient values by 34,000, and it is hardly surprising that an error of such magnitude should have led to absurd results. But even his revised value of a in the expression $\Omega = aE_p$ is enormously too large to apply to values of E of the order obtained in my experiments, where no " acoustical degradation " was observed. I still very much doubt whether Mr. Hart's theory and data rest upon a sound experimental basis.

XXI.—ON THE STATIONARY-WAVE METHOD OF MEASURING SOUND-ABSORPTION AT NORMAL INCIDENCE.*

By E. T. Paris, *D.Sc., F.Inst.P.*

Received January 12, 1927.

ABSTRACT.

A description is given of apparatus employed for measuring coefficients of sound-absorption by the stationary-wave method. The apparatus differs from that used by earlier workers in the use of (1) a small tuned hot-wire microphone for determining relative pressure-amplitudes in the sound-waves ; (2) the employment of a steady valve-driven source of sound with arrangements for maintaining the strength at a constant value ; (3) the screening of source and experimental pipe from disturbances due to the movements of the observer. By the employment of a certain procedure the relation between the response of the microphone and the amplitude of the pressure-variation in the sound-wave is eliminated. Some examples of the employment of the apparatus for determining the coefficients of absorption at normal incidence of acoustic plasters and hair-felt are given.

§1. INTRODUCTION.

THE work described in this Paper arose in connexion with investigations initiated by the Building Research Board of the Department of Scientific and Industrial Research into the acoustic properties of certain materials. These investigations were concerned with testing the sound-absorbing properties of various plasters and other materials which could be used for lining the walls of rooms and auditoria for the purpose of reducing the reverberation in cases where it was in excess of that required for good acoustical conditions. The tests conducted by the Building Research Board were made by Sabine's reverberation method, and the results were expressed as percentage reductions in the period of reverberation produced by the introduction of 100 square feet of the material under test into a room of a certain size and shape.†

For some purposes, however, it was desirable that a method should be available for which considerably less material would be required than was needed for Sabine's method. It was, for example, desired to test a series of acoustic plasters in order to trace the effect of varying the proportions of the materials used in their making, and the necessity for preparing the comparatively large quantities needed for a reverberation experiment was an obvious handicap.

For this reason experiments were begun in 1923 with the object of establishing some method whereby a test of sound-absorption could be carried out on a specimen having an area of not more than one or two square feet. The method first suggested was one in which resonance boxes were made up and lined with the materials to be tested, the sound-absorbing properties being deduced from the sharpness of resonance exhibited by the boxes. Several experimental and theoretical difficulties arose, however, which could not be readily overcome, and it was decided to seek for an alternative method of procuring the desired information.

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* The work described in this Paper was carried out in the Acoustical Research Section of the Air Defence Experimental Establishment, Woolwich, with the assistance of the contribution made to the funds of that Section by the Department of Scientific and Industrial Research on the recommendation of their Physics Research Board.

† These experiments are described in "An Interim Note on Acoustic Experiments carried out for the Government of India" by Messrs. Barnett and Glanville, Building Research Board Paper No. 181, January, 1924.

This alternative was found in what is here called the " Stationary-wave Method " —a method which had previously been used for the study of sound-absorption by J. Tuma (1902), F. Weisbach (1910) and Hawley Taylor (1913). Some notes on the work of these investigators are given in § 3. The theory of the method is very simple and is briefly as follows. The specimen to be tested is supplied in the form of a flat disc, and (with suitable backing) is used to close one end of a cylindrical pipe (Fig. 1). The other end of the pipe remains open, and opposite to it is placed a source of sound of constant pitch and loudness. Sound-waves from the source travel down the pipe and are reflected back from the specimen. If the latter were a perfect reflector, the amplitudes of the incident and reflected waves would be equal and a stationary wave would be formed inside the pipe, with no pressure-variation at the loops and maximum pressure-variation at the nodes. If, however, the specimen absorbs part of the incident sound-energy, the amplitude of the reflected wave will be less than that of the incident wave, so that instead of a stationary wave being formed inside the pipe there will now be a series of maxima and minima.

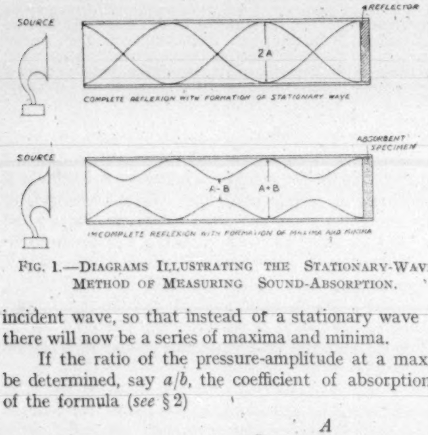
SOURCE
REFLECTOR
2A
COMPLETE REFLEXION WITH FORMATION OF STATIONARY WAVE
SOURCE
ABSORBENT SPECIMEN
A-B
A+B
INCOMPLETE REFLEXION WITH FORMATION OF MAXIMA AND MINIMA

FIG. 1.—DIAGRAMS ILLUSTRATING THE STATIONARY-WAVE METHOD OF MEASURING SOUND-ABSORPTION.

If the ratio of the pressure-amplitude at a maximum to that at a minimum be determined, say a/b, the coefficient of absorption can be calculated by means of the formula (*see* § 2)

$$\alpha = \frac{A}{2 + a/b + b/a} \quad \dots \dots \dots \quad (1.1)$$

It may be noted here that the " absorbed " sound is taken to be that part of the incident sound which is not reflected. Thus, if E_i is the energy-flux in the incident waves and E_r that in the reflected waves, then the rate at which energy is being transmitted across the reflecting surface (the waves being incident normally) is $E_i - E_r$ ergs/cm.^{2} per sec., and the coefficient of absorption is

$$\alpha = \frac{E_i - E_r}{E_i} \quad \dots \dots \dots \quad (1.2)$$

In the case of a porous reflecting surface, therefore, the absorbed sound includes both that which is dissipated into heat by viscous forces acting in the pores and that which is intromitted as wave-motion into the material itself.

The results so far obtained by this method indicate that it is a satisfactory one for testing the sound-absorptive properties of materials at frequencies below 700 vibrations per second, when only small specimens are available. The present Paper contains a description of the apparatus which was employed by the author and some results obtained by its use. The apparatus differs from that used by earlier workers mainly in the following particulars :—

(1) The employment of a small tuned hot-wire microphone for determining the ratio of maximum to minimum pressure-amplitude inside the pipe. This instrument is very simply constructed and easy to use, and requires no accessory apparatus beyond a battery, galvanometer and a few resistance boxes. As it is a tuned instrument, measurements of absorption at a definite wavelength can be made without the necessity of having a pure-tone source of sound.

(2) The employment of a valve-oscillator and loud-speaker as a source of sound, together with a special arrangement described in § 4(*c*) for maintaining the source at a constant strength.

(3) The use of a sound-chamber to enclose the source of sound and the open end of the pipe as described in § 4 (*a*). This is important, as the output of the source may otherwise be influenced by the movement of the observer.

Emphasis is also laid on the use of the procedure (due to Tuma) described in § 5, by which the relation between the pressure-amplitude of the sound and the response of the microphone is eliminated.

The apparatus described was designed primarily for employment with sound of a frequency of 512 vibrations per second, but a few measurements have also been made at 380 and 650 vibrations per second.

The method measures the coefficient of absorption at normal incidence under the condition that transmission is prevented as far as possible by backing the specimen with a suitable material.

§ 2. THEORY OF THE METHOD.

It is assumed that the incident and reflected sounds are plane waves travelling in opposite directions inside the pipe. In practice this condition can be secured by choosing a pipe of suitable diameter and length and arranging the source at an appropriate distance from the mouth of the pipe. Moreover, a preliminary test can always be made to ensure that a stationary wave of the required type exists inside the pipe when a good reflecting substance is used to close the distant end.

Let the axis of the pipe be parallel to the axis of x, and let the surface of the specimen to be tested lie in the plane $x=0$. The potential of the incident wave (supposed travelling in the direction x positive) can be represented by

$$\varphi=A\cos k(at-x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.1)$$

After reflexion there will in general be a wave of reduced amplitude travelling in the negative direction of x. This can be represented by

$$\varphi'=B\cos k(at+x+\varepsilon) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.2)$$

In practice, of course, repeated reflexions take place both at the open or " source " end of the pipe and at the surface of the specimen ; but this does not necessitate any modification of the above expressions, since the net effect, when a steady state is reached, is that of a single train of incident and a single train of reflected waves.

The total potential within the pipe is

$$\Phi=\varphi+\varphi' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.3)$$

If the reflecting surface at $x=0$ were perfectly rigid, the condition to be satisfied by Φ at $x=0$ would be

$$\left(\frac{d\Phi}{dx}\right)_{x=0}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.4)$$

expressing the condition that there is no motion in the plane $x=0$, so that no vibrational energy can be transmitted into the reflecting substance. Under these circumstances, the amplitudes of the incident and reflected waves are equal ($A=B$) and $\varepsilon=0$. The result is a stationary wave

$$\Phi=2A \cos kx \cos kat \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.5)$$

within the pipe.

If part of the energy of the incident sound is lost at reflexion we have

$$\Phi=A \cos k(at-x)+B \cos k(at+x+\varepsilon) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.6)$$

which, by a simple transformation ($t'=t+\frac{1}{2}\frac{\varepsilon}{a}$, $x'=x+\frac{1}{2}\varepsilon$), may be written

$$\begin{aligned}\Phi&=A \cos k(at'-x')+B \cos k(at'+x')\\&=(A+B) \cos kx' \cos kat'\\&\qquad+(A-B) \sin kx' \sin kat' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.7)\end{aligned}$$

so that the motion within the pipe can be regarded as being due to two superimposed stationary waves, one of amplitude $(A+B)$ and the other of amplitude $(A-B)$, the nodes and loops of the first being one quarter of a wavelength distant from the nodes and loops of the second. The result is that there is within the pipe a series of positions of maximum and minimum pressure-variation proportional respectively to $(A+B)$ and $(A-B)$, spaced at distances of one quarter of a wavelength. It is the ratio of the amplitudes at these maxima and minima which is found in the experiment.

To calculate the coefficient of absorption we note that the energy-flux in the incident wave is proportional to A^2, and the energy-flux in the reflected wave is proportional to B^2. The fraction of the incident sound-energy lost at reflexion is thus proportional to $(A^2-B^2)/A^2$, and this is, by definition, the coefficient of absorption (α). Let the observed ratio of the maximum amplitude to the minimum amplitude be a/b. Then to find α in terms of ratio a/b we have, since

$$\frac{a}{b}=\frac{A+B}{A-B}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.8)$$

$$\alpha=\frac{A^2-B^2}{A^2}=\frac{4ab}{(a+b)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.81)$$

Or,

$$\alpha=\frac{4}{2+a/b+b/a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.82)$$

This last expression is that given by Hawley Taylor, and is the most convenient form for computing α from the observed ratio a/b.

Alternatively, let—

$$A = (1+m)B, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.9)$$

where m is positive, since $A > B$. Then—

$$\begin{aligned}\Phi &= B \cos k(at' - x') + B \cos k(at' + x') + mB \cos k(at' + x') \\ &= 2B \cos kx' \cos kat' + mB \cos k(at' + x'). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.91)\end{aligned}$$

Alternatively, therefore, the motion within the pipe may be regarded as being that due to a stationary wave of amplitude $2B$, on which is superimposed a progressive wave of amplitude mB. The loops and nodes of the stationary wave are at the positions of maximum and minimum pressure-variation which occur when an absorbent substance is placed at the end of the pipe.

Also, according to (2.91), the positions of the maxima and minima differ from the position of the nodes and loops which would occur if a perfect reflector were used to close the pipe, by a distance $\frac{1}{2}\varepsilon$. No special attempt was made to measure this shift in the experiment here described, but it may be of interest to note that it was of the order of 1 cm. for a substance with an absorption-coefficient of about 0·3 at 512 variations per second. The shift was in the direction x positive, and thus indicated a virtual node of the stationary vibration in (2.91) about 1 cm. within the surface of the absorbing material. It is possible that the measurement of this shift might be of service in investigating the mechanism by which absorption takes place in some materials. In the case of resonant reflexion it might happen that the virtual node would be in front of the reflecting surface.

§ 3. PREVIOUS EXPERIMENTS WITH THE STATIONARY-WAVE METHOD.

The first experiments with the stationary-wave method appear to have been made by J. Tuma,[*] in Vienna, in 1902. Later experiments were made by F. Weisbach (1910)[†] and Hawley Taylor (1913).[‡] Taylor appears to have been unaware of the work of Tuma and Weisbach.

Tuma's observations were entirely aural. By means of listening tubes, he observed the loudness of the sound in the maximal and minimal positions in a pipe closed by absorbent materials and then found positions of the same loudness in a stationary-wave. Since the amplitude of the pressure-variation in a stationary-wave is proportional to a simple sine or cosine function, he could in this way find the ratio of the maximum to the minimum amplitude when absorption was taking place. Thus suppose the positions corresponding to minimum and maximum loudness were distant x_1 and x_2 from the nearest loop in the stationary-wave, then the ratio of maximum to minimum pressure-amplitude would be $\sin kx_2/\sin kx_1$, where $k = 2\pi/\lambda$. From this ratio a quantity α was calculated such that $(1-\alpha)$ was the ratio of the reflected to the incident amplitude. The actual results obtained were very meagre, and the interest of the Paper lies in the description of the method.

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[*] "Eine Methode zur Vergleichung von Schallstärkung und zur Bestimmung der Reflexionsfähigkeit verschiedenen Materialien," J. Tuma, Sitzungsber. d. K. Akad. d. Wissenschaften, Vol. III, Pt. 2A, pp. 402-410 (1902).

[†] "Versuche über Schalldurchlassigkeit, Schallreflexion und Schallabsorption," F. Weisbach, Ann. d. Phys., Vol. 33, pp. 763-798 (1910).

[‡] "A Direct Method of Finding the Value of Materials as Sound Absorbers," Hawley O. Taylor, Phys. Rev., Vol. 2, pp. 270-287 (1913).

The principle of comparing amplitudes by reference to a stationary-wave has been retained in the present work, the method here described being essentially that of Tuma, with the ear and listening tube replaced by a hot-wire microphone.

Weisbach made experiments with a wooden "resonance-pipe," 238 cm. long, 25 cm. diameter, and walls 4 cm. thick. The source of sound was an electrically driven tuning-fork, mounted on a resonance-box, and in order to ensure that the source was steady, observation of the amplitude of the prongs was made by means of a microscope. The amplitude of the sound in the pipe was measured by means of a magnetophone (mounted on a slider), this being connected with a string galvanometer, the deflexion of which was taken to be proportional to the amplitude of the sound.

In the experiments of Taylor, observations were conducted with a wooden "flue," 115 cm. long, and of square cross-section (9 cm. × 9 cm.), the specimen to be tested being held in place by means of a wooden cap. To explore the amplitude of vibration in the flue use was made of a Rayleigh disc instrument, to which the sound from inside the flue was lead by means of a long glass tube. As the Rayleigh disc involves the employment of a delicate suspension, this instrument could not be moved during an experiment, and it was therefore necessary, when comparing amplitudes at a maximum and a minimum, to move the whole of the flue together with the source of sound. This made the method very cumbersome, and it may be noted that the movement of the source during an experiment may introduce errors into the measurements, inasmuch as the output of the pipe may be influenced by its position in the room. The source of sound was a stopped organ-pipe, the note of which was purified by means of a "tone-screen," consisting of a number of resonators arranged at the open end of the flue, each resonator being designed to prevent the passage of one of the harmonics of the organ-pipe note. The absorption-coefficients were calculated from the deflexions shown by the Rayleigh disc, these being assumed proportional to the square of the amplitude of the sound. Taylor found that for 1 inch of hair-felt backed by wood, the coefficient of absorption was 0·51 at 500 vibrations per second.

§ 4. DESCRIPTION OF APPARATUS.

The description of the apparatus employed can be conveniently divided into three parts dealing with—

(*a*) The experimental pipe and the mounting of the specimen ;
(*b*) The hot-wire microphone and its connexions ; and,
(*c*) The source of sound.

(*a*) *The Experimental Pipe and the Method of Mounting Specimens.*—The dimensions of the pipe finally adopted after some preliminary trials were 30·5 cm. diameter and 226 cm. length. This pipe was made from two glazed earthenware drainpipes cemented together, and was mounted, with its axis horizontal, inside a wooden box supported on four steel legs set in concrete. The pipe was insulated from its supports by pieces of hair-felt, and the box was lined with the same material. The object of the mounting was to prevent the communication of vibrations due to traffic, or other local causes, to the walls of the pipe and thence to the contained air and the microphone.

The general arrangement of the pipe and its housing is shown in Fig. 2. One

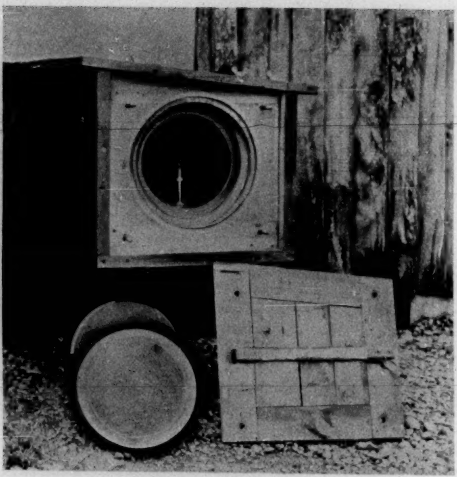

FIG. 3.—VIEW OF EXPERIMENTAL PIPE FROM OUTSIDE LABORATORY.

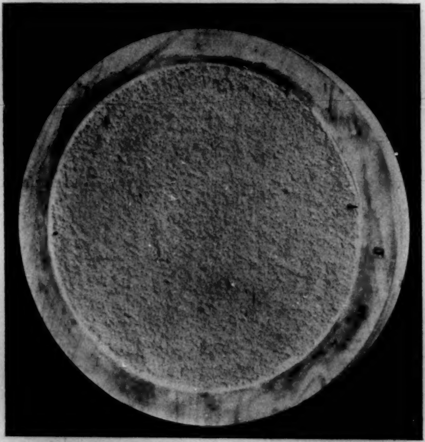

FIG. 4.—SPECIMEN OF AN ACOUSTIC PLASTER MOUNTED FOR TESTING (diameter of Plaster Disc = 12 ins.).

end of the pipe passed through the wall of the laboratory into a sound-chamber, in which was situated the source of sound, the pipe being insulated from the wall by means of felt. The object of the sound-chamber (approximately 7 ft. by 9 ft. by 7 ft.) was to prevent the output of the source from being affected by the movement of the observer in the laboratory. At the other end of the pipe, the specimen, in its mounting, was held firmly in place between the end of the pipe and the detachable cover at the end of the box (*see* Fig. 2). The detachable cover was held in place by four bolts and wing-nuts, and could easily be removed and replaced by hand.

The mountings for the specimens were made of discs (37 cm. in diameter), to which were screwed rings measuring 37 cm. externally, and 30·5 cm. internally. Both disc and ring were made from three layers of five-ply wood glued together. The specimens were provided as discs about 30·5 cm. in diameter, and these were fitted inside the rings, the surface of the backing provided by the disc being first covered with enough cement to fill up any inequalities in the back of the specimen. Any crevices between the specimen and the ring were filled in with plaster of Paris which was allowed to set before a test was made.

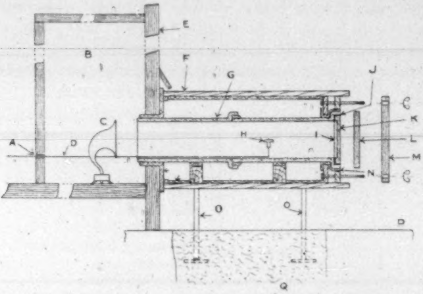

FIG. 2.—ARRANGEMENT OF EXPERIMENTAL PIPE, SOURCE AND SPECIMEN (NOT TO SCALE).

A. Scale Read Here.
B. Sound-Chamber
C. Loud-Speaker.
D. Slider.
E. Wall of Laboratory.
F. Felt-Lined Box Enclosing Pipe.
G. Earthenware Pipe.
H. Microphone.
I. Specimen.
J. Thin Washer of Rubber.
K. Backing to Specimen (3 layers of 5-ply).
L. Packing (1-inch Wood).
M. Cover (held by Bolts and Wing Nuts.)
N. Felt.
O. Steel Supporting Legs set in Concrete.
P. Ground-Level.
Q. Concrete.

The mountings, made as described, were found to keep their shape very well, and were in every way satisfactory.

When fixing a specimen in place for a test a washer of thin leather or rubber was placed between the end of the pipe and the ring of the mounting to ensure an airtight fit.

A view of the pipe in its housing as seen from the outside of the laboratory is shown in Fig. 3, and a specimen of an acoustic plaster in its mounting ready for testing is shown in Fig. 4.

(*b*) *The Hot-wire Microphone.*—The microphones used in these experiments did not differ in any essential detail from the ordinary selective hot-wire microphone,* but, in order that they should offer as little obstruction as possible to the passage of the sound-waves along the experimental pipe, they were made to smaller dimensions than is customary for instruments tuned to the frequencies employed in the experiments. The microphone used for measurements at 512 vibrations is shown in Fig. 5. Its overall length is 6 cm., and diameter of the holder 3·4 cm. The orifice is 1 cm. long and 0·5 cm. in diameter (internal), the conductance being 0·114 cm. The container was made in two parts, which could be slid over one another, so that the microphone was tunable over a moderate range (approximately 460 to 560 vibrations per second). The damping factor at 512 vibrations per second was found to be 81 seconds^{-1} (*see* Appendix I).

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* *Vide* Tucker and Paris, Phil. Trans., A, Vol. 221, pp. 389-430 (1921).

The microphone was carried in the experimental pipe at the end of a long wooden slider (*see* Fig. 2), which passed through a slot in the wall of the sound chamber. At the end which protruded through the wall of the sound chamber, the slider was provided with a millimetre-scale, so that the movement of the microphone in the experimental pipe could be recorded by an observer from outside the sound-chamber. The orifice of the microphone was on the axis of the pipe.

The electrical connexions of the microphone are shown in Fig. 6. The response was measured by a bridge method, the bridge being of the "battery" type with compensating microphone. This type of bridge is preferable to the "Wheatstone" bridge on account of its greater sensitivity, while the use of a compensating microphone makes the balance very steady. The compensating and active microphones were of the same type, but the former was, of course, shielded from the sound.

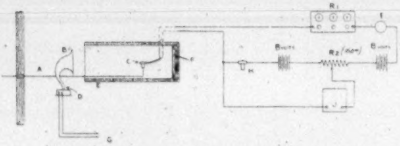

FIG. 6.—ELECTRICAL CONNEXIONS.

A. Slider.
B. Loud-Speaker.
C. Active Microphone.
D. Control Grid.
E. Experimental Pipe.
F. Specimen.
G. To Oscillator.
H. Balancing Microphone.
I. Milliammeter.
J. Micro-Ammeter.

The grid of the microphone carried a heating current of about 28 milliamperes, and had a "hot" resistance of about 320 ohms. When exposed to the sound the fall in the resistance was anything up to 50 ohms, depending on the position of the microphone within the tube and the output of the source of sound. This fall in resistance was measured by introducing resistance in the three-dial resistance box R_1, reading from 0·1 ohm up to 100 ohms (Fig. 6), until the balance of the bridge was restored. The bridge was initially balanced by means of the sliding resistance R_2.

(*c*) *The Source of Sound.*—The provision of a satisfactory source of sound is a matter of importance, as the source must be capable of maintaining an output of constant amplitude and pitch for considerable periods. The oscillator and loudspeaker used in the present experiments were found to be satisfactory in this respect for frequencies between 300 and 1,000 cycles per second.

During the course of an experiment it was necessary to turn the source of sound on or off several times. This was done by means of a switch, which could be used to change over the output terminals of the oscillator from the loud-speaker to an equivalent resistance.

It was found, however, that the amplitude of the sound affecting the microphone in the experimental pipe was often different before and after the source had been

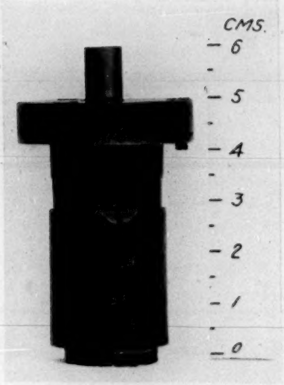
CMS.
6
5
4
3
2
1
0

Fig. 5.—Hot-Wire Microphone Tuned to 512 Vibrns. per Sec.

To face page 276]

switched off, and that adjusting the current in the oscillating circuit was not a satisfactory method of correcting for this variation. To overcome this difficulty and to ensure that the strength of the source remained constant throughout the period during which a set of observations was being made, the following device was made use of. A hot-wire grid of the usual pattern was mounted in the narrow end of the loud-speaker just above the diaphragm. This grid was connected to a battery-bridge of the same pattern as that used for the grid in the active microphone. At the commencement of a set of observations, the oscillator was adjusted so that this control grid showed a resistance change of about 10 ohms, and this change of resistance was maintained throughout the experiment by making small adjustments to the oscillator. This could be done without difficulty, to within about 0·05 ohm, and since experiment shows that when the resistance change in a grid is about 10 ohms, a further change of 1 ohm corresponds to a change in the amplitude of the sound of about 1·9 per cent., it follows that the amplitude of vibration of the source could be maintained constant to within about 1 part in 1,000.

§ 5. METHOD OF OBSERVATION.

The procedure adopted when taking observations was as follows. The microphone was first of all tuned as nearly as possible to the frequency of the note given by the oscillator by altering the volume of the container, a final tuning being made by a slight adjustment of the note of the oscillator. The specimen to be tested was next mounted in its place, the microphone balanced up, and the source of sound turned on. The microphone was then moved along the pipe by means of the slider shown in Figs. 2 and 6 until the position of the minimum nearest to the specimen was found. The resistance-change suffered by the microphone-grid in this position was then noted. The microphone was then moved along (away from the specimen) until a maximum was reached. The resistance-change in this position was also noted, and the specimen was then moved.

Let ρ_1 be the resistance-change at the minimum, and ρ_2 that at the maximum. It is required to find the ratio of the pressure-amplitude at a maximum to that at a minimum (i.e., a/b in the notation of § 2) from the observed resistance-changes ρ_1 and ρ_2.

To accomplish this the experimental pipe was closed with a disc of a good reflecting substance. In the earlier experiments a piece of polished teak 1 inch thick was employed, but later this was replaced by a disc of brass $\frac{1}{8}$ in. thick, backed by three layers of five-ply wood. The position of the loop nearest to the reflector was found by sliding the microphone along the pipe. In this position, if the reflexion were perfect, there should be no change in the resistance of the microphone when the source is switched on or off. In practice the reflexion is, of course, never quite perfect, and sometimes a small change was discerned but this never exceeded 0·005 ohm at 512 vibrations per second. This residual effect at the loop appeared to be too small to be taken into consideration and it was ignored.

One method of finding a/b is to displace the microphone from the loop until it suffers a resistance change ρ_1, the distance through which has been moved (say, y_1) being noted. It is then further displaced until the resistance change is ρ_2, and the distance through which has been moved (say, y_2) again noted. The pressure-amplitude in a stationary-wave being proportional to sin ky, where y is the distance from a loop and $k=2\pi/\lambda$, the pressure-amplitudes producing the resistance-changes ρ_1 and ρ_2 must be proportional to $\sin ky_1$ and $\sin ky_2$, so that

$$\frac{a}{b}=\frac{\sin ky_2}{\sin ky_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5.1)$$

and a can be calculated from (1.1).

In practice, however, it is easier to adjust the strength of the sound, so that $ky_2=90°$, and hence, writing y instead of y_1

$$\frac{a}{b}=(\sin ky)^{-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5.2)$$

and

$$a=\frac{4}{2+\operatorname{cosec} ky+\sin ky}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5.3)$$

To ensure that $ky_2=90°$, the microphone was first placed at a node in the stationary wave, and the strength of the source adjusted until the resistance change suffered by the grid was ρ_2. The microphone was then moved into a loop position and the displacement (y) determined which was necessary to obtain a resistance-change ρ_1. Since y is in practice often less than 1 cm., the microphone was first moved towards the specimen until the resistance change ρ_1 occurred, and then away from it until the same change was repeated, the distance between the two positions being $2y$.

The half-wavelength of the sound was found by observing the position of the second loop from the specimen.

§ 6. CORRECTION FOR THE EFFECT OF THE PRESENCE OF THE MICROPHONE IN THE EXPERIMENTAL PIPE.

It has so far been assumed that the effective pressure-variations actuating the microphone are equal to the pressure-variations which would exist in the pipe if the microphone were absent. In general, however, this will not be the case. If the pipe be narrow, the hot-wire microphone and the pipe together form a well-known type of double resonator. On the other hand, it is obvious that if the pipe be wide and the microphone very small, the coupling between pipe and microphone will be very loose, and the microphone serves merely to indicate what goes on within the pipe.

The theory of the double resonator can be utilised to calculate the effect of the presence of the microphone in the experimental pipe. Details of the calculation are given in Appendix II, where the particular case of the pipe and microphone used in these experiments for observations at 512 vibrations per second is worked out in full. The results show that for all practical purposes the effects of reaction may be ignored. The greatest effect is at a node, where the amplitude recorded by the microphone is 2 per cent. less than it would be if there were no reaction. Midway between a node and a loop the effect is about 0·9 per cent., and this decreases to zero at the loop.

It may be noted that the effects of reaction, small as they are, are largely eliminated by means of the procedure described at the end of the last section, in which, after the resistance changes ρ_1 and ρ_2 have been determined with the specimen in position, the microphone is placed at a node in the stationary wave and the strength of the source adjusted until the change ρ_2 again occurs.

In order to see how this procedure eliminates the effect of reaction, we may refer to equation (2.91) of § 2, which shows that, when the specimen is in position, the potential within the pipe may be regarded as being due to a stationary wave of amplitude $2B$, and a progressive wave of amplitude mB. The effect of reaction in the case of a pure stationary wave is discussed in Appendix II, and, if the small difference between x and x' in (2.5) and (2.91) be disregarded, the same theory is applicable to the stationary wave of amplitude $2B$ occurring in (2.91). The case of a progressive wave is dealt with in Appendix III, where it is shown that the effective amplitude of the potential operating on the microphone, corresponding to an undisturbed amplitude mB, due to a progressive wave is

$$mB/(1+\frac{ac}{4h\sigma}),$$

where a is the velocity of sound in air, c is the conductance of the orifice of the microphone, h is the damping-factor of the microphone, and σ is the cross-sectional area of the pipe.

In the first part of the experiment, with the absorbing material in place, the resistance changes ρ_1 and ρ_2 are observed corresponding to *actual* amplitudes of oscillation in the neck of the microphone of, say, b_0 and a_0. The corrected amplitude corresponding to ρ_1 would be θb_0, where $\theta=1+\frac{ac}{4h\sigma}$. Now a_0 is equal to the sum of the amplitudes of the stationary wave and the progressive wave, say, $a_0'+b_0$. Hence the corrected value of a_0 is $\varphi a_0'+\theta b_0$, where φ is the ratio of the corrected to the uncorrected amplitude at a node—that is, 1·019 (*see* Appendix II).

In the second part of the experiment the sound is adjusted until the resistance change ρ_1 occurs at a node in the stationary wave. This therefore again corresponds to an actual amplitude of $a_0=a_0'+b_0$, but the corrected value is now $\varphi(a_0'+b_0)$.

We wish to find the ratio of the corrected amplitudes at maximum and minimum; that is, $(\varphi a_0'+\theta b_0)/\theta b_0$, by reference to a stationary wave. By taking the reading for ρ_2 at a node we make the correction for reaction nearly the same as it was at the maximum in the first part of the experiment. For the corrected amplitude corresponding to ρ_2 is $\varphi a_0'+\varphi b_0$, which is to be compared with $\varphi a_0'+\theta b_0$, that is, we have to compare $a_0'+b_0$ with $a_0'+\frac{\theta}{\varphi}b_0$. Now $\varphi=1{\cdot}019$, approximately, and $\theta=1{\cdot}016$ (*see* Appendix III), so that, remembering that b_0 is seldom as great as $\frac{1}{5}a_0'$, and frequently less than $\frac{1}{10}a_0'$, we see that the effect of reaction at the node is eliminated for all practical purposes.

It remains to consider the error in measuring the amplitude corresponding to ρ_1. As shown by the figures given in Appendix II, the error involved in matching an amplitude in the stationary wave when ky is small (as it is for the amplitude corresponding to ρ_1) is quite negligible, while Appendix III shows that the effect of reaction on the measurement of the amplitude at a minimum can be corrected for by means of the factor $\theta=1+\frac{ac}{4h\sigma}$. A more exact formula for the absorption-coefficient than (5·3) is therefore

$$=\frac{4}{2+\theta \sin ky+(\theta \sin ky)^{-1}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6\cdot1)$$

where θ, for the apparatus used in the present experiment at 512 vibrations per second, is equal to 1·016. Here again, however, the effect is negligible for most practical purposes, as shown by the figures in the following table :—

ky	α calculated from (5·3).	α calculated from (6·1).
0°	0	0
5°	0·295	0·299
10°	504	510
15°	653	659
20°	760	766
25°	835	841
30°	888	887

The figures in the second column—that is, α as calculated from the formula (5.3)—are plotted against *ky* in Fig. 7. From the curve so obtained values of α can be read off directly by means of the experimentally determined values of *ky*.

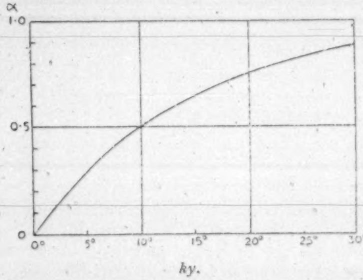

FIG. 7.—COEFFICIENT OF ABSORPTION (α) PLOTTED AGAINST *ky*.

§7. RESULTS OF OBSERVATIONS.

Some examples will now be given of measurements of sound-absorption made with the apparatus described in the preceding sections.

(*a*) *Acoustic Tile.*—A number of determinations have been made of the coefficient of absorption (at or near 512 vibrations per second) of samples of an acoustic tile procured by the Building Research Board from an American source. The readings obtained with this material provide an illustration of the working of the method.

The tile is a proprietary article, believed to consist mainly of pumice and Portland cement.* It has a pleasant stone-like appearance, and when used to line the interiors of halls and auditoria serves to reduce excessive reverberation. Its sound-absorbent properties are due to the porous nature of its structure. This feature is well shown in Fig. 8, which is a reproduction of a microphotograph of a section through a tile

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* For this information and the microphotograph shown in Fig. 8, and also for all details of the composition and manufacture of acoustic plasters, I am indebted to the Officers of the Building Research Station of the Department of Scientific and Industrial Research at Watford, Herts.

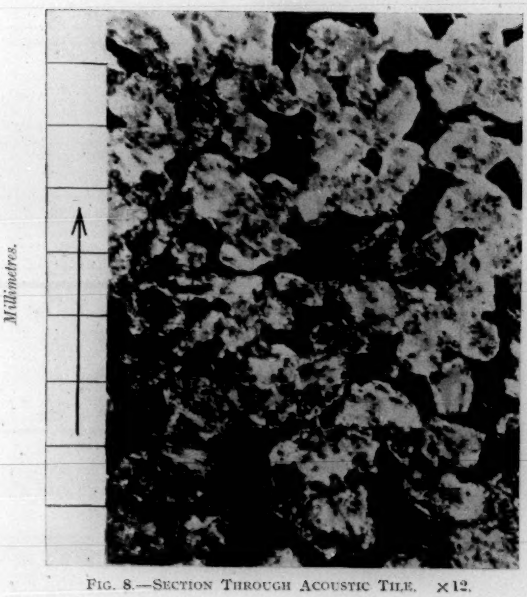
Millimetres.

FIG. 8.—SECTION THROUGH ACOUSTIC TILE. ×12.

[To face page 280

Table of Readings for Acoustic Tile.

Experiment No.	Change of Resistance in Control Grid.	Relative Strength of Source.	Specimen in position			Reflecting plate in position Scale-readings on slider when ohmic change in grid was ρ_1						Mean value of $2y$.	Position of 1st Loop, i.e., $\frac{1}{2}(A+B)$.	Position of 2nd Loop, i.e., $\frac{1}{2}(C+D)$.	$\frac{\lambda}{2}$	ky	α	Temp.
			Deflexion shown by microammeter for ohmic change ρ_1		ρ_2	Near 1st loop.			Near 2nd loop.									
			1st min.	2nd min.		A	B	$2y$	C	D	$2y$							
	ohm.		divs.	divs.	ohm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	cm.	deg.		
I	1	1	2·5 2·5	3.0	8·0	86·05 86·00 86·00	87·50 87·50 87·50	1.45 1·50 1·50	52·80	54·45	1·65	1·53	86·8	53·6	33·2	4·15	0·255	...
II	5	1·8	14 14	14	28·3	86·00 86·00	87·55 87·60	1·55 1·60	52·85	54·50	1·65	1·60	86·8	53·7	33·1	4·35	0·27	...
III	10	2·2	41 40	40 39	38·3	86·00	87·60	1·60	52·85	54·50	1·65	1·625	86·8	53·7	33·1	4·42	0·27	11°C.
IV	15	2·7	60 60	63 62	45·0	86·00 85·95	87·50 87·50	1·50 1·55	52·90 52·90	54·40 54·40	1·50 1·50	1·51	86·75	53·65	33·1	4·11	0·25	...

Mean value of $\alpha = 0{\cdot}26$

$$n = \frac{a}{\lambda} = \frac{33{,}820}{66{\cdot}2} = 511 \text{ vibrations per second.}$$

in a plane perpendicular to the surface which would normally receive the sound. A photograph of a section in a plane parallel to the sound-receiving surface revealed a similar structure, the average proportion of the area occupied by the pores being between 30 and 36 per cent. in each case. The tile is one inch thick.

The details of four determinations of the coefficient of absorption of a sample of this tile are given in the table on page 281.

The figures given in the second column of the table are the ohmic changes which were observed in the resistance of the control grid mounted in the throat of the loud-speaker. By removing this grid and placing it in the holder of a microphone tuned to 511 vibrations per second and calibrating it in a stationary wave, the relative strength (c.c. per sec.) of the source employed in each experiment could be discovered. The figures are given in the third column.

The figures in the seventh, eighth, tenth and eleventh columns are the scale-readings of the slider when the position of the microphone was such that the resistance-change in the grid was equal to ρ_1. Settings were made in the vicinity of the loop nearest the specimen (readings *A* and *B*), and then, as a check, at the second loop, half a wave-length nearer the open end of the pipe (readings *C* and *D*). Generally two or three settings were made in each position, the scale being read to the nearest half millimetre, and the mean of all readings being used to find $2y$. The magnitude of the error in α likely to arise from inaccuracies in the measurement of $2y$ are discussed in Appendix IV.

A number of other determinations, some made with a different sample, gave the same mean value (0·26) for the coefficient of absorption at normal incidence of this type of tile at or near 512 vibrations per second.

This value for the absorption-coefficient is considerably smaller than that quoted for a similar kind of tile by F. R. Watson*—namely, 0·36 at 512 vibrations per second. It is to be noted, however, that the coefficient quoted by Watson was obtained by a reverberation method.

(*b*) *Acoustic Plasters.*—The following table contains the results of tests made on specimens of acoustic plaster submitted by the Building Research Board in 1924. Determinations were made of the absorption coefficients of each specimen at three frequencies—viz., 380, 512 and 650 vibrations per second. Each value given in the table is the mean of four or five separate determinations.

Description of specimen.	Coefficient of absorption.		
	$n=380$	$n=512$	$n=650$
Acoustic Tile 2*A*	0·13	0·26	0·31
Acoustic plaster 4*A*	0·10	0·20	...
Acoustic plaster 5*A* ($3\frac{3}{4}$ parts slag to 1 part magnesite)	0·18	0·25	0·27
Acoustic plaster 6*A* ($4\frac{3}{4}$ parts slag to 1 part magnesite)	0·18	0·27	0·30
Acoustic plaster 7*A* ($6\frac{1}{12}$ parts slag to 1 part magnesite)	0·21	0·31	0·36

All specimens tested were 1 inch thick, and were backed by a thin layer of cement, and either 1 inch of teak wood or three layers of five-ply.

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* "Acoustics of Buildings," p. 25.

The acoustic tile 2*A* was the same as that just described, and its coefficients are reproduced in the table for comparison with those of the acoustic plasters.

The plaster 4*A* was made at the Building Research Station to a specification provided by the Sabine Laboratories in America. The constituents are granulated blast furnace slag and magnesium oxychloride cement. The slag and cement are beaten up with water to a frothy consistency, the included bubbles giving the material its porous and sound-absorbent character. The specification to which this particular specimen of plaster was made has since been considerably modified and improved by the inventor.

The plasters 5*A*, 6*A* and 7*A* are modified forms of the Sabine plaster, and are the subjects of a patent held by the Building Research Board. The constituents are magnesium chloride, granulated slag, magnesium oxide, lime, powdered aluminium and glue. These constituents are beaten up to a froth, the formation of which is assisted by the evolution of gaseous bubbles by the reaction between the lime and the powdered aluminium.

The samples tested show the increase in absorption-coefficient caused by increasing the proportion of slag.

(*c*) *Effect of Varying the Thickness of Acoustic Plasters.*—Three specimens of the acoustic plaster 6*A* were provided by the Building Research Board. These specimens were identical as regards composition, but had different thicknesses—namely, 1, 1½ and 2 inches. The results of tests at 512 vibrations per second were interesting as showing that thickness has a marked influence on the absorption-coefficient. The figures obtained were as follows :—

Thickness of specimen. Inches.	Absorption-coefficient.
1	0·28
1½	0·59
2	0·67

It is clear, therefore, that in making comparative tests it is of the utmost importance to see that the thicknesses of the specimens and their backings are precisely alike.

(*d*) *Effect of Distempering an Acoustic Plaster.*—It is clear that, since acoustic plasters owe their sound-absorbent properties to their porous character, any treatment with paint, varnish or distemper which tends to close up the pores will reduce the amount of sound absorbed. To verify this conclusion, specimens which had been treated with distemper were provided by the Building Research Board. Tests were made at two frequencies ($n=256$ and $n=512$), with the results shown below.

Specimen.		Coefficient of absorption.	
		$n=256$	$n=512$
14*A*	Normal	0·16	0·29
15*A*	One coat of distemper ...	0·06*	0·13
16*A*	Two coats of distemper ...	0·05*	0·11

* These figures are to be regarded as upper limits to the coefficients, since, on account of the small amount of sound absorbed, the measurements became very difficult to take.

(*e*) *Absorption of Sound by Hairfelt.*—The amount of sound absorbed by various thicknesses of hairfelt has been the subject of measurement by Sabine and others, and observations made on two samples of this material are included here. It is obvious, however, that a material such as this is liable to vary considerably in structure and composition, and that no very marked agreement between the observations made by different workers is to be expected.

The felts employed in the tests were cut into circular pieces 12 in. in diameter. A mounting was used similar to that employed with the acoustic plasters, and a preliminary test was made with the mounting empty to ensure that there was no leak of sound. One disc of hairfelt was then fitted in to the mounting, and the absorption-coefficient measured. The measurement was then repeated with two, three or four layers of felt.

The two samples of hairfelt, which will be referred to as " A " and " B," differed somewhat in texture and thickness, though both were nominally ½ in. thick. The following particulars were noted.

Sample A.—Soft, light-coloured felt. Average thickness 0·46 in. (mean of six measurements). Weight per square foot, 8½ oz.

Sample B.—Brown felt, rather coarser texture, and harder than A. Average thickness 0·41 in. (mean of six measurements). Weight per square foot, 7¾ oz.

The sample A was tested at 512 vibrations per second, and sample B at 380 and 512 vibrations per second. The results of the tests are given in the following table :—

Hairfelt.

Number of layers.	Sample A.		Sample B.		
	Approximate thickness.	Coefficient of absorption ($n = 512$).	Approximate thickness.	Coefficient of absorption.	
				$n = 380$.	$n = 512$.
	Ins.		Ins.		
1	0·5	0·21	0·4	0·12	0·17
2	0·9	0·52	0·8	0·28	0·38
3	1·4	0·69	1·2	0·41	0·60
4	1·8	0·69	—	—	—

It may be worth noting that the figures obtained for the felt A are very similar to those found for a ½-in. felt by Sabine* by the reverberation method. This is shown in Fig. 9, in which the coefficients for A are plotted against number of layers employed, and Sabine's results are plotted for comparison. One point of difference may be noted. In the case of the felt A, no appreciable increase in absorption was recorded when four layers were substituted for three layers. The curve plotted from Sabine's results, however, shows that he found a definite increase in absorption.

(*f*) *Cotton Waste.*—The most absorbent surface which has so far been tested consisted of a thickness of 4 in. of loosely packed cotton-waste, held in place by wires stretched across it. This material has been used by Mallett and Dutton to line a sound-chamber in which acoustical experiments were being performed, and they claimed that it was " sufficiently sound-absorbent to prevent any trouble from reflected waves."* Measurements by the stationary-wave apparatus at 512 vibrations per second gave the coefficient of absorption as 0·91.

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* " Collected Papers," p. 213, Fig. 4.

§ 8. CONCLUSION.

The stationary-wave method appears to be one which is well suited to the measurement of absorption-coefficients at normal incidence, when only small specimens of the absorbing material are available. The apparatus described in this Paper is simple and easy to handle, and measurements of absorption can be made very rapidly, the average time required for determining a coefficient, when once the specimen has been prepared, being about 20 minutes. The advantages of the particular instrument (hot-wire microphone) used for comparing the sound amplitudes are its simplicity and easy manipulation, and also the fact that, on account of the microphone being tuned, attention is confined to one particular wavelength. Another advantage is the very small area of the actual receiving surface of the microphone—that is, the cross-sectional area of the orifice. It may be noted that although the reaction of the microphone due to its resonant character has been investigated, no account has been taken of the disturbance which may be caused by the reflexion of sound from the external surface of the microphone. At the frequencies used in the experiments any effect of this kind is probably quite negligible, but at higher frequencies it could not be overlooked. It seems possible that a hot-wire instrument could be constructed which would be sufficiently sensitive for employment up to 1,000 vibrations per second, and yet have dimensions small enough compared with the wavelength for the effect of external reflexion, or scattering, to be ignored. Above this frequency, however, the difficulty of making a small enough instrument rapidly increases, and the same trouble appears to manifest itself with other types of microphone. A possible solution of the difficulty might be found in the employment of a Rayleigh disc, which could be moved about within the pipe. I have tried a disc mounted near the middle of a fine fibre attached at each end to a light metal frame, but so far have not succeeded in obtaining sufficient sensitivity for the arrangement to be of any practical use. A possible alternative is apparatus of the type used by Hawley Taylor, but the disadvantages of moving specimen and source during a set of observations are very great; also it seems likely that the employment of a long sound-duct to lead the sound from the experimental pipe to the measuring instrument may introduce complications into the theory of the method.

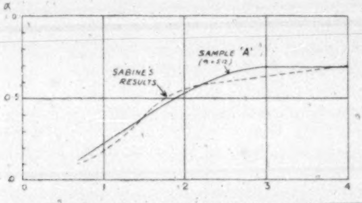

FIG. 9.—ABSORPTION OF SOUND BY HAIRFELT.

NOTE ADDED JANUARY 27, 1927.

The greater part of the work described on the foregoing Paper was completed in the early part of 1924, and was summarised in Report No. 256 (July, 1924), and Report No. 261 (August, 1924) of the Signals Experimental Establishment, Woolwich.

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* Journ. Inst. El. Eng., Vol. 63, p. 504 (1925).

Since that time the work remained practically in abeyance (owing to the demands of other experiments), until the end of 1926, when the above account was written up, and a few additional observations made. At the time of writing I had not seen a Paper by E. A. Eckhardt and V. L. Chrisler, which appeared in April, 1926 ("The Transmission and Absorption of Sound by Some Building Materials," Scientific Papers of the Bureau of Standards, No. 526). In this Paper are given the results of measurements of sound-absorption made with a stationary-wave apparatus resembling that employed by Hawley Taylor (*loc. cit. supr.*). The source of sound was a loud-speaker, and the sound-measuring instrument a telephone, to which the sound was led by a long, narrow exploring tube. The relative amplitudes of the sounds observed were found by comparing the E.M.F.s generated in the telephone by Karcher's method (Sci. Papers of the Bureau of Standards, No. 473, 1923). An advantage of this form of sound-receiver appears to be that it can be used with very high frequencies, Eckhardt and Chrisler quoting absorption-coefficients up to 2,890 vibrations per second, and, in one case, up to 3,210 vibrations per second. The only materials for which they give absorption-coefficients which can be compared with those obtained with the apparatus described above are "Akoustolith" (i.e., the acoustic tile 2A) and 1 in. of hairfelt. For Akoustolith they gave $\alpha=0{\cdot}301$ at 569 vibrations per second, which lies between the values 0·26 at 512 vibrations per second, and 0·31 at 650 vibrations per second, given in § 7 (*b*). It is, however, rather greater than would have been expected, and this may be accounted for by the fact that the specimen appears not to have been backed by a reflecting substance. For hairfelt (1 in. backed by a reflector) they give 0·33 at 297 vibrations per second. This again is rather higher than would be expected, but a comparison is not of much value in the case of this substance.

There is one respect in which the procedure adopted by Eckhardt and Chrisler in making observations differed notably from that described above. A different experimental pipe was used for every frequency employed in the tests, the length of the pipes being such that there was resonance to sound of the particular frequency with which the observations were made. This procedure appears to me to be quite unnecessary and to arise from an incomplete understanding of the theory of the method. The reason given by Eckhardt and Chrisler is, in effect, the following: They say that sound which enters from the source, after passing down the pipe and being reflected from the specimen, returns to the source end, where it may be again reflected with some loss of energy, and that this process may be repeated many times. This, they appear to believe, will vitiate the measurements except in two special cases: (1) When the pipe is in resonance with the sound, and (2) when there is practically no reflexion at the input end. The second case being difficult to realise experimentally, they adopted the first alternative, and constructed pipes to resound to the sound at every frequency at which measurements were made. It should be obvious, however, that when a steady state is reached, the direct and reflected waves coming from the input end will all be reflected by the specimen in the same way and that the net effect is the same as that of a single train of incident and a single train of reflected waves. The *form* of the potential within the pipe is thus determined solely by the reflecting properties of the specimen under test. The *amplitudes* of the pressures and motions, however, are dependent on the length of the pipe and the amount of energy lost at reflexion at the input end. A formal demonstration is not difficult.

Eckhardt and Chrisler also deduce formulæ for calculating the coefficient of absorption when there is loss of energy from the sound during its propagation in the pipe itself. The necessity for taking this loss of energy into consideration appears to arise from the fact that somewhat narrow pipes were used. No dimensions whatever are given in their Paper, but they state (p. 58): "In order to get a tube to resonate satisfactorily its diameter must be relatively small." In the apparatus described above, no appreciable absorption was observed in the pipe itself (*see*, for example, the observations recorded in the table in § 7).

APPENDIX I.

THE DETERMINATION OF THE DAMPING FACTOR OF A HOT-WIRE MICROPHONE.

The stationary-wave apparatus provides an easy means for determining approximately the damping factor of a hot-wire microphone. The procedure is as follows:—

The microphone is placed inside the sound-chamber with the source working steadily at the frequency at which it is desired to measure the damping factor. A series of observations is then made of the ohmic changes in the grid corresponding to different settings of the sliding part of the container. Let Q_1, Q_2, etc., be the internal volumes of the container corresponding to the ohmic changes ρ_1, ρ_2, etc.—the volume being determined in the usual way by weighing the container empty and full of water. Also let Q_m denote the volume for which ρ is a maximum, say, ρ_m.

The microphone is next placed in a stationary wave in the experimental pipe, and the positions found at which the ohmic changes are $\rho_1, \rho_2 \ldots \rho_m$, etc. If these positions are distant $y_1, y_2 \ldots y_m$, etc., from the nearest loop, then the amplitudes of the oscillatory air-currents in the neck of the microphone producing the ohmic changes $\rho_1, \rho_2, \ldots \rho_m$, etc., are proportional in $\sin ky_1$, $\sin ky_2 \ldots \sin ky_m$, etc.

From the theory of the Helmholtz resonator we know that the amplitude of the air-current in the neck, corresponding to a primary potential φ_0, is

$$q_0 = \frac{c}{(\triangle^2 + 4h^2)^{\frac{1}{2}}} \mid \varphi_0 \mid,$$

where c is the conductance of the neck, h is the damping factor, and

$$\triangle^2 = 4\pi^2 n_0^2 \left(\frac{n_0}{n} - \frac{n}{n_0}\right)^2$$

n_0 being the resonance frequency of the resonator and n the frequency of the sound. Hence, if q_a and q_b are the amplitudes corresponding to two different values of n, we have

$$\frac{q_a^2}{q_b^2} = \frac{\sin^2 ky_a}{\sin^2 ky_b} = \frac{\triangle_b^2 + 4h^2}{\triangle_a^2 + 4h^2}$$

so that

$$h = \pm\tfrac{1}{2} \left\{ \frac{\left(\frac{\sin ky_a}{\sin ky_b}\right)^2 \triangle_a^2 - \triangle_b^2}{1 - \left(\frac{\sin ky_a}{\sin ky_b}\right)^2} \right\}^{\frac{1}{2}}$$

If we put $\triangle_a = 0$ (the case for exact tuning) then $\rho_a = \rho_m$, and $y_a = y_m$, and we have

$$h = \pm\tfrac{1}{2}\triangle / \left\{ \left(\frac{\sin ky_m}{\sin ky}\right)^2 - 1 \right\}^{\frac{1}{2}}$$

where $\triangle$ and y correspond to any other tuning of the microphone, the upper or lower sign being closer according as $\triangle$ is positive or negative.

In the expression for $\triangle$ it is to be remembered that n is constant, and equal to $\frac{a}{2\pi}\sqrt{c/Q_m}$, by Rayleigh's formula, a being the velocity of sound in air; while n_0 is varied according to the setting of the sliding part of the container, and is equal to $\frac{a}{2\pi}\sqrt{c/Q}$. Thus

$$\triangle^2=\frac{a^2c}{4\pi^2Q_m}\left\{\left(\frac{Q_m}{Q}\right)^{\frac{1}{2}}-\left(\frac{Q_m}{Q}\right)^{\frac{1}{2}}\right\}^2$$

It is obvious that when Q is near to Q_m, the errors in finding $\triangle$ may be large.

The following figures were obtained by this method with the 512-frequency microphone used for the absorption-tests.

Volume of container.	ky	$\frac{1}{2}\Delta$	h	
c.c.	Degrees.	sec.$^{-1}$.	sec.$^{-1}$.	
11·39	6·45	337·5	79	
11·28	10·5	244·5	86	
11·575	12·1	204·5	73	
11·93	15·05	155	83	
12·325	20·15	104	84	
12·60	27·7	69	109	$\frac{1}{2}\Delta<100$
12·96	32·8	22·5	132	
13·15	33·3 ($=ky_m$)	0	—	
13·25	32·1	13	51	
13·565	27·3	49·5	75	
13·86	20·7	84·5	71	
14·49	13·95	188	76	
15·33	10·5	246	84	

When $\frac{1}{2}\triangle$ is small, the results are somewhat erratic, as would be expected. If these values of h for which $\frac{1}{2}\triangle$ is less than 100 be excluded, the mean of the remaining seven determinations is 81 sec.$^{-1}$.

APPENDIX II.

THE REACTION OF A TUNED MICROPHONE ON A STATIONARY SOUND-WAVE IN A PIPE.

The theory of the reaction of the tuned hot-wire microphone on the stationary sound-waves within the experimental pipe can be dealt with as a special case of the theory of the "Boys" type of double resonator. This theory* shows that if $q_0e^{\iota kat}$ represents the oscillatory air-current (c.c. per sec.) in the neck of a Helmholtz resonator of the type used in hot-wire microphones, placed at a distance l within a stopped pipe of length L, due to a primary potential $2Fe^{\iota kat}$ at the mouth of the pipe, then

$$q_0=\frac{c}{2h-\iota\triangle}\quad\frac{(\beta-1)F}{\beta e^{-\iota kL}-\mu e^{\iota kL}}\left\{e^{\iota k(l-L)}+\mu e^{-\iota k(l-L)}\right\}\quad.\quad\text{(i)}$$

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* Phil. Mag., Vol. II., pp. 751-769 (1926).

where c = the conductance of the orifice of the resonator;
h = the damping-factor of the resonator;
$\triangle = 2\pi n_0(n_0/n - n/n_0)$;
n_0 = the resonance-frequency of the resonator;
n = the frequency of the primary potential;
β = the (complex) coefficient of reflexion of sound-waves, coming from within the pipe, at the open end;
$k = 2\pi/\lambda = 2\pi n/a$;
a = the velocity of sound in air.

The factor μ occurring in (i) represents the "reaction" of the Helmholtz resonator on the potential of the stationary waves within the pipe. For the theory of the Helmholtz resonator shows that the value of q_0 corresponding to a primary potential $\varphi_0 e^{\iota kat}$ is

$$\frac{c}{2h - \iota\triangle} \mid \varphi_0 \mid, \quad \ldots \ldots \ldots \quad \text{(ii)}$$

so that (i) shows that the effective potential in the pipe is

$$\frac{(\beta-1)F}{\beta e^{-\iota kL} - \mu e^{\iota kL}}\left\{e^{\iota k(l-L)} + \mu e^{\iota k(l-L)}\right\} \quad \ldots \ldots \quad \text{(iii)}$$

It can be shown that, in the absence of the resonator, the potential at the same place in the pipe would be

$$\frac{(\beta-1)F}{\beta e^{-\iota kL} - e^{-\iota kL}}\left\{e^{\iota k(l-L)} + e^{-\iota k(l-L)}\right\}$$

$$= \frac{2(\beta-1)F}{\beta e^{-\iota kL} - e^{\iota kL}} \cos k(l-L). \quad \ldots \ldots \ldots \quad \text{(iv)}$$

Hence, comparing (iv) with (iii), we see that the deviation of μ from unity represents the reaction of the resonator on the potential within the pipe.

The value of μ is—

$$\frac{1 + \dfrac{ac}{\sigma(2h - \iota\triangle)} \cos k(l-L) \,.\, e^{\iota k(l-L)}}{1 - \dfrac{ac}{\sigma(2h - \iota\triangle)} \cos k(l-L) \,.\, e^{-\iota k(l-L)}} \quad \ldots \ldots \ldots \quad \text{(v)}$$

where σ is the cross-sectional area of the pipe.

In applying these expressions to the case of the tuned microphone in the experimental pipe, we note first of all that since the Helmholtz resonator which forms the tuned part of the hot-wire microphone is in tune with the sound we have $\triangle = 0$. Also, it is more convenient to measure distances from the stopped end of the pipe, and we therefore put $l - L = -x$, x being the distance from the stopped end. When these alterations have been made, the expression for q_0 and μ become

$$q_0 = \frac{c}{2h} \frac{(\beta-1)F}{\beta e^{-\iota kL} - \mu e^{\iota kL}}\left\{e^{-\iota kx} + \mu e^{\iota kx}\right\}, \quad \ldots \ldots \quad \text{(vi)}$$

$$\mu = \frac{1 + \dfrac{ac}{2h\sigma} \cos kx \,.\, e^{-\iota kx}}{1 - \dfrac{ac}{2h\sigma} \cos kx \,.\, e^{\iota kx}} \quad \ldots \ldots \ldots \ldots \quad \text{(vii)}$$

The case of no reaction is arrived at by making $c_0 \rightarrow 0$, in which case $\mu \rightarrow 1$, and

$$q_0 \rightarrow \frac{c}{2h} \frac{2(\beta-1)F}{\beta e^{-\iota kL} - e^{\iota kL}} \cdot \cos kx \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(viii)}$$

Now, (vi) can be written

$$q_0 = \frac{c}{2h} \frac{2\,(\beta-1)F}{\beta e^{-\iota kL} - e^{\iota kL}} \cdot \frac{1}{2} \left\{ e^{-\iota kx} + \mu e^{\iota kx} \right\} \frac{\beta e^{-\iota kL} - e^{\iota kL}}{\beta e^{-\iota kL} - \mu e^{\iota kL}} \quad . \; . \; . \; . \; . \quad \text{(ix)}$$

Hence, comparing (viii) with (ix), we see that, to estimate the effects of reaction on the amplitude of the air current in the node of the resonator, we must compare the modulus of

$$\frac{1}{2} \left\{ e^{-\iota kx} + \mu e^{\iota kx} \right\} \frac{\beta e^{-\iota kL} - e^{\iota kL}}{\beta e^{-\iota kL} - \mu e^{\iota kL}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(x)}$$

with cos *kx*.

It has been assumed in the above calculation that the primary potential $2F$ is constant and independent of the position of the resonator within the pipe. In the case of the apparatus as arranged for absorption measurements, however, this is not strictly true, unless special precautions are taken. The source of sound (loud-speaker) being close to the open end of the pipe is easily influenced by changes in the disposition of the apparatus within the pipe. Thus, if the source of electrical supply to the loud-speaker remained untouched, it was found, by observation of the resistance of the control grid in the throat of the loud-speaker, that the strength of the source of sound underwent changes as the microphone was moved along the axis of the pipe. These changes were quite small (of the order of 0·01 ohm), and could easily be compensated for by adjusting the electrical supply so that the resistance of the control grid was always the same when a reading was being taken.

For a numerical calculation of the effects of reaction in the case of the apparatus employed for absorption measurements at 512 vibrations per second, we have the following values for the various quantities in the expressions given above :—

$c = 0{\cdot}114$ cm.

$h = 81$ sec.$^{-1}$.

$a = 33{,}760$ cm./sec.

$\sigma = \frac{1}{4}\pi \times (30{\cdot}5)^2$ cm.^{2}.

$L = 226$ cm.

$k = 2\pi/\lambda = 2\pi \times 412/33{,}760$ cm.$^{-1}$.

μ is first calculated by means of (v), for $kx = 0°$, $15°$, $30°$, etc., up to $90°$.

kx	μ
0°	1·0724
15°	$1{\cdot}0625 + \iota \times 0{\cdot}00052$
30°	$1{\cdot}0500 + \iota \times 0{\cdot}000723$
45°	$1{\cdot}0331 + \iota \times 0{\cdot}000549$
60°	$1{\cdot}0163 + \iota \times 0{\cdot}000232$
75°	$1{\cdot}0045 + \iota \times 0{\cdot}000036$
90°	1·0000

If the calculation is continued between $kx=90^\circ$ and $kx=180^\circ$ by steps of 15°, it is found that the numerical values of the real and imaginary parts of μ are the same for 105°, 120°, etc., as for 75°, 60°, etc., but the sign of the imaginary part is reversed. Thus, when $kx=105^\circ$, $\mu=1{\cdot}0045-\iota\times0{\cdot}000036$. The result of this is that the effects are not quite symmetrical about a loop position ($kx=90^\circ$), but the difference is very small, as will be seen from the figures given below.

The next step is to calculate $\frac{1}{2}(e^{-\iota kx}+\mu e^{\iota kx})$ and its modulus, use being made of the value of μ given in the above table.

kx	$\frac{1}{2}(e^{-\iota kx}+\mu e^{\iota kx})$.	Modulus.
0°	1·0362	1·0362
15°	$0{\cdot}9961+\iota\times0{\cdot}0083$	0·9961
30°	$0{\cdot}8874+\iota\times0{\cdot}0128$	0·8875
45°	$0{\cdot}7176+\iota\times0{\cdot}0119$	0·7187
60°	$0{\cdot}5040+\iota\times0{\cdot}0071$	0·5040
75°	$0{\cdot}2594+\iota\times0{\cdot}0021$	0·2594
90°	0	0

It will be seen that the effect of the imaginary part is quite negligible. If the calculation is continued between $kx=90^\circ$ and $kx=180^\circ$, the sign of the real part becomes reversed.

The remainder of the calculation of reaction effects consists (*see* (x)) in evaluating

$$\frac{\beta e^{-\iota kL}-e^{\iota kL}}{\beta e^{-\iota kL}-\mu e^{\iota kL}}, \quad \ldots \ldots \ldots \quad \text{(xi)}$$

or,

$$\frac{(\beta-1)\cos kL-\iota(\beta+1)\sin kL}{(\beta-\mu)\cos kL-\iota(\beta+\mu)\sin kL}. \quad \ldots \ldots \quad \text{(xii)}$$

In order to do this it is necessary to decide on a numerical value for β—that is for the coefficient of reflexion of sound-waves from the open end of the pipe next to the source.

In the case of a pipe the diameter of which is small compared with the wave-length of the sound the coefficient of reflexion from a flanged open end is*

$$\beta=-\frac{1-\iota k\sigma\left(\frac{1}{c_0}-\frac{\iota k}{2\pi}\right)}{1+\iota k\sigma\left(\frac{1}{c_0}-\frac{\iota k}{2\pi}\right)}, \quad \ldots \ldots \ldots \quad \text{(xiii)}$$

where c_0 (the "conductance" of the open end) is approximately equal to $3{\cdot}8\times$the radius of the pipe.

The diameter of the pipe, however, in the present experiments was of the order of half a wavelength (actually 0·46 of a wavelength at 512 vibrations per second), and the above expression for β can no longer be regarded as strictly applicable.

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* cf. Phil Mag., Vol. II, p. 756 (1926).

In the absence, however, of a more satisfactory theoretical expression for β it was assumed that (xiii) remained of the same form for wider pipes and that the effect of the increased wideness could be allowed for merely by choosing a different value for c_0.

In order to find an appropriate value for c_0 the following procedure was resorted to. The proportion of sound which escaped from the open end of the experimental pipe (away from the source) was determined experimentally by the method used for measuring absorption. The results of the observation showed that the amplitude of the wave reflected from the open end was about $\frac{1}{5}$ of that of the incident wave, that is, that the modulus of β was about $\frac{1}{5}$. It was then found by trial that the value of c_0 which must be inserted in (xiii) in order to give this value of mod β was about $10R$ (R=radius of pipe).

This value of c_0 is to be looked upon as upper limit, for it is the escape of sound from the open end of the pipe next to the source of which account has to be taken, and this must be considerably impeded by the loud-speaker trumpet, and the walls and floor of the sound-chamber. It seemed probable, from an inspection of the apparatus, that about half this value, that is, about $5R$, would be nearer the truth. On account of the uncertainty as to the proper value which should be taken for c_0, the calculation of (xii) was made for three different values of β corresponding respectively to $c_0=3R$, $c_0=5R$, and $c_0=7R$. The results, which are given in the following tables, show that the modulus of (xii) is not greatly affected by taking different values for β within the range shown in the first table.

TABLE *of* β.

c_0	β
$3R$	$0\cdot3715+\iota\times0\cdot4652$
$5R$	$0\cdot1875+\iota\times0\cdot3609$
$7R$	$0\cdot1161+\iota\times0\cdot2804$

TABLE *of Modulus of* (xii).

kx	Modulus of (xii).		
	$c_0=3R$.	$c_0=5R$.	$c_0=7R$.
0°	0·952	0·947	0·944
15°	0·958	0·954	0·951
30°	0·966	0·963	0·961
45°	0·978	0·975	0·974
60°	0·989	0·988	0·987
75°	0·997	0·997	0·996
90°	1·000	1·000	1·000

The final result of the calculation is shown in the following table, where $C(kx)$ is used to denote the quantity indicated in (x). $|C(kx)|$ has been calculated for the case when $c_0=5R$; but it can easily be seen, by reference to the preceding table, what the effect would be of making c_0 equal to $3R$ or $7R$.

kx	$\lvert C(kx) \rvert$	$\lvert \cos kx \rvert$
0°	0·9809	1·0000
15°	0·9501	0·9659
30°	0·8544	0·8660
45°	0·7006	0·7071
60°	0·4977	0·5000
75°	0·2585	0·2588
90°	0·0000	0·0000
105°	0·2585	0·2588
120°	0·4976	0·5000
135°	0·7004	0·7071
150°	0·8539	0·8660
165°	0·9495	0·9659
180°	0·9809	1·0000

It will be seen that the greatest deviation of $|C(kx)|$ from $|\cos kx|$ is at the nodes ($kx=0°$ and $180°$), where it amounts to just under 2 per cent. When $kx=45°$ it is less than 1 per cent., and falls to zero when $kx=0°$. The influence of this deviation on the measurement of absorption-coefficients is quite unimportant, and for all practical purposes it is eliminated by the procedure described in § 5.

In calculating $|C(kx)|$, four figures have been retained in order to show the slight asymmetry which exists about the loop position ($kx=90°$), due to the reversal of the sign of the imaginary part of μ which occurs when kx passes through the value 90°.

APPENDIX III.

REACTION OF A TUNED MICROPHONE ON A PROGRESSIVE WAVE IN A PIPE.

Let the microphone lie in the plane $x=0$, and let the potential of the incident wave be $\varphi=Ae^{\iota k(at-x)}$. The presence of the microphone will give use to a reflected wave of small amplitude $\varphi'=A'e^{\iota k(at+x)}$, while the amplitude of the wave transmitted across the plane $x=0$ will be less than $|A|$. Let the transmitted wave be $\psi=Be^{\iota k(at-x)}$.

On the assumption that the disturbance caused by the microphone causes a departure of the waves from their plane form only for a very short distance on either side of the plane $x=0$, the conditions to be satisfied by φ, φ' and ψ when $x=0$ are

$$\varphi+\varphi'=\psi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

$$\sigma\left\{\frac{d\psi}{dx}-\frac{d}{dx}(\varphi+\varphi')\right\}=\frac{dq}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

$$\frac{d^2q}{dt^2}+2h\frac{dq}{dt}+4\pi^2n_0^2q=c\frac{d\psi}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (c)$$

$\frac{dq}{dt}$ is the current, directed inwards, through the orifice of the microphone. For exact tuning, (c) becomes

$$\frac{dq}{dt}=\frac{c}{2h}\cdot\frac{d\psi}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (d)$$

Hence, substituting for ϕ, ϕ' and ψ the values given above, and eliminating $\frac{dq}{dt}$, we find

$$A+A'=B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

$$A-A'=\left(1+\frac{ac}{2h\sigma}\right)B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

Thus

$$2A=\left(2+\frac{ac}{2h\sigma}\right)B,$$

and

$$B=A\Big/\left(1+\frac{ae}{4h\sigma}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (g)$$

Therefore the effect of the reaction of the microphone is to reduce the original potential A to an effective potential $A\Big/\left(1+\frac{ac}{4h\sigma}\right)$. From the figures given in Appendix II we find that $1+\frac{ac}{4h\sigma}=1{\cdot}016$.

APPENDIX IV.

Error Due to Inaccurate Setting of the Slider.

As stated in § 7, the position of the microphone in the experimental pipe could be read on the slider to the nearest half-millimetre. There is, therefore, a possible error in $2y$ of $\pm 0{\cdot}05$ cm., and in y of $\pm 0{\cdot}025$ cm. To find the effect of this error on the coefficient of absorption we differentiate the expression (5.3) of § 5 and obtain

$$\frac{da}{dy}=\frac{1}{4}\,ka^2\cos ky\,.\,\cot^2 ky,$$

and hence, for an increment $\delta y=0{\cdot}025$ cm., approximately,

$$\delta a=\frac{1}{4}\,ka^2\cos ky\,.\,\cot^2 ky\times 0{\cdot}025.$$

Some values of δa for various values of a are shown in the following table. It is assumed that the frequency of the sound is 512 vibrations per second.

ky	α	$d\alpha/dy$	$\delta\alpha$ ($\delta y=0{\cdot}025$ cm.)
1°	0·070	0·379	0·0095
2°	0·130	0·330	0·0083
4°	0·244	0·287	0·0072
6°	0·343	0·251	0·0063
8°	0·429	0·219	0·0055
10°	0·504	0·191	0·0048

Thus, when $a=0{\cdot}25$, the possible error in a due to an error $\pm 0{\cdot}025$ cm. in y is $\pm 0{\cdot}007$. The error is, of course, reduced by taking the mean of a number of determinations.

The arrangements for setting and reading the slider in these experiments could readily be improved and more accurate values of y, and hence of a, could be obtained than have hitherto been attempted.

It was ascertained, for example, that when $y=0{\cdot}75$ cm. a perceptible change in resistance could be produced by moving the microphone a distance of only 0·01 cm., a movement which could not be read on scale on the slider.

Errors in the readings arise principally from: (1) The difficulty of setting the position of the microphone by operating the slider by hand; (2) the unevenness of the bottom of the experimental pipe, which tended to tilt the microphone slightly in some positions, and thus gave rise to false scale-readings on the slider; (3) slight lateral movements of the slider.

The table given above indicates that the errors in α due to inaccuracies in measuring y may be considerable when α is 0·1 or under. On the other hand, when α is above 0·5 small errors in y are not of much importance.

DISCUSSION.

Dr. A. H. **Davies** inquired what type of loud-speaker had been used? The hot-wire microphone had the disadvantage that it could not be used for the higher speech frequencies, and at the National Physical Laboratory it had been found convenient to use as detector a magnetic telephone with electric circuits whose resonance could be varied at will.

Mr. R. S. **Maxwell** said that Sabine had found that the coefficient of absorption varies with the intensity of the sound employed, and had taken as standard an intensity one million times as great as that for the threshold of audibility. He had an impression that Sabine's results for felt were taken as 256 cycles, whereas those of the author and others were taken at 512. If so, the quantities plotted in Fig. 9 were not comparable, since some materials have a resonance of their own and the absorption consequently varies with frequency.

Dr. E. G. **Richardson** urged the value of non-resonant measuring apparatus. A simple hot-wire would avoid resonance and also enable the size of the pipe to be reduced. He had used the stationary-wave method with the hot-wire anemometer for measuring absorption coefficients (Proc. Roy. Soc., A, 112, 522, 1926).

Major W. S. **Tucker** said that the size of the apparatus was determined by the size of the specimens to be tested. He would like to join in congratulating the author: he had been impressed by the speed with which a variety of specimens could be tested in succession.

Prof. E. N. **da C. Andrade**: An interesting method of measuring sound intensities is used by Messrs. Siemens & Halske, of Berlin. When sound issues from a small opening there is a partial separation of the two opposite phases of the vibration, so that at certain places there is on the whole a pressure, at others a suction effect. Just in front of the opening there is formed a narrow jet of air, whose energy depends on the energy of the sound vibration, and this jet is powerful enough to blow aside a light suspended system (of no particular sensitiveness) so as to produce deflections giving a quantitative measure of the sound intensity. The jet may also be used in conjunction with a bolometer or hot-wire microphone.

Mr. L. E. C. **Hughes** said that, when working with Dr. Mallett, he had measured the attenuation of sound along the axis of a loud-speaker in a region screened with cotton waste, assuming an inverse square law for the geometrical attenuation. An energy loss of 12 per cent. on reflection had been found at a frequency of 500 cycles, the loss increasing with frequency.

Dr. A. B. **Wood** asked whether the specimen discs vibrate like a diaphragm? Such behaviour would have a considerable effect on absorption: the point could be tested by varying the diameter as well as the thickness of the discs.

The **Author** said that a loud-speaker of the electromagnetic type was used in the experiments. In reply to Mr. Maxwell, the intensity of the sound could only be varied over a limited range, and no variation of the coefficient of absorption had been detected within this range. The quantities plotted in Fig. 9 were taken from Sabine's results for felt at 512 vibrations per second. With regard to the point raised by Dr. Wood, care had been taken throughout the experiments to test the specimen-holders before the materials were inserted. No appreciable loss of energy could be observed with the holders alone, and it was inferred that any vibration of the discs as diaphragms was quite unimportant. A device of the type mentioned by Prof. Andrade was used some years ago by Mr. E. S. Player, of the Air Defence Experimental Establishment, for exploring the resonance characteristics of pipe resonators. It consisted of a fine hole in the wall of the resonator, the issuing jet impinging on a hot-wire microphone grid.

XXII.—A BALL AND TUBE FLOWMETER SUITABLE FOR PRESSURE CIRCUITS.

By J. H. Awbery, *B.A.*, *B.Sc.*, and Ezer Griffiths, *D.Sc.*, *F.R.S.*, Physics Department, The National Physical Laboratory.

Received February 11, 1927.

Abstract.

The Paper describes a robust form of the Ewing Ball and Tube Flowmeter, suitable for the metering of gases or liquids under pressure, as, for example, the ammonia in a refrigerating plant. The necessary pressure-tight joints for connecting the conical tube to the circuit are described, and also a device for cutting off the flow should the tube fail. An investigation is included of the behaviour of the instrument under practical conditions of pulsating flow.

A STUDY has been made recently of the Ewing type of flowmeter, with a view to adapting it for use in metering fluids under pressure (for example, the ammonia in a refrigerating machine).

This flowmeter consists of a sphere in a conical tube whose narrower end is downward; the fluid to be metered enters at the lower end, and carries the ball up the tube to a point determined by the velocity of the fluid stream. Thus the height of the ball gives a measure of the instantaneous rate of flow, provided the tube has been calibrated for that fluid.

Experience shows that, if the tube is vertical, the ball may take up various types of motion, the height being different for the same flow, according to the type. This effect, which would be a source of difficulty in practical applications, may be eliminated by inclining the tube, in which case the ball rolls on the wall of the tube, and is restricted to one type of motion, to which, of course, corresponds one definite calibration.

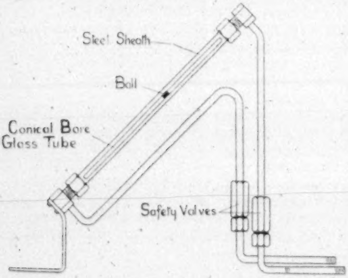


Fig. 1.

An instrument designed for use with liquids under pressure is illustrated in Fig. 1. The conical tube is of heavy walled glass tubing, and is sheathed in thick steel with two longitudinal slits, so that, should the glass give way, the fragments cannot be projected about. The junctions between the glass tube and the rest of the apparatus are the only other points at which special precautions are needed for dealing with high pressures.

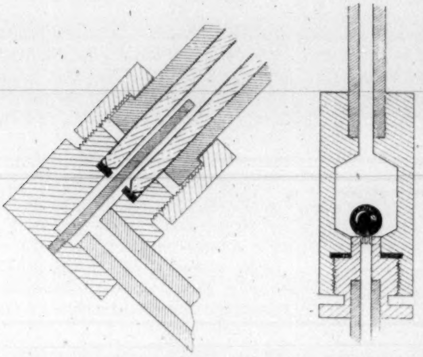
The ends of the glass tube project slightly beyond the sheath, and are ground to a bevel edge. The steel sheath is provided with a collar, and is attached to the connecting tube by an ordinary union nut, which forces the end of the glass tube down to a lead washer, thus making a pressure-tight joint. A small metal peg projects in the centre, to prevent the ball leaving the flowmeter tube.

The device for cutting off the flow in the event of breakage is inserted in the connecting tubes, and consists of a ball valve with a relatively large chamber, the valve seating being uppermost. Normally, the ball remains at the bottom of the chamber, the liquid flowing round it, but should a sudden rush of fluid occur the ball is forced upwards into its seating, so cutting off the flow.

LIMITATIONS OF THE INSTRUMENT.

The ball and tube meter shares with the Venturi the disadvantage of having one point where the passage for the flow of liquid is greatly constricted. This involves a considerably increased velocity at this point, and therefore by Bernoulli's theorem the pressure there is much lower, so that with a highly volatile liquid, such as liquid ammonia, there would be danger of evaporation and consequently erroneous readings.

Another difficulty is one which the meter shares with all types which measure rates of flow and not total quantities—viz., if the flow varies periodically, as is the case if the liquid is driven by certain types of pump, the position of the ball is continually varying, and the question arises as to whether the mid-point of its path may be taken for deducing the rate of flow. If there is no time lag between the establishment of a rate of flow, and the taking up of the corresponding position by the ball, it may be anticipated that the position of the ball which corresponds to the actual mean rate of flow will be the time-average of its path; this will also be true in the case of periodical variations, provided they are truly cyclic, since the displacement-time curve for the ball will be similar to the curve giving rate of flow against time for the liquid, but displaced along the time axis (principle of forced vibrations).



(A) Pressure-tight Joint. (B) Ball Valve.

FIG. 2.—INDUSTRIAL FORM OF INSTRUMENT.

An experimental study of this phenomenon has been carried out. A slow, single acting, one plunger pump was used to pump water round a circuit in which was included a ball and tube meter. The variations were much larger than would usually be encountered in practice, the period of the oscillation being 2 seconds, and the amplitude of the motion of the ball 6 inches, nearly half the length of the meter tube.

Cinematograph photographs of the ball were taken, so as to obtain a record of its position at a uniformly distributed series of times, and the film records measured up. From this a time-position curve was constructed, and is shown in Fig. 3, which is the mean of a number of successive " waves." It will be seen that the motion obtained was distinctly unsymmetrical, so that the case chosen is an extremely severe test of the error which may be introduced by this cause. Nevertheless, the following table shows that the time-average position corresponds very closely to the mean rate of flow measured experimentally, whilst the mid-position would lead to a considerable error if accepted as the reading. In general, the difference between the time-average and the mid-position would be but a small fraction of its value in this case, and no error would arise in most practical cases from a varying rate of flow.

Position of ball.	Height in tube (cm.).	Corresponding rate of flow, deduced from calibration for steady flows (c.c. per sec.).
Highest	37·2	9·3
Lowest...	19·15	4·3
Mid-point	28·2	6·6
True mean	26·5	6·1
(Actual rate of flow)...	—	6·0

The investigation described in this Paper arose out of an inquiry by the Engineering Committee of the Food Investigation Board into the general question of meters suitable for use with refrigerating plants. The chairman, Sir Alfred Ewing, referred to this particular type of meter as one which merited further study, and we are indebted to him for his keen interest in the work. Our thanks are also due to Mr. W. F. Higgins, M.Sc.,, who took the cinematograph film of the motion of the ball under pulsating flow. Mr. A. Snow, observer in the Physics Department, has rendered valuable assistance with the design and construction of the industrial form described.

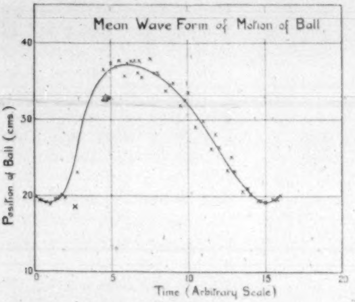


FIG. 3.

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Further Experiments with the Ewing Flowmeter, by J. H. Awbery and Ezer Griffiths, Proc. Roy. Soc., Edin., December (1926).

DISCUSSION.

Prof. E. N. DA C. ANDRADE said that he had seen an instrument with a conical float, giving readings free from ambiguity, used at Heidelberg in 1911 under the name of "rotameter." The float had only one position of stability for a given velocity of flow.

Prof. F. L. HOPWOOD said that it is entertaining to watch the behaviour of a number of balls in a cylindrical tube up which water was flowing vertically. Thus two balls will remain close together and roll on one another; if three are used, the outer two remain at some distance apart and the middle one oscillates between them ; while larger numbers arrange themselves in various other ways. If a cylinder be used instead of a ball it will set itself obliquely across the tube and revolve about the axis of the latter.

Mr. W. A. **Benton** said that he had recently been interested in engineering measurements in which it had been found that the readings of a rotameter are greatly affected by changes in viscosity. The present paper would enable such instruments to be used with the security conferred by fuller knowledge.

Dr. E. H. **Rayner** referred to a flowmeter at least 40 years old, consisting of a conical tube having a diaphragm perforated with a central hole, and a piston whose stem projects downwards through this hole.

Dr. **Griffiths'** reply to the discussion : We have not had any experience with the instrument referred to by Prof. Andrade.

A useful feature of the ball and tube meter is the ease with which suitable balls can be obtained ; ball bearings are available in diameters from $\frac{1}{32}$ in. in diameter up to $\frac{1}{2}$ in. in steps of $\frac{1}{64}$ in., from $\frac{1}{2}$ in. to 1 in. in steps of $\frac{1}{32}$ in., from 1 in. to 2 in. in steps of $\frac{1}{16}$ in., etc. Balls are also available in metric sizes which help to fill in gaps between these sizes.

Then again it is a simple matter to make a hollow glass sphere of approximately the desired weight for metering gas flow. A feature of the instrument not mentioned in the Paper is the possibility of extending its range by the use of two spheres of different sizes ; the lower range is covered by the larger sphere and when this is displaced beyond the working length of the tube the smaller sphere comes into view.

In the other Paper to which a reference is given, a detailed account will be found of the theory which takes account of the effect of viscosity, etc.

The meter referred to by Dr. Rayner is extensively used for measuring water. It was invented for metering waste water a long time ago by G. F. Deacon, and improved by Lord Kelvin, who introduced the integrating device.

XXIII.—A GAS ANALYSIS INSTRUMENT BASED ON SOUND VELOCITY MEASUREMENT.

By Ezer Griffiths, *D.Sc., F.R.S.*, Physics Department, National Physical Laboratory.

ABSTRACT.

The instrument described is based on a principle which has not hitherto been utilised in the design of gas analysis instruments. A quartz crystal is maintained in vibration piezo-electrically, and stationary waves are set up in the gas between the flat surface of the crystal and a movable reflector. The position of the nodes is recognised by the reaction on the quartz crystal, resulting in an increase of the current in the maintaining circuit. The distance from node to node is a measure of the composition of the gaseous mixture, assuming it is composed of two gases which do not react.

THE instrument described in this Paper is based on a principle which has not hitherto been utilised in the design of gas analysis instruments. Recent work on the piezo-electric resonator by Nicolson, Cady, Pierce and Dye has shown that a quartz crystal is an excellent frequency standard when maintained in vibration by an alternating E.M.F. of the right frequency. Pierce used such a piezo-electric crystal to measure the velocity of sound in different gases by the stationary wave method.

Since the velocity of sound in air is about 332 metres per second, and that in carbon dioxide about 259 metres per second, it is obvious that, if the velocity of sound in a mixture of carbon dioxide and air can be measured with sufficient accuracy, it should afford a method of analysing the gaseous mixture. Such a physical measurement has one great advantage in that it does not involve the absorption of one constituent out of the mixture, and thus alter the composition.

It is not claimed that it is to be preferred to the simple direct chemical method for ordinary routine tests, but is simply put forward as a supplementary method for use in special cases. It might, for example, be useful in making measurements of the composition of the atmosphere in a small enclosure without opening the enclosure, when carbon dioxide and air alone are present.

The instrument to be described has the merit that its accurate functioning does not depend on the constancy of the calibration of electrical instruments, for the operation resolves itself into a length measurement—the displacement of a reflector from one position to another, which is determined by the deflection of an electrical instrument used solely as an indicator of successive maxima.

A photograph of the instrument is shown in Fig. 1, and the crystal is shown removed from the brass cylinder in Fig. 2. A diagram of the electrical circuit is given in Fig. 3.

The quartz crystal, by its longitudinal vibration, produces a train of waves in the air of frequency of about 40,000 per second, which is well above the range of audible frequency. In the present apparatus the block of quartz measures approximately $6{\cdot}5 \times 3{\cdot}6 \times 2$ cms. The block is lightly held between two brass electrodes, and it is brought into a state of vigorous longitudinal vibration when the electrodes are connected to an oscillatory valve circuit. The train of sound waves sent out

FIG. 1.

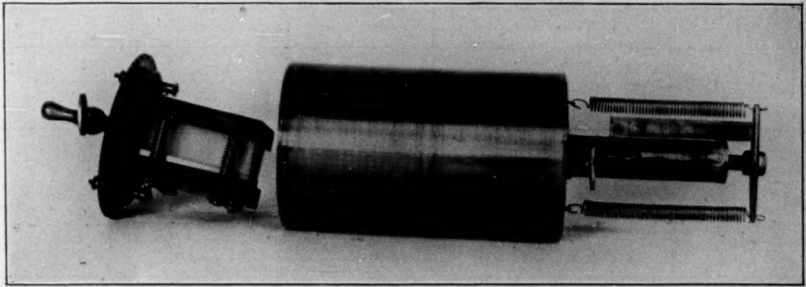

FIG. 2.

To face page 300]

from the crystal face are reflected back from the plane face of a reflector carried, by a micrometer screw. When the reflector is at such a distance that it corresponds to a node, the reaction of the reflected wave on the crystal results in a sharply defined peak in the anode current.

To illustrate the magnitude of the change, the following results may be quoted:— When this crystal was oscillating normally there was a current of half a milliamp. flowing between the anode and the filament. This current increased suddenly to about one and a half milliamp. when the amplitude of the oscillations of the crystal was reduced due to the reaction of the reflected wave on the crystal face. If the instrument is working normally, the position of the reflector can be defined to within one-fiftieth of a millimetre. Pierce gives a curve showing the relation between anode current and the position of the reflector, which illustrates the sharpness with which successive nodes are defined. The displacement of the reflector between successive nodes is of the order of 4 millimetres, and the change in wavelength from carbon dioxide to air is about 1 millimetre, so an accuracy of 2 per cent. can be obtained on one setting. An inspection of Fig. 4 will show that considerably greater accuracy is obtained when the mean of several successive displacements is determined.

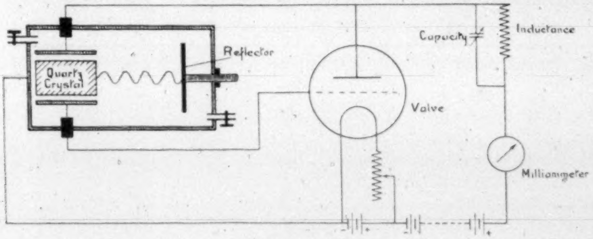

FIG. 3.

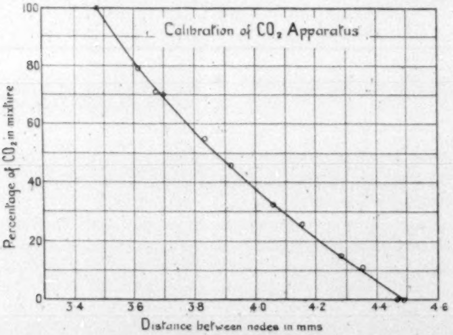

FIG. 4.

The present instrument has been calibrated empirically by measurements on various carbon dioxide-air mixtures, and the calibration curve obtained is shown in Fig. 4. It will be noted that this curve applies only to the particular instrument studied, for the calibration would take account of errors in the micrometer, screw, etc.

The velocity of sound in a gas is a function of temperature, and is given by the formula

$$v=v_0\sqrt{1+\alpha t}$$

where v is the velocity at temperature t, v_0 is the velocity at temperature 0°C., α is the coefficient of expansion of the gas at constant pressure.

Now for any gas $\alpha=0{\cdot}00366$ (approx.), or 1/273 per 1°C. nearly. For air $v_0=33{,}150$ cm./sec. Hence, from the above formula

$$v=(33{,}150+61t)\ \text{cm./sec.}$$

If d is the distance between successive nodes when the gas sample is at temperature t

$$d=\frac{\lambda}{2}$$

where λ is the wavelength of the sound.

Let n be the frequency, which for quartz may be taken as a constant for moderate changes of temperature.

Then $$v=n=2nd.$$

So that d increases by 0·18 per cent. per 1°C. rise of temperature.

The table below shows the value of the velocity of sound in different gases (at 0°C.), and hence indicates the various combinations of two gases which can be analysed by means of a sound velocity determination, assuming, of course, that the two gases taken do not react chemically.

Gas.	Velocity, cms./sec.
Air	$3{\cdot}32\times10^4$
Hydrogen	12·86
Oxygen	3·17
Nitrous oxide	2·60
Ammonia	4·16
Carbon monoxide	3·37
Carbon dioxide	2·59*
Coal gas	4·9 to 5·15
Sulphur dioxide	2·09

One peculiarity of the instrument should be noted. The surface of the crystal has to be kept scrupulously clean and free from moisture deposit, otherwise the crystal will not oscillate satisfactorily. Consequently, the instrument is sometimes troublesome to set in operation.

The work was carried out for the Engineering Committee of the Food Investigation Board, who kindly sanctioned the publication.

In conclusion, the author desired to acknowledge his indebtedness to Dr. Dye, who kindly supplied the crystal and gave much helpful advice. His thanks are also due to Mr. A. Snow, Observer in the Physics Department, for his skill in the construction of the apparatus.

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* Pierce.

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DISCUSSION.

Dr. A. H. **Davis** asked how the author's results compare with theoretical results, such as the ratio of the specific heats of a gas. He thought that the word "sound" should not be used for supersonic disturbances. It might seem that increased accuracy could be obtained by lowering the frequency; but a limit would be set in this direction by loss of sharpness due to curvature of the wave front.

Mr. C. R. **Darling** asked whether the apparatus would be sufficiently accurate for use in estimating the percentage of CO_2 in flue gases (in the neighbourhood of 16 per cent.) for the control of stoking.

Mr. J. H. **Awbery**: I think I can probably answer one of the questions raised by Dr. Davis. The figures given in the Paper do not include the exact frequency of the oscillations, so that the actual values of γ, the ratio of the specific heats, cannot be calculated; but from the calibration curve it is easy to obtain numbers proportional to them. I find that if the (moist) air is taken as unity, then the 100 per cent. mixture has a γ of 0·917, and the 50 per cent. mixture one of 0·943, the question being whether these are consistent. According to the formula of Richarz, they are, since γ for the 50 per cent. mixture, calculated from the extremes, is 0·947, whereas calculated on a simple proportion it would come out to 0·964. I should like to add that this method of investigating gases seems to promise to be of use in various ways. Thus, it might be used to examine the variation of γ with pressure and temperature; arising out of this, it may even prove possible to devise a means of measuring high temperatures, since only the reflector, and not the crystal, need be exposed to the heat.

Dr. E. G. **Richardson** remarked that a method used by Boyle in Canada for detecting the approach of ice-bergs depended on the velocity of sound in the water. In that case supersonic frequencies had to be used in order to secure a directed beam of sound.

Dr. D. **Owen** expressed interest in the application of the quartz resonator. A far higher figure of accuracy, however, seemed attainable than that mentioned in the Paper. The proper procedure of measurement seemed to be to begin with the micrometer set at the zero corresponding to a resonance in air, as obtained by a preliminary set of experiments, at a distance, say, of some 6 cm. between crystal and reflector, at some stated temperature. Then in using the instrument to determine CO_2 content the micrometer should be moved to the first resonance point, then to the second resonance point, both positions being taken as means of several settings, the temperature, of course, also being noted. The distance found between successive resonance points gives an approximate value for the half wavelength. On dividing this with the total distance between quartz and reflector a close approximation to some integer will be obtained, and on dividing the total distance by this integer a far more accurate estimate of the half wavelength results, on which the determination of CO_2 content should be based. Thus the error of setting on a resonance point is distributed over the full distance from quartz to reflector, instead of being debited to a half wavelength only, and, theoretically, a 12-fold or higher gain in accuracy is secured.

Mr. R. S. WHIPPLE suggested that the apparatus might be arranged to give a differential measurement by comparison with an air standard.

Mr. D. J. BLAIKLEY (communicated): An acoustic method of utilising the principle of the stationary wave was used in 1884 by Prof. Geo. Forbes and myself in an experimental apparatus designed for the detection of small quantities of firedamp or other gases in air. This apparatus was shown to the Accidents in Mines Commission in 1885, and exhibited in the Inventions Exhibition of that year. Briefly, the apparatus consisted of two harmonium reeds, one without a resonating tube, speaking its natural pitch; and the other, associated with a tube, speaking at a pitch varying with the density of the air passing through the tube. The difference in pitch between the simple reed and the reed-pipe (shown by beats) affords a ready and sensitive means of determining the amount of foreign gas in the air. The conclusion arrived at by the Accidents in Mines Commission, that the presence of coal-dust was an important factor in the risk of explosion, independently of fire-damp, led to the abandonment of the experiments and development of the apparatus at the time.

AUTHOR'S reply: I agree with Dr. Davis that the word "sound" is somewhat inappropriate, but the proposed substitute, "supersonic disturbance," does not appeal to me. One would naturally prefer to use lower frequencies, but this involves large quartz crystals, which are not readily procurable. It is conceivable that a steel bar or valve-driven tuning-fork could be devised to produce vibrations of sufficient amplitude and of definite frequency. In reply to Mr. Darling, the instrument could be used for the estimation of carbon dioxide in flue gases provided there is not present an unknown quantity of a third gas in which the velocity of sound is different. The apparatus referred to by Dr. Richardson is probably that patented by Prof. Langevin and M. Chilowsky; those interested in this subject should consult the article on "Echo Sounding" in Nature, May 9, 1925, p. 689. Special Publication No. 14 of the International Hydrographic Bureau, Monaco, August, 1926, gives a general account of the practical form of that apparatus. In reply to Dr. Owen, the procedure recommended might be employed if the apparatus could be washed out with air from time to time; but in many experiments this is inadmissible. Moreover, the fiducial setting with air cannot be determined once for all, as the crystal must not be rigidly fixed, or it will be impossible for it to vibrate longitudinally. When investigating the relation between carbon dioxide content of the atmosphere in the chamber and distance between nodes, measurements were made over about 15 half wavelengths, which virtually amounts to the procedure advocated by Dr. Owen. The utility of the instrument is determined by the difference in the velocity of sound in the two gases under consideration, and consequently it should be admirably suited to hydrogen-air mixtures for the ratio of the velocity of sound in hydrogen to the velocity of sound in air is 3·9, whereas in the case of carbon dioxide and air this ratio is only 0·77. Mr. Whipple's suggestion of modifying the apparatus so as to utilise a differential method is attractive. Possibly this could be achieved by using an electrically-driven tuning-fork, one prong being connected to the diaphragm closing one end of the gas chamber, and the other to a diaphragm of a chamber containing air. The displacement of the node with the change of composition of gas mixture might conceivably be measured by the aid of a hot wire microphone connected up by the usual bridge arrangement to a similar fixed microphone in the air chamber. I am very interested in the communication from Mr. Blaikley, as the method he describes has possibilities when a supply of the gaseous mixture can be freely drawn upon. Precautions would have to be taken with organ pipes to avoid variations in the pitch with the pressure of blowing.

XXIV.—THE SCATTERING OF X-RAYS AND THE "J" PHENOMENON.

By B. L. WORSNOP, *B.Sc.*, Lecturer in Physics, King's College, London.

ABSTRACT.

An account is given of experiments which have been carried out using a "balance method" to obtain the "J" discontinuity which Barkla has found with heterogeneous X-rays scattered by elements of low atomic weight.

In a series of experiments, extending over several months, no such discontinuities have been found.

In view of the repeated occurrence of the phenomenon in the laboratories of Prof. Barkla, it is suggested that some condition which has not yet been published is required for its production. The intention is expressed of further investigating this effect by the method which is described in the Paper.

INTRODUCTION.

A CONSIDERATION of the work of Prof. Barkla and his collaborators* on the "J" phenomenon led the writer to perform some experiments which were designed to show the source of the electronic emission which those experimenters had found to be associated with the "J" discontinuity. These experiments, carried out early in 1926, yielded negative results, and it was then thought that this might be due to wrong conditions in the exciting radiation.

Experiments, which are here summarised, were therefore undertaken with a view to obtaining the conditions necessary for the production of the phenomenon. In these investigations as many of the conditions of the experiments of Barkla as possible were reproduced, as will be seen in the following account—e.g., similar heterogeneous beams were used, and the penetration was measured in terms of a mean absorption coefficient, which was obtained from a 50 per cent. absorption.

As is well known, one of Barkla's methods of showing the "J" discontinuity was to direct a primary beam of X-rays on to a scatterer of low atomic weight, and to compare the radiation scattered in a direction at right angles to the incident beam, producing an ionisation S in an ionisation chamber arranged to receive it, with the ionisation P produced by the direct beam which had passed through the scatterer. The ratio S/P was measured directly.

Absorbing sheets of aluminium of equal thickness were then placed in the path of both beams, and the ratio of the ionisation then produced (S'/P') was obtained.

Barkla and Khastgir found that if S/P was arranged to be of the order of unity for a soft radiation it maintained that value as the average penetration of the incident beam was increased. The ratio S'/P', on the other hand, was found to remain constant for the softer radiation, but at certain critical penetrations, corresponding to well defined magnitudes for (μ/ρ), the ratio S'/P' was suddenly decreased. Thus, when aluminium of 0·05 cm. was used, a 10 per cent. drop in S'/P' occurred at $\mu/\rho=3{\cdot}8$, and again at 1·9 for the primary radiation.

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* Phil. Trans. (1916); Phil. Mag., November and May (1925).

APPARATUS.

For the experiments described below the apparatus was designed to detect small *differences* in the primary and the scattered beams: the general arrangement is shown in Fig. 1.

The source of X-rays was a tungsten target Coolidge tube, which was excited by a transformer, controlled by means of an independently regulated field ensuring a similar secondary wave-form for all potentials. The unrectified secondary was applied directly to the tube, and the effective potential was maintained constant for each observation by a slight change of the field current when necessary.

To detect any potential changes during an observation the following arrangement of apparatus was used. A narrow strip of tin foil, *F*, was wrapped round the thick ebonite insulator which surrounded the lead from the secondary of the transformer. This was connected to two insulated valves as shown. The induced potential due to one half of the wave sends a current through one of the valves to earth, and the potential due to the other half, which is effective in the production of X-rays, sends a current through the other valve. This current passes through the galvanometer *G*, and is very nearly proportional to the peak value of the potential of the inner wire *D*.* The galvanometer reflected a spot of light on to the same scale as that

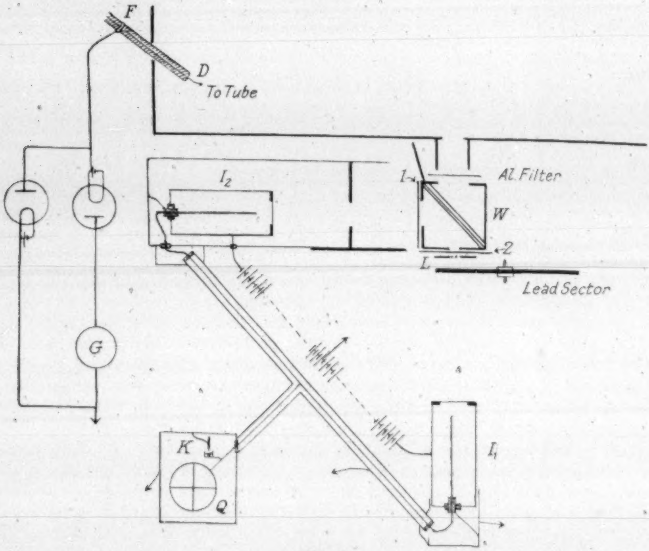

FIG. 1.

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* The writer is indebted to Mr. F. S. Robertson for information, which he has not yet published, showing that under suitable conditions this proportionality holds.

due to the quadrant electrometer, Q. Any variation in potential applied to the tube was therefore immediately detected and controlled.

The scattering substance was contained in a cubical lead box, W, of about 12 cms. side, in the first experiments. The radiation scattered at 90 degrees passed through a circular aperture, of about 5 cms. diameter, to an ionisation chamber I_2. The primary radiation continued through a 1 cm. aperture, and a perforated lead screen, to a second ionisation chamber I_1.

The ionisation chambers were of identical design, and were placed at a distance of not less than 20 cms. from the apertures in the lead box.

A battery of dry cells of some 360 volts was earthed at the mid-point and the two ends were connected to the outer walls of the ionisation chambers in the usual way: the collecting rods were joined together to one pair of quadrants of a quadrant electrometer, which therefore gave a deflection proportional to the difference of the ionisation in I_1 and I_2.

In order to avoid any possible effects due to the gas in the chambers I_1 and I_2 were filled with air and closed by thin gold-beaters' skin.

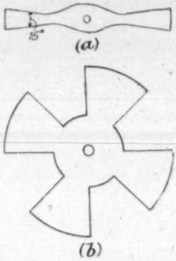

FIG. 2.

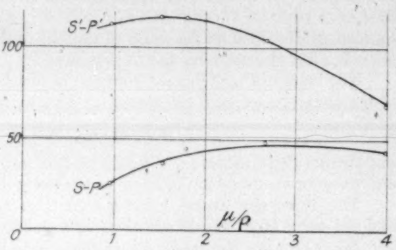

FIG. 3.

Lead sectors of the forms shown in Fig. 2 could be rotated by means of an electric motor, so that the primary beam was cut off intermittently by them.

METHOD OF WORKING.

A heterogeneous beam of X-rays from the tube was limited by lead screens and then fell on the scatterer, either directly, or else after passing through an aluminium filter.

By means of a lead screen perforated by some 10 pin holes, the primary beam was further limited so that the ionisation in I_1 was very nearly equal to that produced in I_2 by the wide beam which was scattered at right angles to the initial direction; i.e., the quadrant electrometer had a slow rate of movement corresponding to $P-S$ approximately equal to zero. Sheets of aluminium of equal thickness were then placed in the positions 1 and 2 of Fig. 1, and the process was repeated so that $P'-S'$ was obtained.

The potential applied to the tube was then changed and a similar set of observations were taken. Throughout the experiments, the filament current through the

Coolidge tube was maintained constant (at 3·7 amperes). The current through the tube was from 3·5 to 4·2 milli-amperes.

Curves showing the relation between $S-P$ or $S'-P'$ and μ/ρ were continuous, as seen in the typical example in Fig. 3. They seemed to indicate that in all the experiments undertaken, the value of S'/P' did not suddenly change.

It was, of course, realised that unless P and P' were constant (which is obviously not the case in a Coolidge tube when the potential is increased) the $S-P$ value does not give the magnitude of S/P. For this and other reasons, it appeared desirable to reduce the observations to the same form as those of Barkla, whilst at the same time retaining the advantages of the balance method. To do this and also to obtain precise values for the penetration of the rays used, the lead sectors shown in Fig. 2 were introduced.

The method of working now had the following sequence. The tube was excited by a definite potential and the quantity $P-S=x$ was measured; a lead sector of two arms, each 15°, was then rotated by a small motor, so that the rays which had passed through the holes in the lead screen, L, were cut off for $30/360=1/12$ of the time. It is therefore apparent that the quadrant electrometer recorded the difference between $11/12\ P$ and S, equal to y, say. (x and y were measured as the number of divisions on the scale of the electrometer moved over in 30 seconds, 10 seconds after the earthing key K was raised.)

The value of P and S could therefore be obtained in terms of the rate of movement of the electrometer, and not merely as a ratio, for,

$$1/12\ P=x-y\text{; and } P-S=x\text{;}$$

and further these values are compensated for any variations during the observations, and were for the same initial beam at the same time.

The aluminium sheets 1 and 2 were then introduced and the process repeated and the value of P' and S' were similarly deduced.

To determine the mean absorption coefficient of the radiation used, the 15° sector was replaced by the one shown in Fig. 2 (*b*): the sheets 1 and 2 were removed and in place of 1, a sheet of aluminium of known thickness was set up in the path of the scattered rays, the motor was set in motion and so cut off half the primary beam. The electrometer therefore now measured $P/2-S_1$, where S_1 is the residual scattered radiation which had passed through the aluminium. A second piece of aluminium sheet was then added to the first and the rate of movement again measured. This process was repeated with a sufficient number of aluminium sheets to enable a curve to be plotted showing the relation between t the thickness of aluminium and the rate of movement. The range of thickness was so chosen that the rate $x/2$ was included.

From this curve the thickness of aluminium which would cut down the rate of movement to $x/2$ was read off.

From these observations it is a simple matter to find the exact value of the equivalent μ/ρ which corresponds to a 50 per cent. absorption, for we have

$$P-S=x$$

$$P/2-S_1=x/2$$

where S_1 is the ionisation produced by the scattered rays which have passed through a thickness t of aluminium, such that the rate is $x/2$.

From the preceding equations $2\,S_1 = S$

Also, $S_1 = S\,e^{-\mu t}$

Therefore $1 = 2e^{-\mu t}$

or, $\mu = \dfrac{2{\cdot}3 \log_{10} 2}{t}$

whence

$$\left(\frac{\mu}{\rho}\right)_{ae} = \frac{2{\cdot}3 \log 2}{2{\cdot}7t} = \frac{0{\cdot}256}{t}$$

The equivalent absorption coefficients, calculated by the above method, were thus compensated for any variations during the observations, as were P, P', S and S'.

It will be noticed that the μ/ρ calculated is for the scattered radiation, but as this is not very different from the primary, it serves as a measure of the penetration of the radiation used.

RESULTS.

Table I is given to show the kind of results obtained. It will be noticed that the values of S/P and S'/P' are mostly consistent to 1 per cent., and that in very few cases are variations greater than 2 per cent. to be found.

TABLE I.

$\frac{\mu}{\rho}$	$P-S$ (x)	$\frac{11}{12}P-S$ (y)	$P'-S'$ (x')	$\frac{11}{12}P'-S'$ (y')	$x-y$	$x'-y'$	P	S	P'	S'	$\frac{S}{P}$	$\frac{S'}{P'}$
3·04	47	−21			68		816	759			$0{\cdot}93_5$	
	44	−19			63		756	712			0·94	
			16·5	−19		35·5			426	409·5		0·96
			14	−29		43			516	502		$0{\cdot}97_4$
			19	−17		36			432	413		$0{\cdot}95_5$
1·96	−131	−240			109		1308	1439			$1{\cdot}10_2$	
	−133	−239			106		1272	1405			$1{\cdot}10_5$	
			−141	−223		82			984	1125		$1{\cdot}14_3$
			−137	−232		95			1140	1277		1·12
2·2	−85	−197			112		1344	1429			1·06	
	−81	−202			121		1452	1533			$1{\cdot}05_5$	
	−101	−220			119		1428	1529			$1{\cdot}07_2$	
			−102	−200		98			1176	1278		$1{\cdot}08_6$
			−110	−200		90			1080	1190		1·10
			−108	−189		81			972	1080		1·11
3·88	65	23			42		504	439			$0{\cdot}87_2$	
	66	23			43		516	450			$0{\cdot}87_3$	
			39	14		25			300	261		$0{\cdot}87_1$
			39	15		24			288	249		$0{\cdot}86_6$
2·82	35	−42			77		924	889			0·96	
	27	−55			82		984	957			0·97	
			12	−40		52			624	612		$0{\cdot}98_2$
			9	−38		47			564	555		$0{\cdot}98_5$
		*	4	−45		49			588	584		$0{\cdot}99_1$

*** For this observation the sheets 1 and 2 were interchanged.**

Figs. 4(*a*), 4(*b*) and 4(*c*) are three curves showing the behaviour of S/P and S'/P' as the mean μ/ρ is changed. They are actual curves for three sets of observations, but also are typical of the results in general.

Curve 4(*c*) is for an initial beam falling directly on the scatterer; curves 4(*a*) and 4(*b*) are for radiation which has passed through a preliminary aluminium filter before impinging on the wax.

These curves are for a scatterer placed in the "reflecting" position, as in Fig. 5 (*a*). It was realised that in this position the scattering from increasing depth as the incident beam became more penetrating, would cause a relative hardening of the scattered rays, but as a sudden change of the order of 10 per cent. was looked for, this was not considered a serious disadvantage.

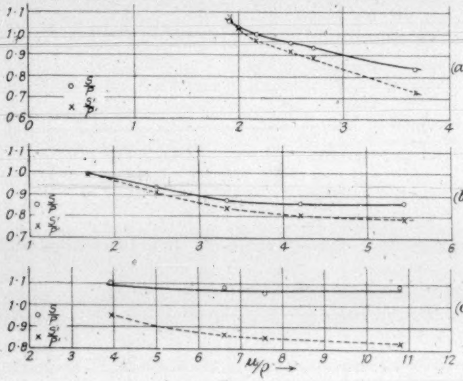

FIG. 4.

It will be seen that there is an upward slope in all the curves at the smaller μ/ρ end; but an examination of the curve for S'/P' in the region where the "J" discontinuities have been previously reported, viz., at $\mu/\rho = 1{\cdot}9$ and $3{\cdot}8$ does not reveal any such change. This was the case in every experiment performed, covering in all the range of μ/ρ from 11 to 1·6.

The next set of results were similar to the above, but were obtained with the wax scatterer at right angles to the former position, as illustrated in Fig. 5(*b*). Here again the curves for S/P and S'/P' for different μ/ρ, slopes upwards as μ/ρ decreases, also the curves are free from discontinuities, at least to within the limits of error of observation.

Figs. 6(*a*) and 6(*b*) are typical of these results. In the former case the initial beam falls directly on the scatterer, whereas, in the latter, the rays pass through a preliminary filter of aluminium.

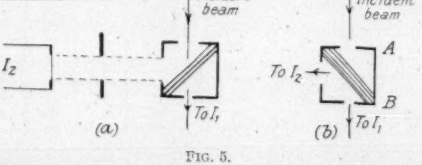

FIG. 5.

Finally, a set of observations was taken with the side AB (Fig. 5) of the lead box removed: this was to see if the rise in the curves at the lower μ/ρ values was due to radiation scattered from the wax to this side and then through the wax to I_2.

The results of this set of observations are summarised in Fig. 7.

Whereas the curves do not show the same steady rise as before, they do not show any indication of the phenomenon sought: where S/P changes, the corresponding S'/P' changes in a similar way.

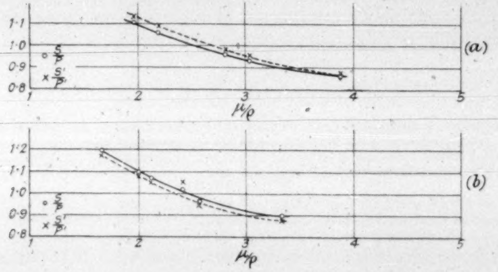

FIG. 6

SUMMARY AND CONCLUSIONS.

The above experiments are an elaboration of what was intended to be a preliminary experiment for another investigation. They were intended to obtain the correct penetration of the incident radiation to produce the " J " discontinuities which Prof. Barkla and his collaborators have repeatedly found by their method, but in no experiment has a discontinuity been found, in a region which should have shown at least two jumps.*

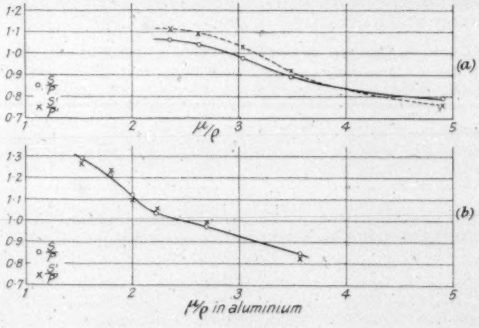

FIG. 7.

The only conclusion to be drawn seems to be that there must be some condition necessary, additional to those in these experiments, and to the published accounts. It seems somewhat premature to offer suggestions as to the nature of those conditions, but it seems desirable to apply this method to the later experiments of Barkla and Khastgir* and Barkla and Watson.† This will be done at the earliest opportunity.

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* Barkla and Watson, Phil. Mag., Nov. (1926), have found discontinuities at the following values of : 3·8, 3·24, 2·44, 1·94, all within the range of these experiments.

It is felt that the results as expressed in the curves shown are not suitable to detect any gradually varying change such as would be anticipated from the Compton theory.

In the experiments which are to be undertaken an attempt will be made to include this, by using beams of a less heterogeneous nature.

In conclusion I would like to take this opportunity of thanking Prof. O. W. Richardson for the interest he has taken in these experiments.

DISCUSSION.

Prof. E. A. OWEN said that Mr. Dufton and he had photographed the shadow of a wedge, using both direct and scattered radiation, and had found no evidence of discontinuity on measuring the variation in intensity along the shadow. The work had been abandoned under the impression that some requisite experimental condition must have been missed; but in view of the negative result obtained by the author also, further evidence from different laboratories was desirable.

Mr. J. GUILD said that both Barkla and the author used the formula $I = I_0 . e^{-\mu t}$, but, strictly, this was inapplicable to a heterogeneous beam in a selectively absorbing medium.

Dr. LEWIS SIMONS (communicated): The whole argument seems to me to turn on the somewhat doubtful assumption that the average quality of the primary rays is the same on both sides of the 10 pin-holes in the lead screen *L*. It is assumed that these holes act merely as a stop. In view of the obliquity of the primary rays in passing through these holes, it would appear that considerable transformations would occur on their walls. Have any preliminary experiments been performed to test the similarity of the primary radiation on both sides of this group of pin-holes for any given condition of the X-ray tube?

The AUTHOR, in reply to the discussion, said that he was much interested to hear that Prof. Owen had obtained negative results with a wedge. He was himself now trying the effect of equal increments of screen thickness. He used a heterogeneous beam because he wished to reproduce the conditions described by Barkla. He did not quite follow Dr. Simons' point about the pin-holes: they could scarcely account for the 10 per cent. discontinuity, and in any case they had been used by Barkla.

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* Barkla and Khastgir, Phil. Mag., Sept. (1926).

† Loc. cit.

XXV.—THE CHARACTERISTICS OF THERMIONIC RECTIFIERS.

By Prof. C. L. FORTESCUE.

ABSTRACT.

This Paper extends the results obtained in a previous Paper to the case of rectifying valves working at low voltages with unsaturated electron currents. The most economic conditions are briefly discussed, and it is shown that, as far as the limited information with respect to the life of the modern valves is concerned, it is probable that long life should be provided for.

INTRODUCTION.

IN a Paper published in 1919* methods were given for calculating the behaviour of thermionic rectifiers working at high voltages and employing filaments with sharply defined saturation values for the emission current. Valves are now largely used on low-voltage circuits where the saturation value of the emission current is not reached, and even with some modern designs of high power, high-voltage rectifiers, the conditions of operation are such that the total emission from the filament is considerably greater than the maximum current through the valve. Shearing† has suggested an approximate method of dealing with these cases, but the characteristic performance may be calculated easily from the Langmuir $\frac{3}{2}$-power law, and it is surprising that this has not been done. This law is applicable to most valves for all currents up to the normal saturation current of the cathode except for the very smallest values. Beyond the normal saturation current the conditions are too complex for any law to be applicable and it is possible that these may be the conditions contemplated by Shearing.

The methods adopted in the present Paper are similar to those of the 1919 Paper, and the performance of the rectifier is expressed in terms of the ratio of the steady voltage on the output side (V_0) to the peak value of the alternating input voltage ($\hat{v}$). The present Paper is, in fact, merely an extension of the earlier one.

THE MEAN CURRENT.

If v_a is the instantaneous voltage between the anode and filament, and i is the instantaneous current, then

$$i=k\hat{v}_a^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

is the expression of the $\frac{3}{2}$-power law. In the present case the instantaneous current through one rectifying valve will be

$$i=k(\hat{v}\sin\theta-V_0)^{\frac{3}{2}},$$

where $v=\hat{v}\sin\theta$ is the applied alternating voltage, it being assumed that the output voltage, V_0, is sensibly constant owing to the necessity of using smoothing devices. (*See* Fig. 1.) The mean value of this current over the whole cycle is

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* Fortescue, Proc. Phys. Soc., Vol. 31, Pt. 5, p. 319.

† Shearing, Journal Inst. El. Eng., Vol. 63, p. 309 (1925).

$$I_0=\frac{1}{\pi}\int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}} k(\hat{v}\sin\theta-V_0)^{\frac{3}{2}}d\theta$$

$$=\frac{k\hat{v}^{\frac{3}{2}}}{\pi}A_1, \quad \ldots \ldots \ldots \ldots \quad (2)$$

where

$$A_1=\int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}}\left(\sin\theta-\frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}d\theta$$

and can be evaluated.

From equation (2)

$$k\frac{V_0^{\frac{3}{2}}}{I_0}=\frac{\pi}{A_1}\left(\frac{V_0}{\hat{v}}\right)^{\frac{3}{2}} \quad \ldots \ldots \ldots \ldots \quad (3)$$

and if V_0, I_0, and a particular valve having a particular value of k are given, then only one value of $\frac{V_0}{\hat{v}}$ will satisfy equation (3). It is convenient to plot the values of $\frac{\pi}{A_1}\left(\frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}$ in the form of a curve, as in Fig. 2.

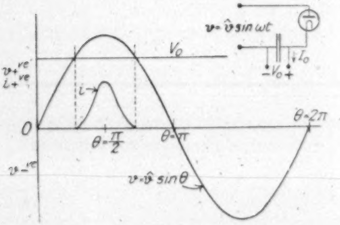

FIG. 1.

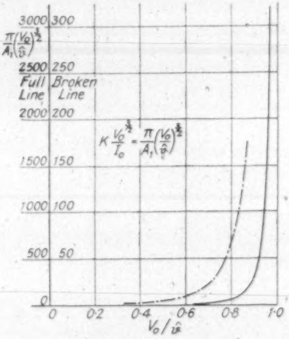

FIG. 2.

For any particular value of $V_0/\hat{v}$ the maximum value attained by the current is

$$i_{\text{max.}}=k(\hat{v}-V_0)^{\frac{3}{2}}.$$

The normal total emission current from the filament should be at least equal to this. Putting

$$I_e=i_{\text{max.}}=k(\hat{v}-V_0)^{\frac{3}{2}},$$

it follows from (2) that

$$\frac{I_e}{I_0}=\frac{\pi}{A_1}\left(1-\frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}$$

This ratio, again, may be plotted as in Fig. 3. It is noticeable that with high values of $V_0/\hat{v}$, the necessary emission current is many times more than $2I_0$.

THE EFFICIENCY.

The instantaneous power taken from the alternating supply by one rectifying valve during the conductive period is

$$i v_a = k(\hat{v}\sin\theta - V_0)^{\frac{3}{2}}\hat{v}\sin\theta.$$

The mean power during one cycle is

$$\frac{k\hat{v}^{\frac{5}{2}}}{\pi}\int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}}\left(\sin\theta - \frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}\sin\theta d\theta$$

$$=\frac{k\hat{v}^{\frac{5}{2}}}{\pi}A_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where
$$A_2 = \int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}}\left(\sin\theta - \frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}\sin\theta d\theta,$$

and can be evaluated.

The power output is I_0, V_0, and expressing (4) in terms of I_0 and V_0, the efficiency is

$$\eta = \frac{A_1}{A_2}\cdot\frac{V_0}{\hat{v}}$$

This is plotted in terms of $V_0/\hat{v}$ in Fig. 3.

THE POWER TO BE DISSIPATED FROM THE ANODE.

The power wasted in the valve is

$$W_a = I_0 V_0\left(\frac{1}{\eta} - 1\right)$$

$$= I_0 V_0\left(\frac{A_2}{A_1}\cdot\frac{\hat{v}}{V_0} - 1\right)$$

The value of W_a/I_0V_0 is also plotted in Fig. 3.

THE R.M.S. VALUE OF THE CURRENT FROM THE ALTERNATING SUPPLY.

At any instant during the conductive period

$$i = k\hat{v}^{\frac{3}{2}}\left(\sin\theta - \frac{V_0}{\hat{v}}\right)^{\frac{3}{2}}$$

Hence

$$\text{mean } i^2 = \frac{k^2\hat{v}^3}{\pi}\int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}}\left(\sin\theta - \frac{V_0}{\hat{v}}\right)^3 d\theta$$

$$= \frac{k^2\hat{v}^3}{\pi}A_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$A_3 = \int_{\sin^{-1}\frac{V_0}{\hat{v}}}^{\frac{\pi}{2}}\left(\sin\theta - \frac{V_0}{\hat{v}}\right)^3 d\theta$$

and can be evaluated.

If I is the R.M.S. current, it follows from (5) that

$$I = k\hat{v}^{\frac{3}{2}}\sqrt{\frac{A_3}{\pi}}$$

whence

$$\frac{I}{I_0} = \frac{\sqrt{\pi A_3}}{A_1}$$

This ratio is plotted in Fig. 3.

THE OVERALL EFFICIENCY.

A consideration of the curves of Fig. 3 shows that in practice there must be a compromise between efficiency and emission current. The power expended in the filament for the production of the emission current varies widely with the different conditions and is determined by the length of life that it may be considered advisable to provide for. For filaments of the same diameter the watts necessary per ampere of emission current increase with the length of life to be given. With varying diameters of filament the same general conditions are applicable, viz., that the longer the life the larger the filament power for a given emission current. Assuming, therefore, that the watts per ampere of emission current are a reasonable basis of comparison it becomes possible to calculate the overall efficiency of the rectifier and so to get some guidance as to the best compromise between efficiency and emission current. If w denotes the watts per ampere of emission current then the filament power is wI_e and the overall efficiency is from equation (4)

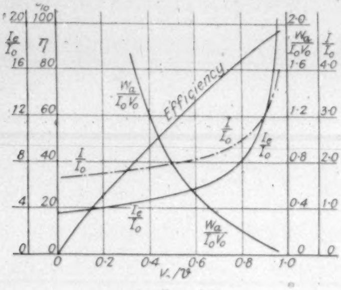

FIG. 3.

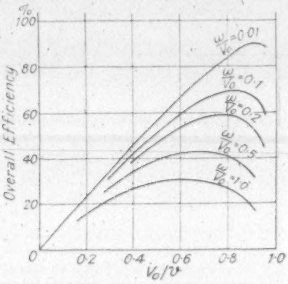

FIG. 4.

$$\eta' = \frac{I_0V_0}{\frac{kA_2}{\pi}\hat{v}^{\frac{5}{2}} + wI_e}$$

Expressing the denominator in terms of I_0 and V_0

$$\eta' = \frac{A_1}{A_2\frac{\hat{v}}{V_0} + \frac{w}{V_0}\left(1 - \frac{V_0}{v}\right)^{\frac{3}{2}}}$$

The values of η' may be plotted in terms of V_0/v for various ratios of w/V_0 as shown in Fig. 4.

ECONOMIC CONSIDERATIONS.

Further progress with the design of a valve for a given output depends upon a knowledge of the way in which the life of the valve depends upon the value of w. Assuming that a curve is available connecting the life in hours, H, and w; then if D is the cost in pence of a valve for any particular purpose and d is the cost per unit of the power supplied, the cost in pence per hour of this valve during its working life is

$$\frac{D}{H} + \frac{I_0 V_0}{\eta'} \cdot \frac{d}{1000},$$

the values of H and η' being those corresponding to particular values of w. This quantity may be worked out and the minimum value found. This minimum gives the most economical valve provided that the value of k demanded by equation (3) can be satisfied. Unfortunately no very complete data connecting H and w are available.

It seems probable from such information as is available that for small low voltage rectifying sets the valves should be designed for long life and low overall efficiency. The ratio of η' may be less than 0·5 and a relatively low value of $V_0/\hat{v}$ will be desirable. For instance, for an output of 10 milliamps at 200 volts from a single valve, the maximum current from the cathode would be in the neighbourhood of 50 milliamps and the voltage available to set up this current would probably exceed 200 volts. Such a valve would cost about £1, and if the cost of power is 1s. per unit, the life should be from 6,000 to 10,000 hours. The filament watts should be about 14 in a bright emitter valve. If a dull emitting filament is employed, with a corresponding reduction of the filament watts, a still longer life should probably be provided for.

DISCUSSION.

Dr. E. H. RAYNER said that it would be interesting to have data applicable to the design of rectifiers working at 4 to 6 volts, in addition to those for the 200-volt rectifiers considered in the Paper. He thought that in estimating the most economical design the life of a valve should be put at considerably less than the 6,000 to 10,000 hours assumed, since the mortality of valves due to accident is much greater than that due to wear of the filament.

Mr. R. P. FUGE said that the estimate would also be affected by the greater cost of using batteries instead of the mains for the filament supply, in accordance with a common practice.

The AUTHOR, in reply to the discussion, said that he had no statistics available as to the mortality of valves due to accidents. He had assumed that transformers would be used for the filament supply, as batteries were unnecessary.

XXVI.—THE THEORY OF LUMINESCENCE IN RADIOACTIVE LUMINOUS COMPOUND.[*]

By J. W. T. Walsh, *M.A., M.Sc., F.Inst.P.*

Received December 20, 1926.

Abstract.

Previous work on this subject is reviewed, and the results of brightness measurements on zinc sulphide compounds containing radium are given for periods up to 4,000 days. From a comparison of the brightness curves of compounds made with the same luminescent material, but with different radium concentrations it is shown that the brightness-time relationship is of the form $B = rf(rt)$, where r is the radium content.

The experimental facts available for developing and testing a theory of the luminescence are summarised, and a number of different theories, including a modified form of the recovery theory, are considered.

Finally it is shown that the observed brightness curves are in excellent agreement with Rutherford's original theory of the destruction of active centres, provided this be combined with a simple hypothesis as to the cause of the progressive increase in the light absorption of the material which has been found experimentally. This leads to the following form of the brightness-time relationship—

$$\log \{B/(b+B)\} + kt + a = 0,$$

where a, b and k are constants, of which the last two are proportional to the radioactive concentration for any given grade of luminescent material.

From the results obtained on two grades of compound it is concluded that (*a*) the rate of destruction of active centres is six to nine times that of the ionisation of inactive molecules, and (*b*) in the new material used for the compounds measured, 20 to 50 per cent. of the molecules are in the active state.

Introduction.

The luminosity exhibited by certain specially prepared chemical compounds, when mixed with any radioactive material emitting alpha-particles at a constant rate, has been found to diminish gradually with lapse of time. The curve of decrease of brightness was first studied by Marsden in the case of willemite, zinc sulphide, and barium platinocyanide.[†] Marsden found, further, that—

(*a*) The brightness and not the number of the individual scintillations diminished with lapse of time.

(*b*) The luminosity was only slightly affected by change of temperature or by exposure to infra-red radiation.

(*c*) The luminosity was principally due to the alpha-particles, and only very slightly to the beta and gamma-rays.

His brightness curves were satisfactorily explained by Rutherford[‡] on the assumption that the luminosity was due to the collision of the alpha-particles with certain "active centres" in the material, each collision giving a flash of light, and resulting in the destruction of the active centre.

Luminous compound consisting of a specially prepared zinc sulphide mixed with a radium salt was much used during the War, and measurements were made at the

[*] Thesis approved for the Degree of Doctor of Science in the University of London.

[†] E. Marsden, "The Phosphorescence Produced by the Alpha- and Beta-Rays," Proc. Roy. Soc., 83, p. 548 (1910).

[‡] E. Rutherford, "Theory of the Luminosity Produced in Certain Substances by Alpha-Rays," Proc. Roy. Soc., 83, p. 561 (1910).

National Physical Laboratory on a number of samples of compound, some of which, by the kindness of the manufacturers* have been retained for measurement ever since they were made in 1915.

The results of measurements extending over a period of some 450 days were given in a previous Paper,† which also contained a description of the method employed for making the measurements of brightness and of radium content.‡

It was found that the simple exponential brightness curve resulting from the application of Rutherford's theory did not fit the observed results for more than a very limited period of time, as the percentage rate of decrease of the brightness gradually diminished. This led to the formulation§ of the "recovery" theory, in which it was assumed that the active centres were not finally destroyed after collision, but were restored at a rate proportional to their concentration. This theory was found to fit the observations then available and was therefore adopted as a working hypothesis to be subjected to the test of a more prolonged series of measurements when opportunity offered.

Since that time measurements have been made at intervals, particularly on Samples 1 and 2 of the original Papers (loc. cit. Notes † §), so that now curves covering a period of about 4,000 days are available. The object of the present Paper is to show the failure of the recovery theory to represent these long-period curves, and by a discussion of them and of other results obtained more recently, to obtain a more satisfactory theory of the decay of luminosity.

THE EXPERIMENTAL DATA: CORRECTION FOR GLASS ABSORPTION.

Previous work on the subject has always, of necessity, been based on comparatively short period brightness curves (usually one year or less), so that this is the first occasion on which it has been possible to make a thorough test of any proposed formula. Long period curves are indispensable for making such a test, since, partly on account of the uneven distribution of the radium emanation through the bulk of the mixture, and partly on account of the difficulties of photometry at such low intensities, the accuracy of the measurements made on a single occasion is probably not greater than about 10 per cent. Figures read from a smoothed curve should, however, be correct to about 5 per cent., except at the very low values which have to be measured when the compound is old. The order of accuracy may be gauged from Figs. 3 and 4, in which the actual readings are shown by the points.

There is a further source of error which cannot be eliminated or even reduced by smoothing. This is the progressive coloration of the glass walls of the vessel used for containing the compound, and a consequent continual increase in the light absorption. This must be allowed for in the final brightness curves.

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[*] Mr. F. Harrison Glew and Messrs. W. Watson & Sons (Electro-Medical), Ltd.

[†] C. C. Paterson, J. W. T. Walsh and W. F. Higgins, "An Investigation of Radium Luminous Compound," Proc. Phys. Soc., 29, p. 217 (1917); N.P.L. Collected Researches, Vol. 15, p. 289 (1920).

[‡] The same methods have been used in the present work, but the brightness unit has been changed to the candle per square metre. The results given in the previous Papers referred to above were expressed in "equivalent foot-candles"—i.e., the brightness of a perfectly diffusing surface of 100 per cent. reflection factor having an illumination of 1 foot-candle. One equivalent foot-candle equals 3·425 candles per square metre.

[§] J. W. T. Walsh, "The Theory of Decay in Radioactive Luminous Compounds," Proc. Roy. Soc., 93, p. 550 (1917); N.P.L. Collected Researches, Vol. 15, p. 313 (1920).

It is probably safe to assume that this effect is due to the destruction of certain molecules within the glass* at a rate proportional to the radioactive concentration. If these molecules absorb light in accordance with Beer's law, the transmission factor τ follows an exponential law with respect to both time and radioactivity.

Although τ is difficult to measure accurately, and is, moreover, not uniform over the walls, measurements have been made in the case of two of the original samples containing respectively 200 and 100 micrograms of radium element per gram. The measured transmissions at the end of 3,900 days were respectively 70 and 84 per cent., after allowing for losses by reflection at the glass surfaces. Since $\sqrt{(\log 0{\cdot}70)/(\log 0{\cdot}84)}$ is very nearly equal to 2 it follows that, to the necessary degree of approximation, it is sufficient to regard the transmission factor as exponential with respect to the radium content, and therefore also with respect to the time since $\tau = f(rt)$. The same conclusion was arrived at in the case of the other samples for which it was possible to make measurements of τ. The expression $\tau = e^{-pt}$ has, therefore, been used throughout this Paper for correcting the observed values of brightness, the value of p adopted for each sample being that deduced from measurements on the tube containing that sample, or on a tube made from the same specimen of glass.

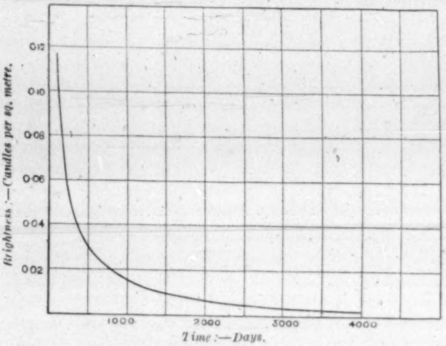

FIG. 1.—OBSERVED BRIGHTNESS CURVE FOR SAMPLE 1 (0·2 MG. Ra/GM.).

RELATION BETWEEN THE BRIGHTNESS CURVES FOR SAMPLES HAVING DIFFERENT RADIOACTIVE CONCENTRATIONS.

From what has been said above with regard to the accuracy of the observations it will be clear that even the long period brightness curves are, by themselves, insufficient to provide a really rigid test of any theory that may be proposed. It is therefore fortunate that a further and more certain test can be obtained by comparing the brightness curves of different samples, made up from the same batch of luminescent material, but with different proportions of the radioactive constituent. Such a comparison may be made in the case of Samples 1 and 2 of the original investigation, which contain respectively 200 and 100 micrograms of radium element per gram, the luminescent material being the same for both. The corrected brightness curves of these samples are shown in Figs. 1 and 2.

The circles in Fig. 2 indicate the course of the curve of Fig. 1 when its abscissæ have been multiplied, and its ordinates divided by 2, i.e., in the ratio of the radium contents of the samples. The difference between the two is everywhere very small, and is, in fact, generally less than the experimental error, so that it would appear that the brightness curves for a set of samples made up with a given luminescent material may be represented by the equation $B_t = r\,f(rt)$ where r is the radium content and t the time.

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* It should be mentioned that the coloration may be completely destroyed by heating the glass in a Bunsen flame.

This conclusion is of such importance, that it is desirable to confirm it by means of samples made with other zinc sulphides, and, where possible, by results published by other experimenters. It is therefore fortunate that an opportunity has lately been presented for making such a confirmation in connection with an investigation of different grades of zinc sulphide which has recently been carried out at the National Physical Laboratory.* Six pairs of samples were made in the Chemical Department of the Laboratory, the samples in each pair being made from the same consignment of zinc sulphide, but with radium contents in the ratio of approximately 2·5 to 1. Measurements on some of these samples have been continued for over 1,200 days.

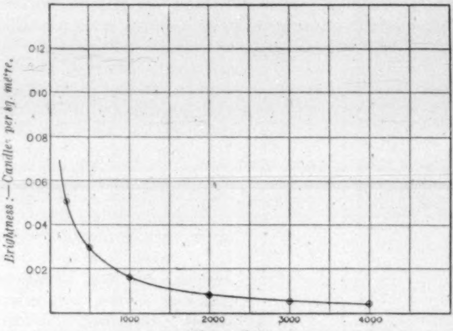

FIG. 2.—OBSERVED BRIGHTNESS CURVE FOR SAMPLE 2 (0·1 MG. Ra/GM.), WITH VALUES CALCULATED FROM FIG. 1 (CIRCLES).

The samples were all mixed by the "wet method," i.e., the zinc sulphide was mixed with the desired quantity of radium bromide in the form of an aqueous solution just sufficient in volume to moisten the whole of the sulphide, and the superfluous moisture was then evaporated. The samples were hermetically sealed in glass tubes of 8 mm. diameter immediately after mixing, and their radium contents were determined in the Radiology Division of the Laboratory using the gamma-ray method. Allowance was made for the absorption of the gamma-rays in the substance of the compound.†

The confirmation obtained with one pair of these samples is shown in Fig. 3, where the full-line represents the brightness curve observed in the case of Sample A, containing 193 micrograms of radium per gram, while the circles indicate points obtained from the smoothed curve for a second Sample, H, made with the same zinc sulphide, but containing 552 micrograms of radium per gram (Fig. 4). The positions of these circles have been calculated from the points marked in Fig. 4, by dividing the ordinates and multiplying the abscissæ by the ratio 552/193. The other pairs of samples made at the same time from other grades of zinc sulphide show a similar agreement.

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* This was carried out for the Chemistry Research Board of the Department of Scientific and Industrial Research, and the writer's thanks are due to the Board for their permission to make use of the results obtained in the course of the investigation.

† E. A. Owen and Winifred E. Fage, "The Estimation of the Radium Content of Radioactive Luminous Compounds," Proc. Phys. Soc., Vol. 23, p. 27 (1921).

In a report of the U.S. National Advisory Committee for Aeronautics* brightness curves, extending over a period of about 260 days, are given for compounds containing respectively, 100, 150, 220, and 300 micrograms of radium element per gram of compound. The relationship already noted is strikingly confirmed in the case of these samples, the agreement being generally within about 3 per cent.

The results given in the previous Paper by the writer and others† also confirm this conclusion although the curves as published only extend over about 500 days.

The only published observations which appear to contradict this relationship are those given by G. Berndt for a number of samples made with the same zinc sulphide, but with radium contents varying from 0·02 to 0·21 mg. radium element per gram.‡ Berndt, however, took as the starting point of his time scale the time of maximum brightness and not the actual time of mixing, and if the origin of the time scale be shifted back 20 days so as to allow for this, Berndt's results are found to be in excellent agreement with the relationship $B/r=f(rt)$. The agreement is, in fact, generally better than that obtained by Berndt for his proposed formula. These observations, therefore, afford further confirmation of this important relationship.

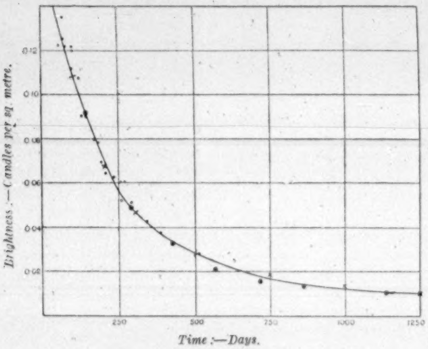

FIG. 3.—SAMPLE A.

Observed Brightness Curve (Full line).
Values Calculated from Fig. 4 (Circles).
Values Calculated on Light Absorption Theory (Crosses).
Observed Values are Indicated by Points.

It may be mentioned in passing, that there is an important practical application of this principle in the life-testing of luminescent zinc sulphides. For, clearly, the brightness curve of a sample containing any given amount of radium can be obtained in, say, one third or even one quarter of the full time by measuring a sample containing three or four times the amount of radium per gram.

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* Report No. 33, "Self-Luminous Materials," by N. E. Dorsey. It may be noted in passing that a purely empirical expression for the brightness curve was obtained at the Bureau of Standards, viz., $B_t=(a+bt)^{-1}$, where a and b were constants (*see* N. E. Dorsey, Proc. Washington Acad. Sci., 7, p. 1 (1917), and Trans. Am. Electrochem. Soc., 32, p. 389 (1917), in discussion). No attempt has apparently been made to establish this formula on a theoretical basis, and it fails, in fact, to represent the results over more than a very limited period of time.

† Paterson, Walsh and Higgins, loc. cit., p. 237.

‡ G. Berndt, "Der Helligkeitsabfall radioaktiver Leuchtfarben," Z. f. techn. Physik., Vol. 1, p. 102 (1920).

OTHER OBSERVED FACTS.

In addition to the relationship noted above, the facts observed by Marsden (loc. cit.) as well as the following have to be considered when testing any theory of luminosity :—

(1) Pure zinc sulphide is not radio-luminescent. Active zinc sulphide is crystalline and in the samples used for the present work, the average linear dimension is of the order of 4×10^{-4} cm., i.e., about one tenth of the range of an alpha-particle in ZnS. Crushing the crystals in a mortar, either before or after mixing with the radium, causes a marked reduction in the brightness.

(2) There are two principal bands in the spectrum of the light emitted by a zinc sulphide compound. The one which contributes most to the luminosity has its maximum in the neighbourhood of 550 $m\mu$. The other is much less luminous and considerably broader. Its maximum is in the neighbourhood of 460 $m\mu$. It is the diminution of brightness of the band of longer wave-length which is responsible, at any rate in the main, for the decay of the luminescence of the compound.*

(3) The photo-luminescence (response to light) of zinc sulphide mixed with radium gradually diminishes even though the sample be kept always in the dark.

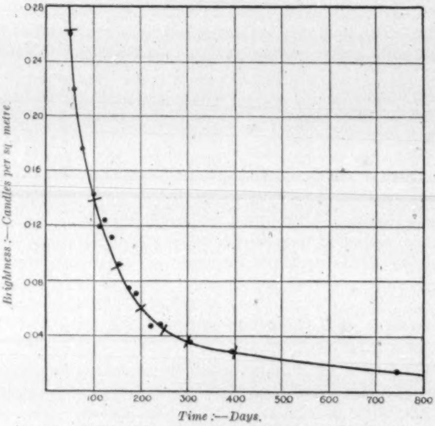

FIG. 4.—SAMPLE H. OBSERVED BRIGHTNESS CURVE WITH ACTUAL OBSERVED VALUES INDICATED BY POINTS.

(4) The observed curves show that (*a*) B_t tends to zero as t increases, and (*b*) after about 700 to 800 days the brightness of a sample containing 100 micrograms of radium per gram is definitely brighter than that of a sample containing twice this amount or more.

THEORIES OF THE LUMINOSITY : (*a*) THEORIES LEADING TO A DOUBLE EXPONENTIAL.

The most natural form of expression after a simple exponential is the compound exponential, $B_t = \sum_1^n B_1 e^{-k_1 t}$. This is of the form $rf(rt)$, as long as B and k are proportional to r. In order to test the agreement of a formula of this type with the observed brightness curves a method was developed for analysing any curve into its component exponentials.[*] Since each component has two parameters the observed curves are not definite enough to test any hypothesis leading to more than two exponentials. Two such hypotheses at once suggest themselves: (i.) The active centres may be of two kinds, each having its own characteristic rate of decay; or (ii.) the active centres, originally all of one kind, may be transformed, after the impact of an alpha-particle and the emission of light, into a second variety of active centre, which, if struck by another alpha-particle, will emit a second flash of light. It is clear that the second hypothesis leads to an expression for B_t exactly analogous to that obtained for the activity curve of a pair of successive radioactive elements.[†]

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* *See* G. Berndt, "Radioaktive Leuchtfarben," (Vieweg, 1920), p. 91 and Figs. 22 and 24. Marsden, loc. cit., p. 553. H. Herszfinkiel and L. Wertenstein, "Phosphorescence du Sulfure de Zinc sous l'action des Rayons Alpha," J. de Phys., 2, p. 31 (1921).

The only direct evidence which can be brought forward to support either of these theories is the progressive change in the relative intensities of the two bands in the spectrum of the emitted light (*see* above), which might very naturally be expected to result from a gradual change in the relative proportions of two kinds of active centres, or from a gradual substitution of "secondary" for "primary" active centres.

The earlier portion of the brightness curve of Sample 1 can be represented satisfactorily by the double exponential[‡] $B_t = 0{\cdot}1335 \operatorname{antilog}_{10}(-0{\cdot}00326t) + 0{\cdot}0474 \operatorname{antilog}_{10}(-0{\cdot}000525t)$.

This expression, calculated from the values of brightness at 100, 300, 500 and 700 days, gives the broken line curve of Fig. 5, while the actual brightness curve is shown in full line. It will be seen that the theoretical curve is not really of the shape required to represent the observations. No doubt a sufficiently good agreement could be obtained with a triple exponential, but as this would contain six independent parameters, the adoption of a theory based on such an expression would appear to be highly artificial.

THEORIES OF THE LUMINOSITY: (*b*) THE THEORY OF SURFACE ACTION.

Berndt found[§] that his curves were well represented by $B_t = B_0(1 - e^{-At})/At$, where A and B_0 are constants. This is the formula given by Rutherford's theory on the assumption that the radium is not distributed uniformly through the material, but is confined to the surface of a sheet or screen;[‖] it might, for instance, be considered as deposited on the external surfaces of the zinc sulphide crystals.

A formula of this type fits the curves fairly well for a considerable part of the time, as may be seen from Fig. 5, where the circles represent the formula

$$(14{\cdot}23/t)\{1 - \operatorname{antilog}_{10}(-0{\cdot}004687t)\}.$$

The values at 150 and 300 days were used to determine the constants. The formula fails progressively after about 2,000 days, however, and this is only to be expected, since it is not in agreement with the observed fact that after long periods of time a sample of lower radium content has a higher luminosity. It will be seen that according to this formula the sample of higher content has always the higher luminosity.

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[*] J. W. T. Walsh, "The Resolution of a Curve into a Number of Exponential Components," Proc. Phys. Soc., 32, p. 26 (1919); *see* also H. Levy, Proc. Phys. Soc., 34, p. 108 (1921).

[†] E. Rutherford, "Radioactive Substances and their Radiations," p. 421 (1913); H. Bateman, "The Solution of a System of Differential Equations Occurring in the Theory of Radioactive Transformations," Proc. Camb. Phil. Soc., 15, p. 423 (1910).

[‡] The common antilogarithm has been used instead of the exponential in order to facilitate calculation. The above expression is equivalent to $0{\cdot}1335e^{-0{\cdot}0075t} + 0{\cdot}0474e^{-0{\cdot}00121t}$.

[§] G. Berndt, "Der Helligkeitsabfall radioaktiver Leuchtfarben," Z. f. techn. Physik., Vol. 1, p. 102 (1920); "Radioaktive Leuchtfarben," pp. 86 to 90.

[‖] This was, in fact, the case actually investigated by Marsden (loc. cit.).

Further, as has been mentioned, the linear dimensions of the crystals are very much less than the range of the alpha-particle, so that a true surface effect can only be postulated on the assumption that the power of the alpha-particle to destroy an active centre decreases much more rapidly along its path than its ionizing power in the case of a gas.

THEORIES OF LUMINOSITY: (*c*) THE MODIFIED RECOVERY THEORY.

The simple recovery theory fails to represent the long period brightness curves, and the same is true of a modification proposed by Witmer,* who assumed that the rate of recovery was affected by the radiation emitted within the material. Witmer's equation for B was $\log(B-B_\infty)-\log(B+B')=-kt+l$. He found this was in good agreement with the brightness curve obtained by Rodman for pure radium bromide, and for a mixture of radium and barium bromides.† Rodman's radioactive concentrations were, however, extremely high, and it is not surprising to find that the same form of equation does not represent the results on ordinary zinc sulphide compounds. It leads, for instance, to a value of B_∞ which is greater than zero.

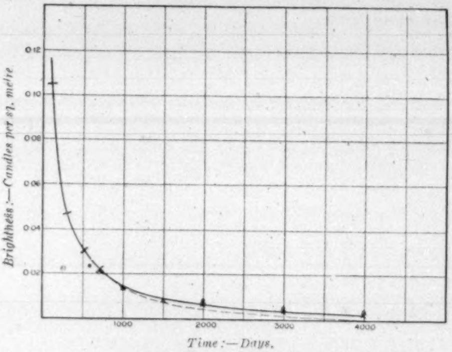

FIG. 5.—SAMPLE I.

Observed Brightness Curve (Full line).
Double Exponential Curve (Broken line).
Surface Action Theory (Circles).
Light Absorption Theory (Crosses).

This objection may be overcome by assuming that the recovery of the active centres is entirely due to the emitted radiation. This assumption is in accordance with the well-known hypothesis that the speed of molecular reactions is dependent on the flood of radiation of a particular frequency which pervades the reacting system. If the energy emitted when an active centre is destroyed is, as seems reasonable on this theory, of the frequency necessary to produce recovery, and if it be assumed that the rate of recombination is proportional to the number of destroyed centres and to the intensity of the radiation, the equation giving the nett rate of decrease of active centres is

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* E. E. Witmer, "The Theory of Alpha Ray Luminescence," Phys. Rev., Vol. 24, p. 639 (1924).

† J. A. Rodman, "The Effect of Temperature on the Luminosity of Radium Compounds," Phys. Rev., 23, p. 478 (1924).

$$\frac{dn}{dt}=-k_1n+k_2(n_0-n)\,.\,k_1n,$$

where n is the number of active centres present at time t and n_0 is the number present initially. The brightness, being proportional to the nett energy emission, is given by

$$B=-m(dn/dt).$$

These equations give for the brightness-time relationship

$$be^{ct}=\frac{\sqrt{1+aB}-1}{\sqrt{1+aB}+1}$$

where a, b and c are independent constants of positive sign. This relation between B and t fails to represent the observed brightness curve of Sample 1 with any real positive value of a even with the values of B_{100}, B_{300} and B_{500} changed by amounts which are quite outside the experimental error.

THEORIES OF THE LUMINOSITY : (*d*) THE THEORY OF SECONDARY IONISATION.

In discussing Marsden's results Rutherford showed that the active centres were destroyed at about 15 times the rate of the inactive molecules. He suggested that this might be due either to greater cross-section or to an instability such that destruction might be caused by the secondary radiation set up by the alpha-particle in the molecules it bombarded. This hypothesis does not change the form of the equation for B, but such a change is brought about if it be supposed that the destruction of one active centre affects others in its neighbourhood, and, in fact, leads to their destruction at a rate proportional to their concentration in the material.

On this hypothesis the aggregate rate of destruction

$$-dn/dt=k_1n+k_1n\,.\,k_2n+k_1k_2n^2\,.\,k_2n+\;\ldots$$
$$=k_1n/(1-k_2n),$$

where k_1 and k_2 are constants, of which k_1 is proportional to the radioactive content.

This gives, on integration,

$$\log_e(n/n_0)-k_2(n-n_0)+k_1t=0,$$

and, since $B=-m(dn/dt)$, $\qquad n=B/(k_1m+k_2B)$.

Hence, $\qquad \log\{B/(k_1m+k_2B)\}-k_2B/(k_1m+k_2B)+k_1t=\text{a constant}.$

This may, clearly, be rewritten thus :—

$$\log B/(b+B)-B/(b+B)+at+c=0,$$

where $\qquad a\equiv k_1,\ b\equiv mk_1/k_2,$

and c is given by the condition that $B=B_0$, when $t=0$.

It is clear that a and b are essentially positive, and proportional to the radioactive concentration, so that the equation is of the form $B=rf(rt)$.

Also $B_\infty=0$.

The relation, in fact, represents the observed brightness curves to a very reasonable degree of accuracy, and the chief objection to it is the high rate of secondary ionisation which it is necessary to assume during the early part of the life. In the case of Sample 1, for instance, the best values of the constants are $a=7{\cdot}2\times10^{-4}$, $b=0{\cdot}051$, and $c=1{\cdot}000$. From these values it is easy to deduce that even after the lapse of 100 days, each destruction of an active centre leads to the destruction of a second centre in about four cases out of every five.

An alternative hypothesis which leads to an exactly similar form of brightness curve is that the radiation emitted as a result of the destruction of active centres by bombardment causes the formation of active centres from inactive centres, and that the radiation emitted as a result of this recovery action results, in its turn, in a further destruction of active centres, and so on, it being assumed that the number of inactive centres is so great that it may be regarded as sensibly constant.

THEORIES OF THE LUMINOSITY: (*e*) THE LIGHT ABSORPTION THEORY.

The above brief outline of some of the more promising theories which have been tested will serve to show the many different directions in which an explanation of the observed phenomena has been sought. With the exception of the two alternatives described in the immediately preceding section, none of the theories gives an expression which is in satisfactory accord with the observed brightness curve.

The chief objection to both the theories outlined in Section (*d*) above is the somewhat artificial nature of the hypotheses upon which they are based. In consequence they have now been discarded in favour of a much simpler and more natural theory, which leads to an expression of very similar form and which gives at least as good, if not better, agreement with the observed brightness curve. It gives, in addition a simple explanation of the decay of photo-luminescence in a radio-active compound (*see* § (3), p. 323), a fact which had not previously been accounted for.

Several determinations have been made at different times[*] of the transmission of the zinc sulphide itself for the light emitted as a result of the radioactive bombardment. If the absorption factor α be defined by the equation $B_p=Be^{-\alpha p}$ where B_p/B is the fraction of light transmitted by a layer of the material of thickness p cm.,[†] then α ranges from about 30 to 100 cm.$^{-1}$ in the case of new compound. It has been found by Clinton,[‡] however, that α increases rapidly with lapse of time. In the case of the compound he used, for which the radium content was about 150 micrograms per gram, the value of α at 0, 100 and 200 days was 32, 46 and 52 cm.$^{-1}$ respectively. This rapid increase in the absorption factor of the material itself seems to have escaped notice as a possible cause of the rapid initial rate of decrease of the brightness. It will be shown in the next section, however, that it gives, when applied to Rutherford's simple decay theory, a brightness curve which is in excellent agreement with observation.

It may be noted in passing that, as mentioned above, this effect explains very simply the gradual decrease of photo-luminescence in a radioactive compound, for the penetration of the incident light to the interior of the salt, and the emission of the light produced by the photo-luminescent action, are both rapidly reduced by the increase in the absorption factor of the material.

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[*] F. Bahr, "Die Oekonomie der radioaktiven Leuchtfarben," Zeits. f. Beleuchtungswesen, Vol. 22, p. 153 (1916). G. Berndt, "Radioaktive Leuchtfarben," p. 66.

[†] *See*, e.g. J. W. T. Walsh, "Photometry," p. 116 (London, 1926).

[‡] W. C. Clinton, "Some Photometric Tests of the Brightness of Radioactive Self-Luminous Materials," Illum. Eng., Vol. 11, p. 260 (1918).

EXPERIMENTAL PROOF OF THE LIGHT ABSORPTION THEORY.

It is clear that if the absorption factor of the luminescent material at any time be α, and if the total light flux emitted from an elementary volume dv of the material be Fdv lumens, the candle-power of the surface due to this element of volume will be $(Fdv/4\pi)e^{-\alpha x}$, where x is the distance of the element behind the surface*. Thus the candle-power per unit area, due to a layer of thickness dx at a distance x behind the surface, is $(Fdx/4\pi)e^{-\alpha x}$. It follows that the total brightness of a sheet of material of thickness x is $(F/4\pi)\int_0^x e^{-\alpha x}dx=(F/4\pi\alpha)(1-e^{-\alpha x})$.

Now if it be supposed that the ionization of both the active and inactive molecules of the material results in an increase of α by an amount which is proportional to the concentration of ionized molecules in the material (Beer's law), it is clear that instead of α there must be written an expression of the form $\{\alpha+\beta(n_0-n)+\gamma t\}$ where (n_0-n) is the number of destroyed active centres, and α, β, γ are constants. Although the third term should, strictly, be of the same form as the second, the rate of decrease in the number of the inactive centres is so slow that the rate of destruction may be regarded as sensibly constant. It follows that the brightness of an infinitely thick layer of material is given by $B=F/4\pi\{\alpha+\beta\ (n_0-n)+\gamma t\}$ or, writing $F/4\pi=-m\ (dn/dt)$

$$B\{\alpha+\beta(n_0-n)+\gamma t\}=-m\ (dn/dt)$$

Rutherford's simple destruction theory gives $n=n_0e^{-kt}$,

so that
$$dn/dt=-kn_0e^{-kt}$$

and, finally,
$$B\{\alpha+\beta n_0(1-e^{-kt})+\gamma t\}=mkn_0e^{-kt}$$

or
$$\log_e\frac{B\ (1+ct)}{b+B}+a+kt=0$$

where
$$a\equiv\log\ (1+\alpha/\beta n_0)$$
$$b\equiv mk/\beta$$

and
$$c\equiv\gamma/(\alpha+\beta n_0)$$

It will be seen that this expression for the brightness curve contains four constants, viz., a, b, c and k. Since m, α and β are, *a priori*, independent of the radioactive content, while k and γ are proportional to it, it follows that b and c are also proportional to this quantity, and that the equation is of the form $B=r\ f(rt)$, while $B_\infty=0$.

Further, it is clear that α and k are essentially positive on the basis of the original assumptions, while a, b and c can also be shown to be positive on *a priori* reasoning. For B decreases to zero as a limit so that, since $\log B(1+ct)/(b+B)$ must always be real, if b is negative, c is also negative, and, further, $ct=-1$ when $B=-b$. Moreover under these conditions β is negative, but it is clear that βn_0 must be numerically less than α since the absorption factor can never be less than zero. Hence for c to be negative, γ must be negative.

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* *See*, e.g. J. W. T. Walsh, "Photometry," pp. 86 and 116.

Now it is easy to show (vide infra) that the rate of ionization of the inactive molecules is about 0·0069 per cent. per day, so that if γ is negative as well as β, α must, as before, be numerically greater than $\beta n_0+\gamma/0{\cdot}69\times10^{-4}$ and therefore c must be numerically less than $0{\cdot}69\times10^{-4}$. It follows that when $ct=-1$, t is greater than 14,500, so that B_t and therefore b is so small as to be negligible for at least the first 500 days. This is not in accordance with the observed form of the brightness curve, so that it must be concluded that both b and c are positive in sign. Since α and β are thus both positive, so also is a.

It is found that the expression

$$\log\frac{B\,(1+ct)}{b+B}+a+kt=0$$

represents the observed brightness curve of sample l to a very satisfactory degree of accuracy, when c is put equal to zero, and the fit of the curve is not improved by adopting a finite positive value for c. It follows that the expression for the brightness curve of this sample of compound may be written

$$\log\frac{B}{b+B}+a+kt=0\cdot$$

The values of the constants calculated from the observed values of B at 100, 300 and 500 days are as follows :—

$$a=0{\cdot}030$$

$$b=0{\cdot}0077$$

$$k=4{\cdot}08\times10^{-4}$$

Exactly similar expressions are found to fit the observed brightness curves of all the samples studied. The constants for Sample A, for instance, are as follows :—

$$a=0{\cdot}043$$

$$b=0{\cdot}012$$

$$k=6{\cdot}15\times10^{-4}$$

The degree of approximation with which this expression represents the observations in these two cases, is shown in Figs. 5 and 3 respectively, where the full line curves are the observed brightness curves, while the theoretical values deduced from the algebraic expression are indicated by crosses. It will be seen that the agreement is everywhere exceedingly good, and well within the limit of 5 per cent. mentioned in the early part of the Paper. This fact, combined with the simplicity of the underlying assumptions, gives the light absorption theory considerable value as a working hypothesis of the mechanism producing the luminosity in a radioactive compound. It will therefore be of interest to make some deductions which, if the necessary assumptions be valid, give interesting information as to the stability and concentration of the active centres in a luminescent zinc sulphide.

THE RATE OF DESTRUCTION OF ACTIVE CENTRES.

If it be assumed that the number of molecules of zinc sulphide ionized by an alpha-particle is the same as the number of ions produced by the same alpha-particle in a gas,[*] it is possible to deduce at once the relative rates of destruction of the active and inactive molecules. For in one gram of compound containing 200 micrograms of radium element, the number of alpha-particles emitted per second from the radium and its products as far as Radium *C* is $4 \times 3{\cdot}4 \times 10^{10} \times 2 \times 10^{-4}$, i.e., the number per day is $2{\cdot}35 \times 10^{12}$. The number of ions produced in the same time is, therefore, $4{\cdot}4 \times 10^{17}$. The total number of molecules in a gram of zinc sulphide is $6{\cdot}38 \times 10^{21}$. The rate of destruction of inactive centres is, therefore, 0·69 per cent. per 100 days.

Now the rate of destruction of the active centres is clearly equal to k, i.e., in the case of Sample 1 to 4·1 per cent. per 100 days. It must, therefore, be concluded that for this sample of compound the active centres are about six times as liable as the inactive molecules to destruction by the bombardment of the alpha-particles. For Sample A the ratio is about nine. It is interesting to compare these figures with that of 15 obtained by Rutherford (vide supra), who did not allow for the effect of light absorption.

THE CONCENTRATION OF THE ACTIVE CENTRES.

Another interesting estimate which can be made from the values of the constants in the expression for the brightness curve is that of the value of n, i.e., the number of active centres present in the luminescent material. For if it be assumed that each active centre emits, on destruction, one quantum of energy in the form of radiation of wavelength 550 $m\mu$, the mean wavelength of the emission band of a zinc sulphide compound,[†] the energy emitted per second is $\frac{-dn}{dt \times 24 \times 3{,}600} \times 3{\cdot}6 \times 10^{-12}$ ergs, i.e., the luminous flux emitted is $(-dn/dt) \times 660 \times 3{\cdot}6 \times 10^{-19} \div 86{,}400$ lumens, since the mechanical equivalent of light of wavelength 550 $m\mu$ is about 660 lumens per watt.[‡] But the luminous flux emitted in one cubic centimetre of the compound is clearly $4\pi B\{\alpha + \beta n_0(1 - e^{-kt})\} \times 10^{-4}$, if B is expressed in candles per square metre and α and β are in cm.$^{-1}$. Also, since $a = \log(1 + \alpha/\beta n_0)$ this expression may be written

$$4\pi B\alpha\left\{1 + \frac{1 - e^{-kt}}{e^a - 1}\right\} \times 10^{-4} \text{ lumens.}$$

Equating these two results, the number of active centres destroyed per day in one cubic centimetre of the material is found to be

$$\frac{4\pi B\alpha(e^a - e^{-kt}) \times 8{\cdot}64}{(e^a - 1) \times 660 \times 3{\cdot}6 \times 10^{-19}}$$

i.e., the number present at time t is

$$\frac{\beta\alpha(e^a - e^{-kt})}{k(e^a - 1)} \times 4{\cdot}57 \times 10^{17}.$$

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[*] Rutherford, loc. cit., p. 571.

[†] *See* e.g., E. Marsden, loc. cit., p. 553; H. Herszfinkiel, L. Wertenstein, "Phosphorescence du Sulfure de Zinc sous l'action des Rayons Alpha," J. de Phys., Vol. 2, p. 31 (1921); G. Berndt, "Radioaktive Leuchtfarben," pp. 90-92.

[‡] *See* e.g., J. W. T. Walsh, "Photometry," p. 465.

The most interesting value of n is n_0, i.e., the number of active centres present in the luminescent material before any radioactive bombardment has taken place. Owing to the fact that the radioactive bombardment is not constant, but gradually rises to its equilibrium value during the first 30 days after mixing, the simple form of the brightness curve given above cannot be used for this calculation, but the special form of the curve appropriate to the initial part of the life (for the first 100 days) must be determined.

THE INITIAL PART OF THE BRIGHTNESS CURVE.

It has been shown in a previous Paper* that, during the first thirty days after mixing, the alpha-ray activity of the material at any time t may be taken as $r\left(1-\frac{4}{5}e^{-\lambda t}\right)$, where r is the equilibrium activity, and $\lambda=0{\cdot}180$ day^{-1}. It follows that $dn/dt=-nk\left(1-\frac{4}{5}e^{-\lambda t}\right)$, or $\log n/n_0=-kt+\frac{4k}{5\lambda}(1-e^{-\lambda t})$. From this result it is easy to show that the formula for the brightness curve is $\log\frac{B}{b'+B}+a'+kt=0$,

where

$$a'\equiv a+\frac{4k}{5\lambda}e^{-\lambda t},$$

$$b'\equiv b\left(1-\frac{4}{5}e^{-\lambda t}\right),$$

and a, b and k have the same values as before.

Thus,

$$\log\frac{5B_0}{b+5B_0}+a+4{\cdot}44k=0,$$

which gives for Sample 1, $B_0=0{\cdot}048$ and for Sample A, $B_0=0{\cdot}051$. It will be clear that the expression for n_0 becomes, similarly, $(5B_0a/k)\times4{\cdot}57\times10^{17}$, or, putting $a=50$, $(B_0/k)\times1{\cdot}14\times10^{20}$. In the case of Sample 1 the value of n_0 is $1{\cdot}3\times10^{22}$, so that if the density of the material be put equal to 4, the number of active centres originally present per gram is $3{\cdot}2\times10^{21}$, i.e., about 50 per cent. of the total number of molecules. The corresponding figure for Sample A is 37 per cent. In considering the probability of these figures it has to be remembered that they are directly proportional to the value assumed for the absorption factor of the zinc sulphide before bombardment. Clinton's results gave a figure of 32 for this quantity, instead of the figure of 50 assumed in the above calculation.

In view of the large uncertainty introduced by this factor, no correction has been made for the fact that the values of brightness obtained were as measured through the wall of a glass tube. They should, therefore, be increased by about 10 per cent. on this account.

Although little reliance can be placed on the actual values of n_0 obtained it is satisfactory to find that they are of a reasonable order of magnitude.

In the case of Sample A actual measurements of brightness were made from within a few hours of the actual time of mixing. It is interesting to compare the observed with the calculated brightness curve for the period 0 to 100 days. This has been done in Fig. 6, where the full line represents the calculated brightness curve, using the values of the constants given on p. 12 above. The points represent the observations, and it will be seen that while the theoretical value of B_0 is in excellent agreement with the measured value, the subsequent observations all lie below the theoretical curve for the first 70 to 80 days.

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* J. W. T. Walsh, loc. cit., Note § (p. 319).

Exactly the same thing has been found in the case of the other samples for which observations are available for the initial period, although the difference between the observed and calculated values is not always as large as that in the case of Sample A. The explanation of the discrepancy seems to be that there is a diffusion of radium emanation over the whole tube, so that while the initial values of brightness are in agreement, the subsequent values are lower than would be expected until a state of "equilibrium" has been established.* There is no doubt whatever that emanation escapes from the compound since tubes which were not efficiently sealed at first were found to give consistently low values of gamma-ray activity until re-sealed. The fact that the observed values do not lie on a smooth curve (to the accuracy of measurement) indicates that the escape of the emanation from the compound is, to a certain extent, irregular.

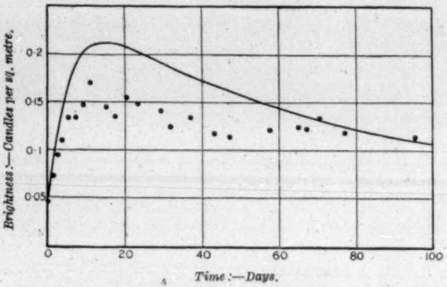

FIG. 6.—INITIAL PORTION OF THE BRIGHTNESS CURVE OF SAMPLE A. THEORETICAL CURVE (FULL LINE) WITH OBSERVED POINTS.

CONCLUSIONS.

From the results given in the Paper, and the confirmation afforded by the other samples of compound studied, it may be concluded that the mechanism of the luminosity production is of the kind originally postulated by Rutherford to explain Marsden's results. In other words, the light emission is the result of the destruction by alpha-particles of certain "active centres" in the luminescent material. The rate of destruction is proportional to the radioactive concentration and to the concentration of active centres in the material. In a compound of constant activity, therefore, the rate of decrease of the active centres follows an exponential curve, and so does the amount of light actually emitted in unit volume of the material.

As time goes on, the transparency of the luminescent material gradually decreases, so that the curve of brightness of the material in bulk is not exponential, but is of the form represented by the equation $\log B/(b+B)+kt+a=0$, where B is the brightness at time t, and a, b and k are constants. The values of b and k are, for a given luminescent materual, proportional to the radioactive concentration, while a is independent of this quantity. It follows that, as found experimentally, the brightness curve of a compound of radioactive concentration r may be obtained from that of a compound of concentration r' made up with the same luminescent material by multiplying the ordinates (B) and dividing the abscissæ (t) of the latter curve by the factor r/r'.

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* *See*, in this connection, E. Rutherford, "Radioactive Substances," pp. 361 *et seq.*

The brightness curves obtained by the application of this theory are in excellent agreement with those found experimentally, except during the first 70 to 80 days after mixing, when the observed values are always lower than those obtained by the modified formula applicable to the initial part of the brightness curve of a radium compound mixed by the wet method (during the attainment of radioactive equilibrium). This difference may be attributed to the diffusion of radium emanation over the interior of the container.

On the assumption that the number of molecules of zinc sulphide ionized by an alpha-particle is the same as the number of ions produced by such a particle in air, it follows that the rate of destruction of active centres is about six to nine times that of ionization of the inactive molecules.

On the assumption that each active centre emits, on destruction, one quantum of energy in the region of the spectrum which corresponds with the maximum of the principal luminous band of a zinc sulphide compound, it can be deduced that about 25 to 50 per cent. of the molecules are in the active state in the case of new material such as that used for the compounds studied.

The writer's acknowledgments are due to Dr. E. A. Owen, lately in charge of the Radiology Division of the National Physical Laboratory, and to Mr. T. E. Rooney, of the Chemistry Department, who were associated with him in the investigation referred to on p. 321. He also wishes to thank Mr. B. J. W. Oram, of the Photometry Division, who co-operated in the somewhat arduous photometric work involved in the determination of the brightness curves.

DISCUSSION.

LORD RAYLEIGH pointed out that since the radio-active material was mixed with the luminescent material throughout the experiment, it had been impossible to test the effect of rest on the latter of these materials. He was glad that the author had succeeded in disentangling the facts in spite of this difficulty, but he would like to suggest that future investigations might be simplified by separating the constituents of the luminous compound. He had recently been using the natural luminosity of certain uranium salts as a photometric standard in the study of the light coming from the sky at night. Uranium nitrate, for instance, is luminescent in response to light or to the action of radio-active substances, and consequently is faintly self-luminous in response to the uranium which it contains. It would be interesting to subject it to the influence of radium emanation or some similar substance, and to find out whether its luminescence can be fatigued. Although the luminosity of uranium salts is much fainter than that of the compound discussed in the Paper, it is practically permanent, and its marked difference in this respect might be worth investigating.

Dr. H. BORNS: Can the author say anything further about the influence of purposely added impurities, to which he has only referred in his introduction?

Dr. D. OWEN asked whether anything was known as to the nature of the "active centres." Did the spectrum of the emitted light give any information on this subject?

The AUTHOR, in reply, to the discussion, said that the separation of the constituents of the compound as suggested by Lord Rayleigh would certainly afford the right method for a more fundamental investigation than that described in the Paper, but the latter had been concerned primarily with a practical problem, and had, therefore, been directed to the compound in the form in which it is usually employed. The luminosity of the compound could be made more permanent while its intensity was diminished by reducing the radium content, and a sufficient reduction might conceivably lead to properties comparable with those of a uranium salt. The nature of the impurities to which Dr. Borns had referred was to some extent a trade secret. The report of the Chemical Research Board mentioned in the footnote on p. 4 of the Paper was a confidential document. Sir Ernest Rutherford had suggested that the "active centres" might be complex molecules, but the author did not himself hold any theory as to their nature.

XXVII.—DISTORTION OF RESONANCE CURVES OF ELECTRICALLY-DRIVEN TUNING FORKS.

By E. Mallett, *D.Sc., M.I.E.E., A.M.I.C.E.*

Received December 10, 1926.

Abstract.

In the first section of the Paper the experimental arrangements employed to obtain accurate resonance curves are described. In the second section resonance curves with increasing exciting currents show increasing distortion until an unstable state of affairs is arrived at in which the amplitude for a given current over a certain frequency range can have two different values, depending upon whether the frequency has been approached from above or below. The indication here is that a decrease of resonant frequency takes place with increase of amplitude, and it further appears that an increase of damping also takes place. Experiments with a free fork showed that these effects were still present, though not to the same extent as with the driven fork. Static experiments showed a departure from the straight line law both in the case of the deflection of the fork prongs for given loads, and the flux change through the core for a given deflection of the prongs. In section III the effect of such departures on the equation of motion is considered mathematically; it is shown that the term depending on the cube of the amplitude is the most important and that its inclusion gives rise to resonance curves of the form observed. The effect of the static square term is shown to be in effect a dynamic cube term. In section IV the experimentally obtained resonance curves are examined in the light of the theory, and substantial agreement is found, while in section V a detailed calculation of the various constants is made. In section VI another effect of the non-linearity is found in the possibility of producing fundamental frequency vibrations in the fork by means of exciting currents of double frequency, and the effect of a first harmonic in an exciting current of fundamental frequency is considered.

In section VII distortions of a second type are shown to consist of "coupled circuit" effects: these at large amplitudes are modified by distortions of the first type.

Contents.

I. Experimental Arrangements.
II. First Experiments.
  A. Resonance Curves.
  B. Free Fork Experiments:—
    (i) Frequency studied by Lissajous Figures.
    (ii) Damping studied by Photographic Decay Curves.
  C. Static Experiments:—
    (i) Mechanical Stiffness of Fork.
    (ii) Flux alterations with Deflection.
III. Theoretical.
  Curves are drawn on the assumption of an additional restoring force and an additional damping term varying as x^3, and found to be similar to the experimentally obtained curves.
  Static x^2 terms lead to dynamic x^3 terms.
IV. Application of Theory to Experimental Results.
V. Calculation of Fork Constants.
VI. Double Frequency Excitation.
VII. Distortions of Second Type—coupled circuit effects.
  Conclusion.

I. EXPERIMENTAL ARRANGEMENTS.

THE first experiments were carried out on a Koenig tuning fork MI3/640 (frequency 320 cycles per second), mounted on a block of wood which was screwed to a fairly massive wooden support standing on rubber blocks. The fork was magnetized and between its prongs was mounted a coil of wire of about 10,000 turns with a core of a bundle of soft iron wires, so that a current of frequency, f, passed through the coil produced an alternating pull on the fork prongs also of frequency, f, the amplitude of the pull being proportional to the product of the current and the permanent flux across the air gaps.

The alternating-current was supplied from an oscillator-amplifier arranged each valve with suitable grid bias and the oscillator only just maintained, so that as pure a wave form as possible was obtained. With such an arrangement the alterations of load due to the impedance changes with frequency of the fork in the secondary circuit of an output transformer in the amplifier anode circuit, produce no appreciable alteration of the frequency-capacity calibration of the oscillator. In drawing resonance curves with a very lightly damped vibrator such as a tuning fork, interest lies chiefly in the accurate determination of the change of frequency corresponding to a given change of capacity, and with the latter a small part only of the total oscillator capacity,

$$\delta f = -\frac{f_0}{2C_0}\,\delta C \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

That this relation holds very closely was verified by setting up an independent oscillator with a telephone in its anode circuit nearly tuned to the fork at resonance, and counting the beats between the telephone and the fork as the condenser of the fork oscillator was varied.

A difficulty arose, however, in the gradual drift of frequency of the oscillator during the course of the experiments, which, though small, was of great importance, since frequency changes of the order of 1 part in 10,000 were being investigated. The difficulty was overcome by using a second fork, loaded so as to be tuned to the first, as a frequency standard. It was driven at a very small amplitude off the oscillator supplying the fork, but through a separate amplifier, and had across its coil a valve voltmeter with a 6μA Paul indicating instrument. A wedge-shaped piece of rubber at the end of a lever could be pushed up between the prongs to damp the fork, and the " standard " frequency was taken to be that at which the " damped " impedance was the same as the " free " impedance, as indicated by the micro-ammeter. In this way, the oscillator could be set by one vernier condenser very accurately to the " standard " frequency, while a second set of vernier condensers provided the means of changing ω by a known amount. The alterations of the first vernier allowing for the " drift " with the second vernier always at a certain setting made quite an inappreciable alteration of C_0 for the standard frequency and were not read, the alterations of the second vernier giving the δC of formula 1.

The small currents used were measured by a valve voltmeter across a non-inductive resistance in series with the fork.

The whole circuit diagram is shown in Fig. 1.

The amplitude of the vibrations, when large enough, was measured by means of a microscope. Initial measurements showed that the amplitudes of the two prongs was the same under the conditions described. When the amplitude was small an attempt was made to measure it by means of a Kennelly mirror, but this was not successful as will be seen.

II. First Experiments.

A. Resonance Curves.

Fig. 2 shows a typical resonance curve obtained on the Koenig fork *MI*3. The core of the exciting coil was 1¼ in. below the tips of the prongs. The double amplitude as measured on the microscope, is plotted against the oscillator capacity in μFs. The amplitude was measured some distance (2·64 cms.) below the tip of the prongs. One thousand divisions on the microscope correspond to 1 mm. actual deflection, or an amplitude of vibration of ½ mm. To find the amplitude at the tip in mms. the deflections must be multiplied by 1·19, or the actual microscope readings by 0·000595.

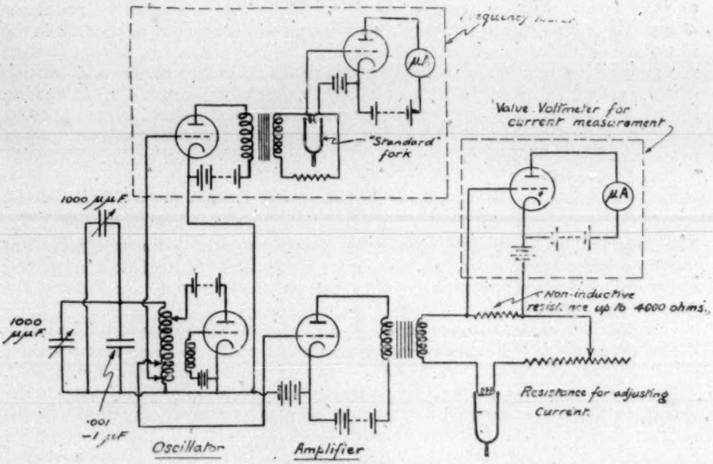
equency
μA
"Standard" fork
Value Voltmeter for current measurement
μA
1000 μμF.
1000 μμF.
·001 –1 μF
Non-inductive resist. nce up to 4000 ohms.
Resistance for adjusting Current.
Oscillator
Amplifier

Fig 1.

The frequency differences are found by multiplying δC in microfarads by 314. The current was kept constant at 1·51 mA.

At first following Kennelly on the receiver diaphragm distortion* it was thought that the shape of this resonance curve was due to some mechanical " coupled circuit " effect. Then it was observed that with larger exciting currents and consequently larger amplitudes, under certain conditions without any circuit alterations the amplitude gradually increased until the fork prongs were striking the core.

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[*] Kennelly, "Electrical Vibration Instruments," Chap. XII.

To elucidate this effect curves were drawn of amplitude against current at different frequencies. Two of these are reproduced in Fig. 3, in which curve (*a*) was taken at a frequency less than resonance (i.e., less than 320), and curve (*b*) at a frequency greater than resonance. It is seen that in the latter case the increase of amplitude with current is less as the latter increases, but in the former it is greater and greater, and finally an unstable position is reached at which the amplitude goes on increasing until the fork strikes the core.

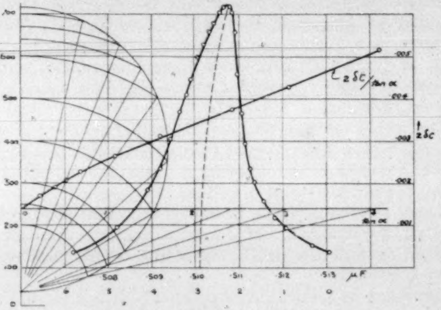

Fig. 2

These results can be explained if it be assumed that the resonant frequency of the fork decreases as the amplitude increases; so that the point at any amplitude mid-way between the two sides of the resonance curve of Fig. 2 gives the capacity value corresponding to the resonant frequency of the fork at that amplitude. Then in curve (*b*) of Fig. 3, as the current increases and the amplitude increases the resonant frequency decreases and so the exciting frequency is still lower than the resonant frequency and the amplitude increase is not so large as it would be were there no change of resonant frequency. But in curve (*a*) where the exciting frequency is below the resonant frequency of small amplitude, an increase of current and amplitude brings the resonant frequency closer to the exciting frequency, and so the amplitude increase is greater than would be accounted for by the increase of current alone.

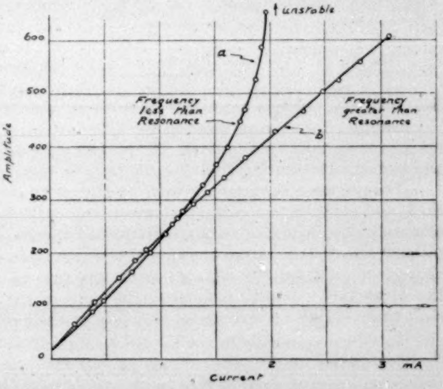

Fig. 3.

The instability would be explained by the change of impedance of the fork as its resonant frequency changes, and the resulting change in current with constant voltage from the oscillator. For instance, let *OP*, Fig. 4, represent the impedance at a particular frequency and amplitude, and *AR* the resonant motional impedance. Then an increase of amplitude resulting in a decrease of natural frequency will move the point *P* round towards *R*, say, to *P'*. Thus the impedance *OP* is reduced to *OP'* with a corresponding increase of current and amplitude, and hence a still further rotation of *P* round the circle. For this reason when the constant current curves such as Fig. 2 were taken, it was found necessary to have a large resistance in series with the fork, and to have a correspondingly powerful amplifier valve.

Another unstable effect was observed with a sufficiently large constant current, and taking care that the current was never increased above the fixed value by suitable manipulation of the series resistances. When the frequency was gradually decreased from a high value, the amplitude rose to a certain value and then after a slight decrease, suddenly decreased by a large amount to a stable value, and then finally gradually decreased. On increasing the frequency again the amplitude retraced the first curve and went beyond the point of sudden decrease before there was a sudden increase, and a final retracing of the initial curve. The process is indicated by the firm lines and arrows in Fig. 5. It was as though the resonance curve had been completely turned over at the tip, but that the dotted part was unstable and incapable of being traversed.

Similar results have been obtained by Prof. E. V. Appleton* with a vibration galvanometer, but the Author was not aware of this until the present Paper was in print.

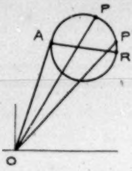

Fig. 4.

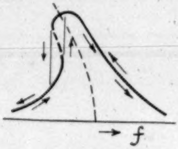

Fig. 5.

To settle finally that there actually was a change of resonant frequency with amplitude a series of peaks was drawn with different exciting currents. This family of curves is shown in Fig. 6, where it is seen that the capacity at which the peak occurs increases progressively as the current is increased; that is to say, there is a marked decrease of resonant frequency with amplitude.

Another interesting point is brought out in Fig. 6 by plotting the maximum amplitude against the current. It is seen that this curve is concave towards the abscissa, thus indicating that the damping increases with increase of amplitude.

This increase of damping with amplitude is also indicated by carrying out the circle and straight line construction† on the resonance curve of Fig. 2. In order to eliminate the change of resonant frequency the construction is modified slightly by plotting the distance from one side of the resonance curve to the other against tan α instead of as usual the distance from one side to the resonant frequency. Since the decay factor is proportional to the slope of the curve $\delta C/\tan\alpha$ obtained, it is clear that there is an increase of damping at the higher amplitudes.

B. Free Fork Experiments.

It was next desired to find out whether these effects were due to the fact that the fork was driven in the manner described, or whether they were present when the fork was vibrating freely without the presence of the bobbin and coil.

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* Phil. Mag., Vol. 47, p. 609.

† J.I.E.E. 62, p. 523, and Expl. Wireless, February, p. 95 (1927).

(i) The fork was clamped so that it could be set in vibration by bowing. Another electrically-driven fork with adjustable riders was set up at right angles so that Lissajous figures were obtained on a screen, using a pointolite lamp and a suitable optical arrangement with small plane mirrors on the prongs of the forks. The driven fork was vibrated by the oscillator first at a slightly lower frequency and then at a slightly higher frequency than the natural frequency of the fork, maximum amplitude being obtained in each case by adjusting the position of the riders. The free fork was vigorously bowed. The times at which the Lissajous figure was a straight line, that is, the half beats, were recorded on a telegraphic "local inker" by tapping a key. The distance between the marks obtained was proportional to the time of a half-beat. The intervals were somewhat erratic, but plotting the mean of each 4 intervals against the time, curves were obtained which gave quite definite indications with each setting of the driven fork. In the first case the time of the beat definitely increased, showing that as the amplitude decreased the frequency of the free fork departed further from that of the driven fork; that is, it increased. In the second the beat time decreased; the frequency of the free fork approached more closely that of the driven fork—it increased.

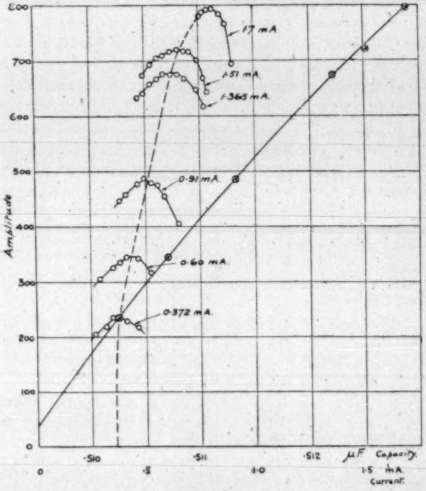

Fig. 6.

The change of frequency as calculated from these curves was of the order of 2 to 3 parts in 10,000, whereas the change in Fig. 10 is about 8 parts in 10,000. The largest amplitude in the two cases was roughly the same. The conclusion is, therefore, that there is a frequency change with amplitude in the free fork. There is further an indication that the frequency change in the driven fork is made up of two parts; one inherent in the fork itself and the other due to the driving arrangements.

(ii) The damping of the free fork was investigated by taking decay curves photographically. Two such curves were studied in this connection, (*a*) obtained with the electrical excitation by breaking the oscillator circuit, and (*b*) obtained with the bobbin removed.

Since the amplitude after any time, t, is $x=x_0t^{-\Delta t}$, $\triangle = -2{\cdot}3\frac{d\log x}{dt}$, for any amplitude is to be found as 2·3 times the slope of the curve obtained by plotting the logarithm of the amplitude against the time.

In this way the curves of Fig. 7 were obtained from the photographic records. In each case there is an increase of damping with amplitude, and not only the initial damping but also the rate of increase of damping with amplitude is greater with the electrical driving arrangements in position than when they are removed.

C. Static Experiments.

(i) Since part at any rate of the change of frequency with amplitude occurs with the free fork, it would be expected that a non-linear curve would be obtained by a static test of deflection for various loads.

The fork was accordingly clamped rigidly by its handle to the bed plate of a drilling machine, and loads were applied by placing weights in scale pans attached by cords passing over suitable pulleys to stirrups on the prongs. In order that the load could be applied both to draw the prongs closer together and to separate them, one stirrup was made longer than the other, and had holes drilled in it through which the cords from the shorter stirrup passed. Deflections were read by the same microscope used previously. To avoid moving the microscope to read the deflection of each prong, a short strip of copper was attached to the end of each prong and marks made on each which were visible at the same time in the microscope field.

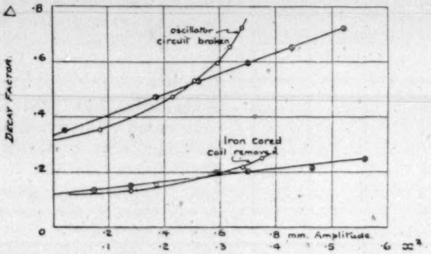

Fig. 7

A distinct lack of symmetry was found in the curve of deflection/load, but sufficient accuracy to evaluate the constants in an expression of the form $f = Au + Bu^2 + Cu^3$ was not attained. It was clear, however, that some such expression would be necessary.

(ii) In the simple linear theory of the telephone receiver, which can be applied with very slight modifications to the driven tuning fork, the driving force is shown to be proportional to the permanent flux, which is taken to vary linearly with the deflection. The flux produced by the current is also taken to be a linear function of the air gap.

The next experiments were directed towards examining the validity of these assumptions. The terminals of the coil of a fork were connected to a galvanometer, and the ballistic deflection observed when the coil and core together snatched away from their position between the prongs of the fork. This was repeated with various air gaps, obtained by clamping the prongs with a brass clamp. The ballistic deflections are proportional to the flux Φ_0 through the core due to the permanent magnetism of the fork.

Next a special 50 turn coil was wound on the bobbin and a current started through this with the core in position between the prongs. The ballistic deflections of the galvanometer were now proportional to the flux Φ_i produced by the current for different air gaps.

In each case there is a marked departure from a straight line. For deflections of 2 mm. on each side of the mean position, the flux can be represented by expressions of the form $\Phi = A + Bx + Cx^2 + Dx^3$, and the values of $\frac{B}{A}$, $\frac{C}{A}$ and $\frac{D}{A}$ were nearly the same for Φ_0 and Φ_i.

III. THEORETICAL.

In the simple theory of the fork, calling M the magnetic motive force of the permanent magnetism, and i the current through the coils (in absolute units), x the displacement from mean position towards the core, R the reluctance in the mean position, S the cross-sectional area of air gap, B_0 the flux density due to the permanent magnet, and N the number of turns on the core, the magnetic pull on each prong $= \left[\frac{B^2 S}{8\pi}\right]$

$$= \left\{\frac{M + 4\pi N i}{S\left(R - \frac{2x}{S}\right)}\right\}^2 \cdot \frac{S}{8\pi}$$

and when x is small so that $\frac{2x}{SR} << 1$,

The pull $=$

$$\frac{\Phi_0^2}{8\pi S} + \frac{B_0^2 x}{2\pi R} + \frac{NB_0 i}{R} + \quad . \quad . \quad . \quad \text{terms in } i^2 \text{ and } i^2 x.$$

The first term is the steady pull, the second a pull depending on the position of the prong, and the third the alternating pull. The remaining terms are small, involve harmonic distortion and are neglected. Thus the equation of motion of the prong can be written :—

$$m\ddot{x} + r\dot{x} + sx = Ai + \frac{B_0}{\pi R}x$$

or

$$m\ddot{x} + r\dot{x} + s_1 x = Ai$$

where $s_1 = \left(s - \frac{B_0}{\pi R}\right)$, i.e., the effect of the positional pull is to modify the stiffness of the prong.

Actually owing to eddy currents and hysteresis, there will be a lag of the flux behind the current and of the positional force behind x, so that A will be complex, and the positional force will involve also an increase of the effective mechanical resistance r. But these changes will not alter the simple linear relation between x and i.

Now when the amplitude is large, it has been seen that statically at any rate, the flux in the air gap can be written $(\Phi_0 + \Phi_i)(1 + bx + cx^2 + dx^3)$, and the pull r is proportional to the total flux squared, or very nearly

$$(\Phi_0^2 + 2\Phi_0\Phi_i + \Phi_i^2)(1 + 2bx + 2cx^2 + 2dx^3) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This expanded gives rise to a number of terms, the interest in which chiefly lies in those containing Φ_i, x, x^3, $\Phi_i x^2$, and $\Phi_i^2 x$, Φ_i gives the driving pull, and x the positional pull of the linear theory, while the remainder, since both x and Φ_i are varying very nearly sinusoidally, contain terms which also vary sinusoidally with the same frequency. We may in fact assume, at any rate, to a first approximation, that the motion of the fork is simple harmonic, and analyse $x^3 \sin^3 \omega t$, $\Phi_i . \sin \omega t . x^2 \sin^2 \omega t$, and $\Phi_i^2 \sin^2 \omega t . x \sin \omega t$ trigonometrically into terms of fundamental frequency, and triple frequency, and neglect all but the terms of fundamental frequency in finding an equation for the motion.

In this way the amplitude of the total force actuating the fork may be written

$$\begin{aligned} &A\Phi_i(\cos\varphi + j\sin\varphi) + Bx(\cos\alpha - j\sin\alpha) + Cx^3(\cos\beta - j\sin\beta) \\ &\quad + D\Phi_i x^2(\cos\gamma - j\sin\gamma) + E\Phi_i^2 x(\cos\delta - j\sin\delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3) \end{aligned}$$

where the phase of the motion x is reckoned as standard, where φ is the angle of lead of the current in front of the motion, less the angle of lag of the flux behind the current, and the angles $\alpha, \beta, \gamma, \delta$ represent the lags of the various forces, produced, as in the simple theory, by the eddy current and magnetic hysteresis losses. Of these forces the first is the main driving force, the second and the last are simple positional forces, which can be taken as modifying r and s, while the third and the fourth can produce distortion of the resonance curve. Of these, the fourth, except at small amplitudes of very large currents, is small compared with the third, and will be neglected at first to simplify the theory. There is left therefore a force $A\Phi_i(\cos\varphi + j\sin\varphi) + \frac{3}{4}Cx^3(\cos\beta - j\sin\beta)$ of fundamental frequency.

On the other side of the equation of motion it has been seen that the restoring force is not linear for large amplitudes, but can be written $sx - s_1 x^2 - s_2 x^3$. Now if x is varying sinusoidally, the x^2 term will introduce no fundamental frequency restoring force, but the x^3 term will. Also if there is mechanical hysteresis, this restoring force will lag behind x, and must be written in the equation

$$\frac{3}{4} s_2 x^3(\cos\xi - j\sin\xi)$$

So that the equations are now

$$\begin{aligned} &-m\omega^2 x + j\omega r x + sx - \tfrac{3}{4}s_2 x^3(\cos\xi - j\sin\xi \\ &\qquad = A\Phi i(\cos\varphi + j\sin\varphi) + \tfrac{3}{4}x^3(\cos\beta - j\sin\beta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) \end{aligned}$$

or

$$\left.\begin{aligned} \frac{-m\omega^2 + s}{\omega r} x - Bx^3 &= A\cos\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ x + Dx^3 &= A\sin\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}\right\} (5)$$

The constant D expresses the increase of resistance with amplitude, and B the decrease of frequency with amplitude, while the new constant A depends upon the current.

In order to find whether these equations can give curves similar to those obtained experimentally, including the unstable effects, D is neglected, as it will only involve a flattening of the curves more pronounced as the amplitude is increased, and a series of curves drawn.

Taking $B=0{\cdot}5$ and $A=1$, $1{\cdot}5$, 2 and $2{\cdot}5$, for various values of x, the corresponding values of φ are found for different values of $\frac{-m\omega^2+s}{\omega r}$. Curves (5) (i) and (ii) are then plotted to the same base φ, and the intersection of the two curves gives x and φ for the value of $\frac{-m\omega^2+s}{\omega r}$ and therefore of ω taken.

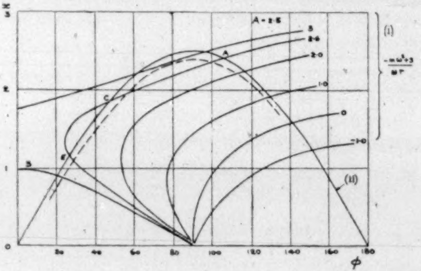

Fig. 8.

As an example the curves with $A=2{\cdot}5$ are given in Fig. 8.

The solutions of x from these curves, and from similar curves drawn with $A=1$, $1{\cdot}5$ and 2, are plotted in Fig. 9 against—$\frac{-m\omega^2+s}{\omega r}$, which for small dampings as obtained with tuning forks is proportional to the frequency.

It is seen that the resonance curves of Fig. 9 are for the smaller amplitudes of the same shape as the experimental curve of Fig. 2, and that the peaks of the curve Fig. 6 also conform to the general arrangement. Further with still larger amplitudes, the tip of the curve actually does turn over as was expected from the unstable experimental result.

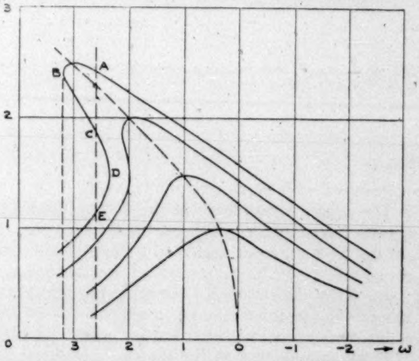

Fig. 9.

That the underside of the curve from B to D in Fig. 18 represents an unstable equilibrium can be readily seen from the constant frequency curves of Fig. 8. For any slight increase in the resistance, r will flatten curve (ii) say to the dotted line, resulting in a reduction of amplitude at points A and E, which is the natural result. But in the case of point C the theoretical conditions can only be met by an *increase* of amplitude, which there is no means of achieving. The amplitude therefore goes on falling to E, where conditions are again stable. On the other hand, a reduction of resistance at C necessitates a reduction of amplitude. This is impossible, and the amplitude goes on increasing until the point A is reached.

The theory developed can, therefore, in a general way, explain the experimental results. The changes of frequency given by the theory, agree with the expression given in Lord Rayleigh's "Sound," Vol. 1, p. 78, for the frequency reduction with amplitude of a freely vibrating fork without damping. Rayleigh's formula 7 is

$$m^2=n^2+\tfrac{3}{4}\beta A^2$$

Here n is the small amplitude value of 2π times the natural frequency of the fork, m is the value at an amplitude A, and β is the co-efficient of u^3 in the equation

$$\ddot{u}+nu+\beta u^3=0$$

for the free vibrations of the fork (undamped).

We started with an equation

$$m\ddot{x}+r\dot{x}+sx+kx^3=Ai$$

which on the assumption of x varying sinusoidally, and of free undamped vibrations, becomes

$$-m\omega^2x-sx+\tfrac{3}{4}kx^3=0$$

so that $\tfrac{3}{4}k$ is what we have called s', and $-\frac{k}{m}$ corresponds to Rayleigh's β.

Now the frequency at maximum amplitude is obtained, as will be seen from Fig. 8, when $\varphi=90°$, and then from (5) with $D=0$,

$$x=\frac{A}{\omega r}=A_0$$

where A_0=the amplitude at resonance,

and
$$m\left(-\omega^2+\frac{s}{m}\right)=s'A_0^2$$

Writing $\frac{s}{m}=\omega_0^2$, the small amplitude value (Rayleigh's n^2) this gives

$$\omega_0^2-\omega^2=\frac{s'}{m}A_0^2=\frac{\tfrac{3}{4}k}{m}A_0^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

whence
$$\omega^2-\omega_0^2=\tfrac{3}{4}\beta A_0^2$$

in agreement with Rayleigh's approximation.

The square terms, both in the actual restoring force on the fork, and in the magnetic forces acting, which can be looked upon as modifying the restoring force, have up till now, been neglected. They can be collected together into one term s_1x^2 and will result in the restoring forces not being symmetrical about the position of rest. Thus the restoring force for a movement one way will be $sx+s_1x^2$, and for a movement the other way $sx-s_1x^2$. The result will probably be that the mean position of the prongs when vibrating will differ slightly from their rest position.

If we assume that at the extreme positions, to the right and to the left, the potential energies are equal, and that the movement from the rest position to the left is x_1 and to the right x_2, we shall have

$$\int_0^{x_1}(sx+s_1x^2)dx=\int_0^{x_2}(sx-s_1x^2)dx$$

or
$$\tfrac{1}{2}sx_1^2+\tfrac{1}{3}s_1x_1^3=\tfrac{1}{2}sx_2^2-\tfrac{1}{3}s_2x_2^3.$$

If still assuming that the fork is vibrating sinusoidally, we write for the vibration amplitude $x_m=\frac{x_1+x_2}{2}$, and for the shift of the mean position $x_0=\frac{x_2-x_1}{2}$ we find that very nearly

$$x_0=\frac{1}{3}\,\frac{s_1}{s}\,x_m^2$$

The restoring force term in the equation of motion becomes therefore, dropping the subscript m,

$$\begin{aligned}&s(x+x_0)-s_1(x+x_0)^2\\&=sx-\frac{2}{3}\,s_1x^2-\frac{2}{3}\,\frac{s_1^2}{s}\,x^3-\frac{1}{9}\,\frac{s_1^2}{s^2}\,x_4\end{aligned}$$

The last term is small and is neglected. The third term is a cube term and can be dealt with as previously. We still have a second term in x^2. Applying this again with $\frac{2}{3}s_1$ instead of s_1, we shall have an additional cube term with co-efficient $\frac{2}{3}\left(\frac{2}{3}s_1\right)^3\cdot\frac{1}{s}$, and a remaining square term of co-efficient $\frac{2}{3}\cdot\frac{2}{3}s$, and so on. The final co-efficient of the cube term will be

$$-\frac{2}{3}\cdot\frac{s_1^2}{s}\left\{1+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^4+\ \ .\ \ .\ \ .\ \ \right\}=-\frac{6}{5}\cdot\frac{s_1^2}{s}$$

Thus the equation of motion becomes

$$m\ddot{x}+r\dot{x}+sx-\frac{6}{5}\,\frac{s_1^2}{s}\,x^3=A$$

and the previous investigation with the cube terms will cover the case of the square terms also.

For instance, the frequency change at maximum amplitude is found by writing

$$k=\frac{6}{5}\cdot\frac{s_1^2}{s}$$

in equation (7), giving

$$\begin{aligned}\omega_0^2-\omega^2&=\frac{3}{4}\cdot\frac{6}{5}\cdot\frac{s_1^2}{sm}A_0^2\\&=\frac{9}{10}\cdot\frac{s_1^2}{\omega_0^2m^2}\cdot A_0^2\end{aligned}$$

Rayleigh gives for the frequency change due to the square term

$$m^2=n^2-\frac{5a^2A^2}{6n^2}$$

where a is defined by the equation

$$\ddot{u}+n^2u+au^2=0$$

In our nomenclature, a is $\frac{s_1}{m}$, and using this Rayleigh's expression becomes

$$\omega^2-\omega_0^2=-\frac{5}{6}\,\frac{s_1^2}{\omega_1^2m^2}A_0^2.$$

It is seen that the correction is of the same order in each expression, but that our co-efficient is too large, being 0·9 as compared with Rayleigh's 0·833. The frequency change introduced by the square term is in any case small compared with that due to the cube term, and it is evident that to a sufficiently close approximation, the cube term investigation should cover the experimental results.

IV. APPLICATION OF THEORY TO EXPERIMENTAL RESULTS.

Since the whole of the resonance takes place with only a small change of w, it is possible to write $\frac{-m\omega^2+s}{\omega r}=K\delta C$, where K is a constant and δC the change of oscillator capacity from the small amplitude resonance value. At equal values of x, the left hand side of equation (5) (ii) is the same and therefore $\sin\varphi$ is the same. At these points (5) (i) gives the two equations:

$$\left.\begin{aligned} K\delta C_1x-Bx^3&=A\cos\varphi\\ K\delta C_2x-Bx^3&=A\cos\varphi\end{aligned}\right\}$$

where $A\cos\varphi$ is in the first positive and equal to $-A\cos\varphi$ in the second.

Adding gives

$$K(\delta C_1+\delta C_2)x-2Bx^3=0\quad .\quad (8)$$

The δC_2 may contain a constant if we do not know the small amplitude value of C at resonance. For instance, suppose the δC_2 measured from 0 where the capacity difference is C_0 to the resonance value (Fig. 10a).

Equation (8) gives

$$2C_0-(\delta C_1+\delta C_2)-2Bx^3=0\quad (9)$$

If, therefore, $\delta C_1+\delta C_2$ is plotted against x^2 a straight line should result, whose slope is $-\frac{2B}{K}$ and whose intercept on the axis of y gives $2C_0$.

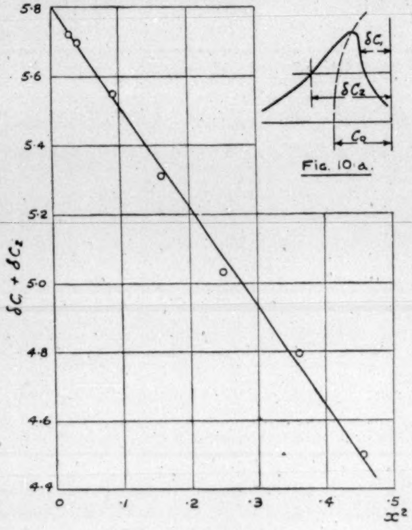

Fig. 10a.

Fig. 10

The following table is drawn up from Fig. 6, taking the origin at 0·513 μF, the unit of δC as 0·001 μF, and a reading of 100 on the microscope as 0·1, and allowing 40 divisions as zero error.

x	δC_1	δC_2	$\delta C_1+\delta C_2$	x^2
0·16	1·00	4·75	5·72	0·025
0·20	1·35	4·35	5·70	0·040
0·30	1·77	3·78	5·55	0·090
0·40	1·91	3·40	5·31	0·160
0·50	2·02	3·10	5·03	0·250
0·60	2·08	2·72	4·80	0·360
0·675	2·25	2·25	4·50	0·455

$\delta C_1+\delta C_2$ is plotted against x^2 in Fig. 10.

It is seen from the figure that the points lie quite well on a straight line, whose intercept on the y axis is $5{\cdot}8=2C_0$ (from which $C_0=2{\cdot}9$ fitting in well with the value indicated by Fig. 6) and whose slope$=2{\cdot}92$, giving $\frac{B}{K}=1{\cdot}46$.

Thus the frequency change checks up with the theory quite well. The damping is most conveniently examined from the family of peaks, Fig. 6. At the peaks $\sin\varphi=1$, and equation (5)(ii) becomes

$$x+Dx^3=A.$$

Now A is proportional to $i=A'i$ say, so that

$$\frac{x}{i}+D\frac{x^3}{i}=A' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where the x's are the maximum values.

This can be checked by plotting $\frac{x}{i}$ against $\frac{x^3}{i}$, when the slope of the line which should be obtained will give D, and the intercept on the y axis will give A'.

The necessary figures from Fig. 10 are collected in the following table :—

i mA	x	$\frac{x}{i}$	$\frac{x^3}{i}$
0·37	0·185	0·495	0·017
0·60	0·295	0·490	0·043
0·91	0·435	0·477	0·091
1·36	0·625	0·459	0·180
1·51	0·670	0·445	0·200
1·70	0·745	0·440	0·244

and the third column is plotted against the fourth in Fig. 11.

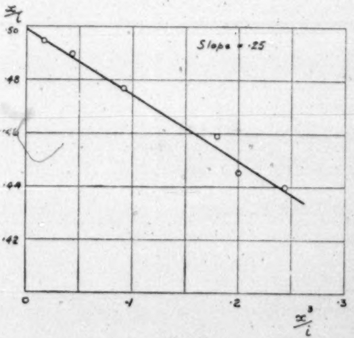

Fig 11

It is seen that quite a good straight line results, that the intercept is $0{\cdot}5=A'$, and that the slope is $0{\cdot}25=D$, so that equation (ii) becomes

$$x+0{\cdot}25x^3=0{\cdot}5i \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Evidently therefore the theory successfully explains also the change of damping found with amplitude.

The decay curves of Fig. 7 also give a confirmation of the theory as regards the law governing the increase of resistance with amplitude, $\triangle$ plotted against x^2 giving in each case a straight line.

The remaining constants may now be found in the equations

$$\left.\begin{aligned} -m\omega^2x+sx-s'x^3&=Ai\cos\varphi \\ \omega rx+\omega r'x^3&=Ai\sin\varphi \end{aligned}\right\}\begin{matrix}\text{(i)}\\ \text{(ii)}\end{matrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where A is now written for the force factor, and where in the previous work we have written

$$\frac{-m\omega^2+s}{\omega r}=K\delta C,\ \frac{s'}{\omega r}=B,\ \frac{r'}{r}=D,\ \frac{A}{\omega r}=A' \quad . \ . \ . \ . \ . \ . \ . \quad (12a)$$

Values have been found for D and B/K and A'.

Going back to equation (12) (i) above and dividing throughout by ωr gives

$$K\delta Cx-Bx^3=A'i\cos\varphi \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (13)$$

from which

$$\frac{x\delta\varphi}{\cos\varphi}-\frac{B}{K}\left(\frac{1}{\cos\varphi}.\,x^3\right)=\frac{A'i}{K} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (14)$$

If, therefore, $\frac{x\delta C}{\cos\varphi}$ is plotted against $\frac{x^3}{\cos\varphi}$, a straight line should result whose slope will be $\frac{B}{K}$ and intercept $\frac{A'i}{K}$. But first the values of $\cos\varphi$ must be found. This may be done from the second equation divided by ωr

$$x+\frac{r'}{r}x^3=\frac{A}{\omega r}i\sin\varphi$$

which in the case of the curve of Fig. 16, with $i=1{\cdot}51$ mA and using the values $\frac{r'}{r}=D=0{\cdot}25$, and $\frac{A}{\omega r}=A'=0{\cdot}5$, becomes

$$x+0{\cdot}25x^3=0{\cdot}755\sin\varphi.$$

From the figure constructed in this way it was found that the slope $=\frac{B}{K}=1{\cdot}4$ and the intercept $=\frac{A'i}{K}=0{\cdot}36$,

whence
$$K=\frac{A'i}{0{\cdot}36}=\frac{0{\cdot}5\times1{\cdot}51}{0{\cdot}36}=2{\cdot}1$$

and
$$B=1{\cdot}4\times2{\cdot}1=2{\cdot}94,$$

and the equations are

$$\left.\begin{array}{ll} & 2{\cdot}1\delta C-2{\cdot}94x^3=0{\cdot}755\cos\varphi \\ \text{and} & x+0{\cdot}25x^3=0{\cdot}755\sin\varphi \end{array}\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (15)$$

two equations for the determination of x and φ for given values of δC.

V. CALCULATION OF FORK CONSTANTS.

Taking the equivalent mass of one prong of the fork as $m=19$ grams (calculated) leads to an equivalent stiffness of

$$s=m\times320^2\times4\pi^2$$
$$=77\times10^6\text{dynes/cm.}$$

In the work which precedes the double amplitude was taken in mm. at a distance of 1·04 in. from the tip of the prong. To convert these to single amplitudes at the prong tips in cms. it is necessary to divide all values of x by 16·85 and correspondingly increase the constants in the equations.

Thus the equations (15) become

$$\left.\begin{aligned} 2\cdot1\delta C-833x^3&=0\cdot0448\cos\varphi \\ x+70\cdot9x^3&=0\cdot0448\sin\varphi \end{aligned}\right\} \qquad . \; . \; . \; . \; . \; . \; . \; . \qquad (16)$$

Now $\delta\omega=320\times2\pi\times\frac{\delta C}{2C_0}$, and the δC unit is $0\cdot001\mu F$, C_0 is $0\cdot510\mu F$, or in the units chosen 510.

Hence $\delta\omega=1\cdot97\delta C.$

Also
$$\frac{-m\omega^2+s}{\omega r}=\frac{1}{r}\left(-m\omega+\frac{s}{\omega}\right)=\frac{m}{\omega r}\left(\omega_0^2-\omega^2\right)$$
$$=\frac{2m}{r}\delta\omega \quad \text{very nearly.}$$

Hence
$$\frac{2m}{r}\delta\omega=\frac{2m}{r}\times1\cdot97\delta C=2\cdot1\delta C$$

whence
$$\frac{r}{2m}=0\cdot94 \text{ and } r=0\cdot94\times2\times19$$
$$=35\cdot7 \text{ dynes per cm. per second.}$$

Further, since $\frac{Ai}{\omega r}=0\cdot0448$, with $i=1\cdot51\times\sqrt{2}\times10^{-4}$ absolute units (the maximum value is used since the maximum value of x is used throughout)

$$A=\frac{0\cdot0448\times320\times2\pi\times35\cdot7}{1\cdot51\times\sqrt{2}\times10^{-4}}$$
$$=15\cdot1\times10^6 \text{dynes/ab. amp.}$$

Lastly,
$$833=B=\frac{s'}{\omega r}$$

$$\therefore \quad s'=833\times2\pi\times320\times35\cdot7=59\cdot6\times10^6$$

and
$$70\cdot9=\frac{r'}{r} \qquad \therefore r'=70\cdot9\times35\cdot7\doteq2{,}530.$$

If the distorting effect is lumped together as $s_0x^3(\cos\xi-\gamma\sin\xi)$,

$$s_0\cos\xi=833\omega r$$
$$s\xi\sin\xi=70\cdot9\omega r$$

giving
$$\tan\xi=\frac{70\cdot9}{833}=\cdot085$$

and
$$\xi=5^\circ,$$

which is an equivalent angle of lag, made up of electrical eddies and hysteresis and mechanical hysteresis.

The final equations for the motion x of the tip of the fork prongs are, therefore,

$$-19\omega^2x+77\times10^6x-59\cdot6\times10^6x^3=15\cdot1\times10^6i\cos\varphi$$
$$35\cdot7\omega x+2530\omega x^3=15\cdot1\times10^6i\sin\varphi$$

x being the amplitude in cms. and i the current amplitude in absolute units.

VI. DOUBLE FREQUENCY EXCITATION.

It appeared on consideration that if the current were of double the fork frequency, then terms in equation (2) with even powers of $\Phi_i x$ would now give a single frequency component of the pull. The most important of these would be $2\Phi_0\Phi_i \,.\, 2bx$, which, introducing the time element, would be

$$4b\Phi_0\Phi_i \sin(2\omega t+\varphi)\sin\omega t=4b\Phi_0\Phi_i x\{\tfrac{1}{2}c\omega(\omega t+\varphi)-\tfrac{1}{2}\cos(3\omega t-\varphi)\},$$

giving a single frequency component of amplitude $2b\Phi_0\Phi_i x$.

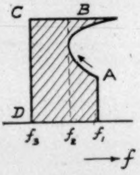

FIG. 12.

One would expect, therefore, that if the coil of the fork were supplied with a sufficiently powerful current of twice the frequency of the fork or round about that value, the fork once set into vibration would be maintained at a frequency exactly one half of that of the exciting current—i.e., at about its own natural frequency.

The experiment was successfully carried out on a 50 cycle double reed, excited by 20mA at about 100 cycles through an 8,000 turn coil, from an alternator in the first instance, but afterwards from a valve oscillator. There was no vibration until the reeds were started by flicking with the finger, and then, within certain frequency limits, the amplitude built up until the reeds were striking the core.

The form of the resonance curve obtained appeared to be somewhat as shown in Fig. 12. To start the vibration the frequency had to be between f_1 and f_2. On decreasing the frequency after starting the vibration, the amplitude increased until the reeds struck the core. At a certain frequency, f_3, the amplitude suddenly dropped to zero. With frequencies between f_2 and f_3 the change from maximum amplitude (striking the core) to zero was always abrupt. If the amplitude was reduced by just touching the reed, the vibration collapsed. But quite stable amplitudes not striking the core existed between f_1 and f_2, and this part of the curve could be traversed backwards or forwards by altering the frequency. If the frequency was lowered below f_2, so that the reeds struck the core and then gradually increased, the reeds still struck, although f_2 had been passed. If now the reeds were very lightly damped by the finger the amplitude fell to a stable position along AB, but if the frequency had been raised beyond f_1 the amplitude fell to zero.

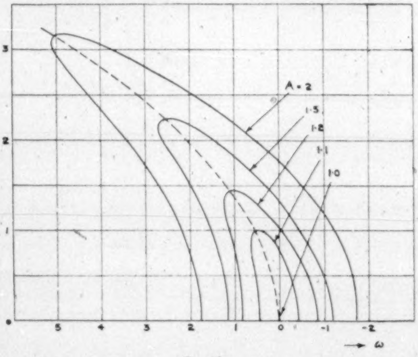

FIG 13.

It was also observed that the greater the exciting current the greater the frequency range over which vibrations could be maintained.

Some progress towards an explanation of these results has been made.

The differential equation is now

$$m\ddot{x}+r\dot{x}+sx=Aix.$$

Considering, as before, x as of standard phase (still assuming that its variations are sinusoidal), and writing the current as $i \sin (2\omega t+\varphi)$, and the displacement as $x \sin \omega t$, the equation becomes

$$-m\omega^2 x \sin \omega t+\omega r x \cos \omega t+ex \sin \omega t,$$

$$=Aix \sin (2\omega t+\varphi) \sin \omega t,$$

$$=\frac{Ai}{2}\,x(\cos \omega t \cos \varphi-\sin \omega t \sin \varphi),$$

on neglecting all forces except those varying with angular frequency ω.

Equating sine and cosine terms gives

$$\left.\begin{aligned}-m\omega^2+s&=-\frac{Ai}{2}\sin \varphi\\ \omega r&=\frac{Ai}{2}\cos \varphi\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (16)$$

Since x does not appear it can have any value whatever by this theory, provided that the second equation is satisfied by making—$Ai \geqq 2\omega r$.

Then for any value of i

$$\left(\frac{Ai}{2}\right)^2=(\omega r)^2+(-m\omega^2+s)^2, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (17)$$

which will fix the frequency, and

$$\tan \varphi=\frac{m\omega^2-s}{\omega r} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

which will fix the phase angle.

Thus a vibration with any given current is only possible at one particular frequency, and it will then have a definite phase angle with regard to the current. It will consequently be unstable.

The condition arrived at now is very similar to that in Melde's famous experiment and other cases of maintenance of vibrations with forces of double frequency, for which Rayleigh* gives the equation

$$\ddot{u}+k\dot{u}+(n^2-2\alpha \sin pt)u=0,$$

with solutions to a first approximation which can be identified with those given above.

A further step is the introduction into the equations of the x^3 terms found experimentally to be successful in the previous sections. The amplitude will now be fixed, and a vibration found to be possible over a range of frequencies and with varying phase angle.

---

* "Sound," I, p. 82.

Equations (16) become

$$-\omega^2mx+sx-\tfrac{3}{4}s'x^3=-\frac{Ai}{2}\,x\sin\varphi$$

$$\omega rx+Dx^3=\frac{Ai}{2}\,x\cos\varphi$$

which may be written

$$\left.\begin{aligned}\frac{-m\omega^2+s}{\omega r}-Bx^2&=-A\sin\varphi\\ 1+Dx^2&=A\cos\varphi\end{aligned}\right\}\quad\ldots\ldots\quad(19)$$

These equations have been examined arithmetically in the manner described in Section III, and the curves in Fig. 13 for values of $A=1{\cdot}1$, $1{\cdot}2$, $1{\cdot}5$ and 2 are the result, taking $B=0{\cdot}5$ and $D=0{\cdot}1$.

These curves explain a number of the experimental results. It is clear, for instance, how the range of frequency over which vibrations are possible is greater the greater the current (and the value of A), and that as the frequency is lowered vibrations will increase in amplitude, and finally will cease abruptly (the undersides of the curves represent unstable conditions as before). But the sudden rise to large amplitude on decreasing the frequency and its persistence on again raising the frequency, as well as the necessity to give a starting flick, are unexplained.

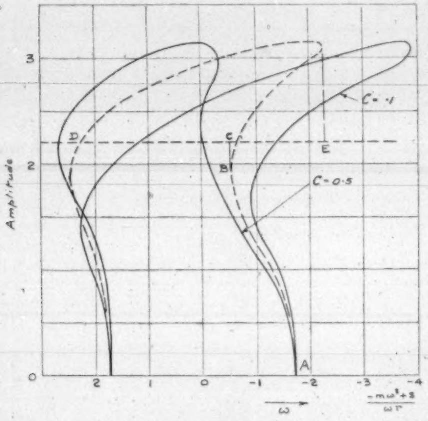

Fig. 14

A still closer explanation can be obtained by considering terms in the forces acting of higher powers of x. As the fork is vibrating at single frequency there will be a single frequency back E.M.F. produced proportional to x, which will cause a current to flow through the coils. The term $\Phi_i{}^2\,.\,2d\,.\,x^3$ in the expansions of equation (2) will therefore contain a term proportional to $x^5\sin^5\omega t$, which will contain a fundamental term. As to its sign it will be more or less opposing the pull, and will be positive on the right-hand side of (19) (i). Its phase will be fixed relatively to x, and there will also be a damping term in equation (4) (ii), but this will be neglected as merely flattening the curve. Making this addition, therefore, equation (19) becomes

$$\left.\begin{aligned}\frac{-m\omega^2+s}{r}-Bx^2+Cx^4&=-A\sin\varphi\\ 1+Dx^2&=A\cos\varphi\end{aligned}\right\}\quad\ldots\ldots\quad(20)$$

with $A=2$, $B=0{\cdot}5$, $D=0{\cdot}1$ and $C=0{\cdot}05$ and $0{\cdot}1$, the curves in Fig. 14 are obtained.

These explain some of the outstanding points quite well if it is imagined that at the dotted horizontal line the prongs strike the core. Referring to Fig. 14, lowering the frequency from *A*, the amplitude will rise steadily to *B* and then suddenly to *C*. On lowering further from *C*, the amplitude collapses suddenly at *D*. On raising the frequency from *C* a little and damping a stable position is found if the fall is on to the curve between *A* and *B*, but the collapse is complete if outside *A*. In any case the amplitude collapses at *E*.

Another effect of the double frequency excitation was found when the amplifier valve was overrun, so that a second harmonic was present in the fork-exciting current. On reversing the leads to the fork the amplitude of vibration changed quite considerably. This effect was noticed at all frequencies, but it was more marked near the resonant frequency. It also occurred with small amplitudes as well as with large ones.

This change of amplitude can be explained by a linear theory. In equation (2) the flux Φ_i due to the current can be written $\Phi_i \sin \omega t + \Phi_2 \sin (2\omega t + \psi_1)$, and in the expansion for the pull there will be terms $2\Phi_0\Phi_i \sin \omega t$ and

$$2\Phi_0\Phi_2 \,.\, 2bx \sin (2\omega t + \psi_1) \sin \omega t,$$

yielding single frequency pulls, besides others in higher powers of x.

When the terminals are reversed we change the phase of the current, and therefore of the flux by 180 deg., so that the flux becomes

$$-\Phi_i \sin \omega t - \Phi_2 \sin (2\omega t + \psi_1).$$

But the sign of x must also be changed because the phase of the vibration will also be changed by 180 deg. Thus the pull becomes

$$-2\Phi_0\Phi_i \sin \omega t + 4\Phi_0\Phi_i bx \sin (2\omega t + \psi_1),$$

instead of

$$+2\Phi_0\Phi_i \sin \omega t + 4\Phi_0\Phi_i bx \sin (2\omega t + \psi_1).$$

The second term is acting in the reverse sense when the terminals are reversed.

To study the effect of such an impurity of wave form on the shape of the resonance curve, the equation of motion may be written

$$m\ddot{x} + r\dot{x} + sx = A_1 \sin (\omega t + \varphi) + A_2 x \sin (2\omega t + \psi + \varphi).$$

With $x = x \sin \omega t$ this becomes

$$-m\omega^2 x \sin \omega t + \omega r \cos \omega t + sx \sin \omega t$$

$$= A_1 \sin \omega t \cos \varphi + A_1 \cos \omega t \sin \varphi + \frac{A_2}{2} x \{\cos \omega t \cos (\psi + \varphi) - \sin \omega t \sin (\psi + \varphi)\}$$

which gives

$$x^2 = \frac{A_1}{\sqrt{(-m\omega^2 + s)^2 + (\omega r)^2 - \frac{A_2^2}{4} \cos^2 \psi + \frac{A_2}{2} \sin \psi}},$$

and

$$\tan (\varphi + \psi) = \frac{\omega r}{-m\omega^2 + s}.$$

From these expressions it appears that the amplitude depends upon ψ. If ψ is $+\frac{\pi}{2}$, the amplitude is decreased, if $-\frac{\pi}{2}$ it is increased, if 0 it is increased.

In the first and second cases $\tan \varphi = \frac{\omega r}{-m\omega^2 + s}$, since $y=0$, but in the third and in all intermediate cases φ will be different, so that at resonance φ is not $\frac{\pi}{2}$. In all cases, however, both the amplitude and the phase changes will be symmetrical about the resonance value of ω, but as in no case is the resonance simple, the circle and straight line construction cannot succeed.

It may be noted that in the second and third cases, if $\frac{A_2}{2} = \omega r$, the amplitude at resonance is ∞.

This corresponds to the conditions examined in equation (16).

When the amplitudes are large, so that the effects of the higher powers of x are to be taken into account, a still more complicated resonance curve must result.

VII. Distortions of Second Type.

Everything that had been done up to the present indicated that with very small amplitudes the resonance curve of the fork would be practically symmetrical, and it was hoped by using a Kennelly mirror to measure the amplitudes and by measuring also the impedance of the fork to calculate the small amplitude fork constants in a similar manner to the methods used by Kennelly on the telephone receiver. But the resonance curve obtained was distorted in the manner shown in Fig. 15. The circle and straight line construction carried out on this curve gave two lines of unequal slopes for the tan a/ω curve meeting near the resonance value, and the same form was given by the construction carried out on the impedance/frequency curve. With the Kennelly mirror removed, however, a perfectly straight line was obtained from the impedance/frequency curve. Evidently the distortion at the very small amplitude (about 40 microns) in the experiment was not inherent in the fork itself, but was produced by the Kennelly mirror.

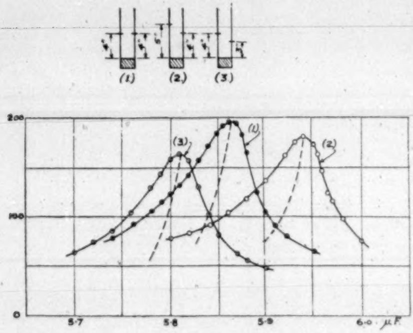

Fig. 15

The same type of resonance curve has also been found with a laboratory-made fork, a vibration galvanometer, telephone receivers, and a single reed of a multiple reed frequency meter. It appears to be a " coupled circuit " effect, with the coupling either insufficient to give a double hump or involving too much addition to the damping. In some cases, however, the double hump does appear, as found by Kennelly for the telephone receiver and R. L. Jones[*] for the vibration galvanometer.

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[*] Proc. Phys. Soc., Vol. 35, p. 67.

(i) *The Laboratory made fork* was simply two bars of hard steel clamped to a fairly massive iron block. The position of the block could be moved so as to alter the prong length, and the fork was supported on a wooden stand. With a fork length of $8\frac{1}{2}$ in. (frequency 120), the curves of Fig. 15 were obtained with riders in different positions along the prongs. It is seen that the main effects of throwing one prong or the other out of balance are to move the curve bodily and to increase the damping a little. The form of the curve as indicated by the slope of the mid-line remains substantially the same. Even when the out of balance was made very much greater this remained the case.

The amplitudes of the curves of Fig. 15 are comparatively small. In Fig. 16 the exciting current has been increased to make the amplitude three times as great, and the unstable effect has just appeared. The shape of the mid-line shows clearly the presence of both types of distortion.

The amplitude of the two prongs as read by the microscope was very nearly the same. A very thin strip of indiarubber was lightly stretched between the two prongs, and attached by rubber solution. The microscope showed clearly that the centre of the rubber—that is, the point mid-way between the two prongs—was stationary, even when the fork was vibrating vigorously. The centre of mass of the system does not move. It does not follow that the actual amplitudes of vibration of the prongs with regard to the block to which they are rigidly clamped are identical. They may be different both as regards amplitude and phase, the vibration of the block on its support to the stand being such as to keep the centre of mass stationary. The amplitudes that are read by the microscope are thus compounded of the motion of the prong and that of the whole fork, and these amplitudes are identical.

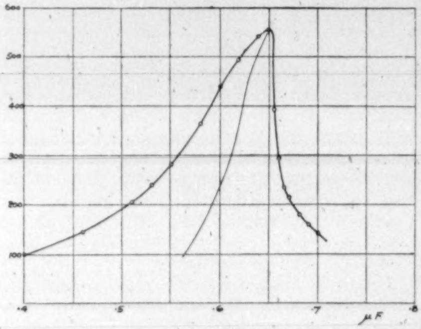

Fig. 16.

If in Fig. 17 (i) a_0, b_0, c_0 represent the positions of the prongs and the centre line at the tip of the prongs at test from some fixed point, then $c_0=\frac{1}{2}(a_0+b_0)$. If the vibration of the left-hand prong is $a \sin(\omega t+\varphi_1)$ and of the right-hand prong $b \sin(\omega t+\varphi_2)$, and of the whole fork about the centre line $c \sin(\omega t+\varphi_3)$, then the position of the centre of the prongs at any moment is given by

$$\frac{a_0+a \sin(\omega t+\varphi_1)+b_0+b \sin(\omega t+\varphi_2)}{2}+c \sin(\omega t+\varphi_3),$$

and the movement by

$$\tfrac{1}{2}\{a \sin(\omega t+\varphi_1)+b \sin(\omega t+\varphi_2)\}+c \sin(\omega t+\varphi_3).$$

This movement as has been shown experimentally is zero, hence

$$a \sin(\omega t+\varphi_1)+b \sin(\omega t+\varphi_2)+2c \sin(\omega t+\varphi_3)=0. \quad . \quad . \quad . \quad . \quad (21)$$

The amplitudes that are measured from a fixed reference point are the maximum values of

$$a \sin(\omega t+\varphi_1)+c \sin(\omega t+\varphi_3),$$

and

$$b \sin(\omega t+\varphi_2)+c \sin(\omega t+\varphi_3),$$

which (using (21)) are equal to

$$\pm\{\tfrac{1}{2}a \sin(\omega t+\varphi_1)-\tfrac{1}{2}b \sin(\omega t+\varphi_2)\},$$

respectively.

This is shown vectorially in Fig. 17 (ii), where Oa and Ob represent the vibrations of the left and right hand prongs about the supporting block, with a vector sum Oc'. Oc drawn in the opposite direction to Oc' and half its length represents the motion of the whole fork about the stand. OA is the vector sum of Oa and Oc and OB that of Ob and Oc. OA and OB are the amplitudes that are measured, and they are equal. It is also clear that OA is the vector sum of $\tfrac{1}{2}Oa$ and $-\tfrac{1}{2}Ob$.

The construction thus found is used in Fig. 18 to find the form of the observed resonance curve when the prongs are out of balance both as regards natural frequency and damping. Rays drawn from O_1 and O_2 to AB represent the mechanical impedances of the two prongs and their intercepts on the circles represent the vector velocities (with phase angles multiplied by -1). Since the excitation of the two prongs is always 180 deg. out of phase the required observed amplitude is to be obtained by adding the circle intercepts instead of subtracting them. Thus, at the point ω', O_1a_1 represents the motion of the first prong and O_2a_2 that of the second. a_2a_1' is drawn equal and parallel to O_1a_1 and O_2a_1' set out horizontally from ω' to give a point r' on the resonance curve.

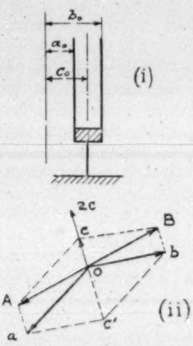

Fig. 17.

It is seen that the shape of the curve is similar to those obtained experimentally, but that the slope of the mid-line is reversed as regards frequency. To obtain a slope in the right sense the prong with the higher natural frequency must have the higher damping, and it would appear that this condition must hold generally.

Explained on this basis the distortion of the curve of the Koenig fork by the Kennelly mirror would be due to an increase of the stiffness of the prong by the spring of the mirror, and at the same time an increase of the damping of the prong.

(ii) Resonance curves obtained from a vibration galvanometer of the bifilar type were of similar shape. As the current is increased the curve is moved bodily to a lower frequency. The conditions here are somewhat similar to those in the home-made fork, the observed amplitude depending on the motion of each of the stretching wires and upon any movement of the whole system. The lowering of frequency can be ascribed in a general way to an increase of coupling with current, but much further work remains to be done before the curves can be explained completely.

(iii) In the tuning fork and the vibration galvanometer there are clearly two separate members of the system which can have different resonant frequencies and produce by their interactions the "coupled circuit" effects. But in the cases of the telephone receiver diaphragm and a single reed, which can give similar curves, it must be imagined that the single member is capable of two slightly different modes of vibration in order to explain the results. Both the telephone receiver and the single reed show also distortions of the first type so that the unstable effect can be produced. This must have a great effect on the performance of loud-speakers when over-loaded.

CONCLUSION.

The resonance curve of apparently simple mechanical vibratory systems are in general distorted; if a vertical line is drawn through the maximum amplitude in a curve of amplitude of vibration plotted against the frequency of excitation, then the curve is not symmetrical about this line. This occurs in such systems as tuning forks, reeds single or double, vibration galvanometers, and telephone receiver diaphragms.

It has been found that the lack of symmetry may be of two different types. The first case is that of a device acting as a perfectly simple or single system, which with very small exciting forces and consequently very small amplitudes, does give a symmetrical resonance curve. But with larger forces and consequently larger amplitudes distortion of the curve appears. In the Koenig tuning fork investigated, the distortion has been shown to be due to a decrease of resonant frequency and an increase of decay factor with amplitude. These changes are brought about by the non-linearity with amplitude of the forces brought into play. The complete explanation is very complicated. Both the mechanical restoring forces and the electro-magnetic forces of the driving arrangement show this non-linearity, so that the distortion of the resonance curve is due in part to one and in part to the other. It is shown, however, that a close approximation to the form of the resonance curve found is obtained if in the equation of a simple driven system $m\ddot{x}+r\dot{x}+sx=f$, s is replaced by $(s_1-s_2x^2)$, and r by $(r_1+r_2x^2)$. The separation of

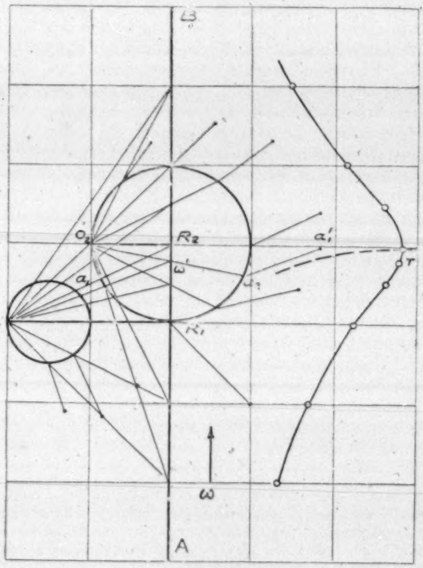

FIG. 18.

the mechanical and electro-magnetic components of the distorting forces has not been completed, and further work remains to be done here. It has, however, been found that all the factors s_1 s_2 r_1 and r_2 depend upon the nature of the support of the fork on the one hand, and on the position of the exciting coil, whether near the tips of the fork or nearer the base, on the other hand, so that the relative magnitudes of the two disturbances will be very different in different cases.

It has further been found as a natural consequence of the non-linearity, that unstable effects may be produced, so that the amplitude of vibration depends upon whether the frequency in question has been approached from a higher or a lower frequency, and that excitation of large amplitude vibrations of fundamental frequency can be carried out by currents of double frequency.

The second cause of dissymmetry of resonance curves is a "coupled circuit" effect, and occurs at small amplitudes as well as large ones. The mechanical system even though apparently single or simple, has certain inherent out of balance which makes it in effect two individual members coupled together. The form of the resonance curve in this case is different from that of the first; the mid-line of the curve bends over at the top in the first case and is vertical at the bottom. In the second case it is vertical at the top and bends over at the bottom.

Even though the system shows this second type of distortion, the first type appears when the amplitudes are large. The mathematical form of the resonance curve must then be very complicated.

It appears, therefore, that for use as a frequency meter, or for fixing the wavelength of a wireless station from a valve-maintained tuning fork, or for any purpose in which it is important that the frequency of the fork should be as nearly absolutely constant as possible, it is necessary to work with very small amplitudes of vibration.

The whole of the experimental work was carried out in the Telegraphy and Telephony Laboratories of the City and Guilds (Eng.) College, where valuable assistance was given by Mr. I.L. Turnbull, a senior student, in many of the experiments and by Mr. F. W. Andrews in the design and construction of the apparatus. The author was throughout indebted to Prof. C. L. Fortescue for ever-ready discussion and suggestion.

DISCUSSION.

Mr. ALBERT CAMPBELL: It is interesting to know that tuning forks when driven at large amplitudes may show the condition of instability described by the author. I have observed similar instability with strip vibrators of thin steel, used in a frequency meter of adjustable reed type (Phys. Soc. Proc., 14, p. 267, 1896). The resonance point is slightly different with ascending and descending pitch. I would also draw attention to the interesting series of resonance curves given in Dr. R. Hartmann-Kempf's book, "Elektro-Akustische Untersuchungen" (Gebr. Knauer, Frankfurt A.M., 1903). Dr. Mallett is to be congratulated.

XXVIII.—ATOMIC NUCLEI AND THEIR TRANSFORMATIONS.

The Twelfth Guthrie Lecture. Delivered February 25, 1927.

By Sir ERNEST RUTHERFORD, *O.M., P.R.S.*

IN the year 1911, when I put forward the nuclear theory of atomic structure based on the interpretation of the simple scattering of α particles by matter, there seemed little hope of any immediate progress in our knowledge of the detailed constitution of the nucleus. So much was this the case that in a discussion on atomic structure before the Royal Society in 1913, in answer to an enquiry as to the constitution of the nucleus, I replied that a consideration of this question had best be left to the next generation. While much more progress has been made in the interval than seemed likely at that time, we cannot but recognise that only a beginning has yet been made in the attack on this most difficult and fundamental of problems in Physics. It may be of interest, however, to give a brief résumé of the main additions to our knowledge in recent years, and then later to consider in more detail some of the more promising lines of attack.

From the earlier experiments on scattering of α particles, it seemed certain that the space occupied by the nucleus of an atom was minute compared with that occupied by its whole structure. Consequently, the constituent parts of the charged nucleus must be held together in this minute volume by such powerful forces that we could not hope to influence them seriously by the physical forces available in the laboratory. The great magnitude of the forces in the nucleus of a heavy atom was clearly manifested by the enormous individual kinetic energy of the α and β particles set free during the radioactive transformations. Later, by the work of Ellis, Meitner and others, definite evidence was obtained that the greater part of the γ radiation also had its origin in the nucleus. The wave-length of some of these γ rays was measured, thus giving us definite information of some of the modes of vibration of the nuclear constituents. The individual quantum energy of some of these γ rays is very high, corresponding in some cases to at least 3 million electron volts.

The wonderful series of transformations manifested in the elements uranium, thorium and actinium provided us with a wealth of data on the modes of transformation of these heavy elements; but, unfortunately, our theories of nuclear structure are as yet in such an embryonic state that we are able to make little if any use of the numerous facts that have been accumulated. I shall refer later to this subject, but it suffices at present to point out that the study of these transformations did give us some general information of great importance about nuclei.

Since both helium nuclei (α particles) and swift electrons (β particles) are hurled from the nucleus, it could safely be deduced that the nucleus of these heavy atoms must contain electrons and helium nuclei as part of their structure, unless it be supposed that the helium nucleus is in some way formed of simpler constituents at the moment of its expulsion from the main nucleus.

In another direction, the study of the radioactive transformations gave us the most convincing evidence in support of the nuclear theory, and brought to light the extraordinarily simple relations which exist between the change in atomic number in a disintegrating element and the nature of the transformation. The expulsion of an α particle, which carries a charge $2e$, lowers the nuclear charge by two units, while the expulsion of an electron raises the nuclear charge one unit. This generalisation, known as the Displacement Law, first put forward in general form by Russell, Fajans and Soddy, gives us at once the nuclear charge and mass of each element in the radioactive series, and thus fixes the ordinary physical and chemical properties of each radioactive element, as well as its atomic weight. This relation is remarkable both for its simplicity and generality, and to my mind represents a contribution to our knowledge of nuclei of the greatest importance and interest.

I can only refer in passing to the great step in advance made by Moseley in indicating that the nuclear charge of an element in fundamental units is represented by its ordinal or atomic number. This deduction has been directly verified by Chadwick in several cases by accurate measurement of the scattering of α particles.

The discovery of elements in the radioactive series, now known as isotopes, which are chemically identical but differ in mass and radioactive properties, was a notable addition to our knowledge. It was at once seen that the identity of chemical behaviour was a definite proof that isotopes had the same nuclear charge, while the difference in mass and radioactivity showed that the nuclei of the isotopes differed in constitution and stability. The difference in mass of some of the radioactive isotopes is noteworthy. For example, in these radioactive series there are believed to be 7 isotopes of lead, which vary in mass between 214 and 206, while their half-periods of transformation for the radioactive isotopes vary between 27 minutes and 16 years. The end products, uranium-lead (206) and thorium-lead (208), are believed to be permanently stable.

I need only refer in passing to the proof by Aston that many of the ordinary elements consist of a mixture of isotopes. One of the important deductions from his work is the "whole-number" rule, for the mass of each isotope is found in many cases to be nearly a whole number expressed in terms of the mass of oxygen taken as 16. This indicates that the ultimate building unit of nuclei is of mass unity in the nuclear structure. This unit, known as the proton, is slightly less in mass than the hydrogen nucleus mass 1·0072, the difference being ascribed to the packing effect—that is, to the interaction of the electromagnetic fields in the highly condensed nucleus. Later experiments have shown, however, that while the whole-number rule holds very approximately in many cases, it is widely departed from in others. The accuracy of the "whole-number rule" is now under examination by Dr. Aston, using a mass spectrograph of much greater resolution, so that we may soon expect to have much more precise information as to the relative masses of the isotopes. If these constants can be obtained with sufficient precison, they must certainly prove of great value in throwing light on many points of nuclear structure. The reason of this will be clear from the discussion later in this Paper.

We have seen that radioactive evidence indicates that helium nuclei and electrons are constituents of the nuclear structure of heavy atoms; but, notwithstanding the fundamental character of the radioactive transformations, in no case is a proton liberated from the nucleus. The general evidence strongly supports the view that the electrons and protons are the fundamental units of nuclear structures, and this has been directly confirmed by the experiments of Rutherford and Chadwick, Pettersen and Kirsch, on the artificial disintegration of light elements by α particles. For twelve elements definite proof has been obtained that protons are ejected at high speed when the material is bombarded by swift α particles. With the exception of carbon and oxygen, on which the experimental evidence is conflicting, all elements from boron to potassium inclusive liberate protons under bombardment.

In the majority of these experiments, the scintillation method has been employed where certain observations can only be made of the emission of particles which have a range greater than the scattered α particles, for the number of the latter are usually greatly in excess of the number of protons ejected. In general, the protons appear to be emitted about equally in all directions relative to the bombarding pencil of α particles; but the speed of the protons is greater in the forward than in the backward direction, probably due to the motion of the bombarded nucleus.

Unfortunately, the amount of disintegration effected by even the swiftest α particles is very small, only one proton being liberated for about 100,000 bombarding α particles. The experimental study of this artificial disintegration of elements, while simple in principle, is in practice beset with many experimental and observational difficulties, while an elaborate technique is required to obtain quantitative results.

DIMENSIONS AND LAW OF FORCE.

We have seen that the study of the scattering of α particles first disclosed the nuclear structure of atoms, and even now this is the only trustworthy method of investigation of the law of force in the neighbourhood of a nucleus, and for giving some idea of the dimensions of the nuclear structure. In the classical experiments of Geiger and Marsden, the scattering of α particles at various angles was found to be in accord with the coulomb law of force for the elements silver and gold. This question has recently been re-examined by Rutherford and Chadwick by a modified method with similar results. Even for the swiftest α particles available, the scattering of α particles through an angle of 135° for the elements copper, silver and gold showed no observable departure from that calculated on the inverse square law. For these elements the closest distance of approach of the α particle to the nucleus using α particles from radium C of velocity $1{\cdot}922 \times 10^9$ cms./sec. is 1·23, 1·94, $3{\cdot}15 \times 10^{-12}$ cms. for copper, silver and gold respectively. Since it is to be expected that the coulomb law of force would be departed from if the α particle entered the main nuclear structure, we can thus conclude that the radius of the charged nucleus, assumed spherical, must be less than the above numbers for copper, silver and gold respectively. Until α particles of higher speed are available there is no way of fixing the minimum limit of the nuclear dimensions by these methods. Similar experiments made with a thin sputtered layer of uranium metal indicated no certain change of the law of force at a distance 3×10^{-12} cms. from the centre of the nucleus. This observation raises a considerable difficulty, for it will be seen later that certain radioactive data indicate that the uranium nucleus, or at any rate some constituents of its structure, must extend to more than twice this distance.

In the case of the light atoms of small nuclear charge, the α particle in a close collision can approach much nearer to the nucleus, and we are thus enabled to study the law of force at much smaller distances. The experiments of Rutherford and the detailed investigations of Chadwick and Bieler early disclosed that the coulomb law of force was no longer applicable when swift α particles collided with hydrogen nuclei. The variation of scattering with angle as well as the variation with velocity of the α particle, are almost the reverse of that to be expected on an inverse square law. For the swiftest α particles, the number of H particles shot forward within a few degrees of the direction of the α particle is about a hundred times the calculated number.

The nature of the results obtained may be illustrated by the following example. The number of H particles projected after collision in directions making angles from 20° to 30° with the line of flight of the incident α particles was counted. When the velocity of the α particle was small, range about 2 cms. in air, the number of α particles scattered in this specified direction was about that to be expected on a coulomb law of force. With increase of velocity of the α particle, the number of H particles observed was greater than the theoretical, and the discrepancy increased very rapidly with rise of velocity of the incident α particles. For example, with α particles of 2·9 cms. range, the observed number was three times the calculated, while for α particles of 6·6 cms. range, the ratio rose to nearly thirty.

This failure of the inverse square law was interpreted by Chadwick and Bieler in the following way: Assuming that the H nucleus could be regarded as a point charge, a model was sought for the He nucleus (α particle), which would account in a general way for all the experimental results. It was found, from the calculation of Darwin, that an α particle behaved in these conditions as a body with properties intermediate between those of a charged elastic sphere and a charged electric plate, and it was compared as a first approximation, with an elastic oblate spheroid of semi-axes about 8×10^{-13} cm. and 4×10^{-13} cm., moving in the direction of its minor axis (see also p. 369). An H nucleus projected towards such an α particle would move under the ordinary electrostatic forces until it reached the spheroidal surface of the above dimensions. It would thus encounter a very powerful field of force, and recoil as from a hard elastic body. It was not possible to make any close comparison with this model, owing to the difficulty of calculating the collision relations for such a spheroid.

Looking at this model from a physical standpoint, we may say that the scattering is normal for all particles which do not enter the spheroidal surface, but any particle which crosses this surface encounters a deflecting field in which the forces on the particles increase much more rapidly than is to be expected on an inverse square law of force, so that the scattering is abnormal.

Before discussing this model in more detail, we shall now consider some experiments which Dr. Chadwick and I have recently made to examine the scattering of α particles in helium by a similar method. In this case, the particles concerned in the collision are identical in charge and mass, and have the same structure. There is, however, one difficulty in these experiments which does not arise in the investigations of the collision between α particles and H nuclei, where the recoiling H particles on account of their greater range can in most cases be observed independently of the incident α particles. In the case of a collision between two particles of equal mass, the angle between the scattered α particle and the recoiling nucleus is always 90°, and there is no way of distinguishing between the two types of particles. For example, suppose we are observing the particles which are scattered at an angle φ with the direction of the incident particles. These will include the α particles scattered through an angle φ and the recoiling nuclei which arise from α

particles scattered through 90° $-\varphi$, and in many cases, where the law of inverse square is no longer applicable, we can only estimate roughly the relative importance of the two groups.

Taken as a whole, the distribution of scattered particles in collisions between α particles and *He* nuclei is very similar in type to that observed between α particles and *H* nuclei. There is a similar concentration of scattered α particles in the forward direction, which is the more marked the swifter the incident α particle. In one respect, these experiments gave us information which could not easily be obtained from the investigation of *H* nuclei. When the scattered particles are observed between 40° and 50° with the incident rays, the number of scattered α particles and the helium nuclei set in motion by recoil are equal in number and velocity. The curve of variation of number with the energy of the incident α particles is indicated in Fig. 1, where the ordinates represent the ratio of the observed to the calculated number and the abscissæ the value of $1/E$, where E is the energy of the colliding α particles. The dotted line for ordinate 1 represents the relation that would be observed for an inverse square law. It is seen that the ratio is large for swift α particles, falls rapidly below unity and then rises again to the calculated value. A much more marked drop below the calculated value is observed for α particles of range about 3 cms. when the scattering is observed between 10° and 20°.

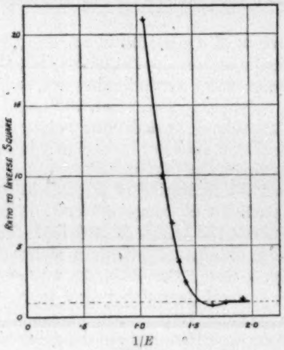

FIG. 1.

It is thus clear that for a definite range of velocity of the α particle, the scattering between certain angles is greater than the "normal" value—i.e., the value calculated for point charges—i.e., obeying the inverse square law. For another range of velocity, the scattering falls below the normal, rising ultimately to the normal value as the speed of the α particles decreases.

This deficiency of scattering between certain angles for α particles of given velocity is to be expected for any type of collision, where the scattering is abnormally large for another range of angles of scattering. As this conclusion is of a very general character and quite independent of any knowledge of the forces involved in a collision, it may be desirable to discuss it in some detail.

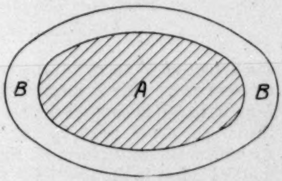

FIG. 2.

In Fig. 2 is represented a central section of an imaginary surface which represents the boundary dividing the region where the forces due to a nucleus are normal and abnormal. Any particle which does not enter this surface suffers normal scattering, but any particle swift enough to penetrate within the surface is deflected from its path by some unknown combination of forces. Suppose, for simplicity, the α particles are projected normally to the section in Fig. 2. This section consequently represents the target area exposed to the α particles. Suppose, for illustration, that all particles which fall within the inner area A are scattered within a certain range of angles, and that the number of α particles so scattered is above normal. Since the number of α particles falling on the surface is fixed by its target area, and each particle is scattered in one direction or another, the excess of scattered particles due to the target area A must be exactly counterbalanced by a defect in the number scattered through another range of angles by the region B. This must always be true for a *given velocity* of incident α particles provided that no α particles are absorbed by the nucleus.

In some respects, this conclusion that an excess of α particles in one direction must be balanced by a defect in another direction is an obvious deduction, but the utility of this conception in interpreting the phenomena of scattering has not, so far as I am aware, been fully recognised. It can only be applied when the angular distribution of scattered α particles has been determined for a definite velocity of incident α particles, and has the great advantage that it does not involve any knowledge of the nature of the collision or of the structure of the nucleus.

For example, Chadwick and Bieler examined the scattering of H particles for α particles of range 6·3 cms. in air between the angles 0° and 60°. The curve obtained by them is shown in Fig. 3, where the ordinates represent the ratio of the number of H particles to incident α particles. Ultimately above a certain angle of deflexion the scattering must become normal. The value calculated on an inverse square law is shown in curve B. Extrapolating the observed curve, the two intersect at P for an angle of about 80°, which represents the division between the two angular regions where the scattering is normal and abnormal. Since there is a large excess of α particles scattered within the angles of 0° to 60°, there must be a corresponding defect in number between 60° and 80°. This is illustrated in Fig. 4, where the number scattered per degree from 0° to 80° is compared with the theoretical number. The area AA above the theoretical curve must be balanced by the area between this curve and BB. Assuming the correctness of the data involved, it is clear that there must be a marked defect in the scattering about 70°, and that the curve rises rapidly about 80°, where the scattering becomes normal.

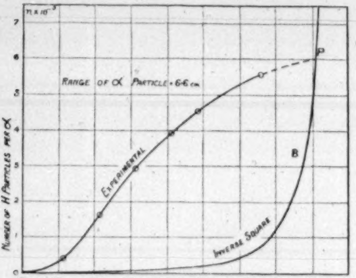

FIG. 3.

On account of the shortness of the range of the H particles scattered at such large angles, it is difficult to verify this deduction experimentally, but there is little doubt of its essential correctness.

In this connection it is of especial interest to consider the scattering of α particles by aluminium, a subject which has been carefully studied the last few years. Bieler compared the scattering by thin sheets of aluminium and gold, and assuming that the scattering of gold was normal, concluded that the number of α particles scattered by aluminium was less than the normal. This defect from the calculated value increased with the speed of the a particles and the angle of scattering of the a particles. Rutherford and Chadwick examined the scattering at larger angles—viz., 135°—and found an unexpected result, which is illustrated in Fig. 5, where the dotted line represents the normal calculated scattering and the full curve the experimental results. The abscissæ represent the reciprocal of the energy of the a particle, so that the left side of the curve represents high velocities of the a particles.

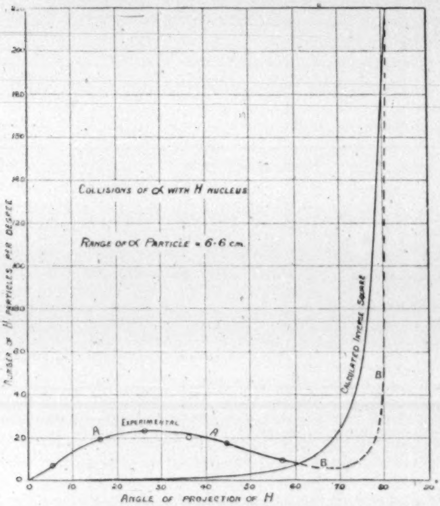

FIG. 4.

For low energies, the ratio is less than unity and decreases with increase of energy of the a particle, passing through a minimum corresponding to a particles of about 5 cms. range. For the highest range of particles used—viz., 6·6 cms.—the scattering curve is rising steeply. Similar results were observed for magnesium. For 90°, the scattering curves showed a marked defect from the normal, but no minimum was observed. There can be little doubt that if still higher range a particles could have been used, the curve would have passed through a minimum corresponding to Fig. 5.

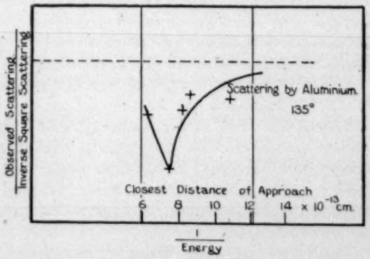

FIG. 5.

Using the data published by Bieler, Rutherford and Chadwick, it is possible to estimate the defect of scattering for a 5 cm. a particle for all angles between 30° and 135°. The results are plotted in Fig. 6. It is seen that there is a marked defect of scattering over this wide range of angles. From what has already been said, it is clear there must be an excess of scattering in another angular region to compensate it. It does not seem likely that the scattering round 180° can be sufficiently large for such a compensation, for it would mean an increase in this region of the order of one hundred times the calculated value. It is not easy to investigate this point, but the experiments now in progress ought to settle whether any marked excess is observed. Failing compensation in this region, there must be an excess of scattering in the region between 0° and 30°. This is under investigation, but it may be difficult to obtain results of an accuracy required to settle this point, since a small percentage excess of scattering at a small angle may easily compensate for the defect over a range of 100° shown in the figure.

The examples referred to all illustrate in an interesting way the usefulness of this principle of compensation and the suggestions it offers of useful directions of investigation.

The principle is only valid if there is no disappearance of α particles in the nucleus. Now it is known that protons are ejected from aluminium and magnesium by bombardment of α particles, and it is quite probable that the collision which leads to the ejection of a proton involves the capture of the α particle.

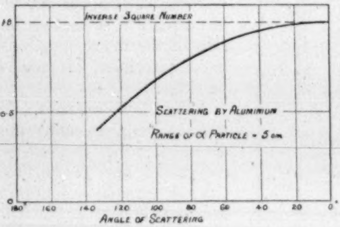

Fig. 6.

Blackett has given strong evidence by examination of the Wilson tracks in a disintegrating collision that capture of the α particles does occur in nitrogen. Unfortunately the data at present available as to the number of protons ejected at various angles with the direction of the α particle in aluminium and magnesium are too uncertain to give a decisive answer as to whether the defect of scattered α particles shown in Fig. 6 can be accounted for by capture of α particles. The evidence available rather indicates that the number of ejected protons is only a fraction of the defect in scattered α particles. If this be the case, we must probably look for the defect of scattering by an excess at other angles.

NUCLEAR FORCES AND STRUCTURE.

We have so far compared the observed scattering of α particles by nuclei with that calculated on classical mechanics. Assuming that the nuclei behave as point charges, any divergence from calculation has been assigned to departure of the forces from a coulomb law or to a nuclear structure comparable in dimensions with the distances involved in close collisions.

During the last year, however, the adequacy of classical mechanics for a solution of atomic problems has been questioned, and a new mechanics has arisen largely based on the work of Heisenberg, Schrödinger, Born, Dirac and others. The new mechanics has already shown its power by giving a correct solution of a variety of atomic problems, and it has been suggested that the abnormal scattering of α particles by light elements may be only apparent and due rather to a failure of the classical mechanics when applied to collisions than to any failure in the coulomb law of force.

Recently, however, Wentzel, Born and Oppenheimer have shown that on certain assumptions the laws of scattering by a central field of force obeying the law of inverse squares are of the same form on wave mechanics as on classical mechanics. A general attack on this problem of collisions is in progress, but I understand that apart from minor perturbations it seems unlikely that the wave mechanics can offer any explanation of the abnormal scattering of high velocity α particles by the light elements hydrogen, helium and aluminium.

Schrödinger and De Broglie have given certain criteria for the conditions under which marked diffraction patterns in collisions may be expected. In a Paper in course of publication,* kindly shown me by Mr. Blackett, these criteria have been interpreted in terms of collision-relations founded on classical mechanics. Unfortunately the criteria are not very precise, and until some scattering phenomena have been completely explained on the wave mechanics, so that the requisite comparative data are available, it is difficult to be certain where marked diffraction patterns may be expected to disclose themselves in the type of collision we have been investigating, where swift massive particles collide with heavy nuclei. In general, if we try to fix the conditions by assuming, say, that the abnormal scattering of α particles by aluminium is explicable on wave mechanics, it is found that the criteria break down completely for collisions of α particles with hydrogen and helium nuclei.

It thus appears probable that we have to seek for an explanation of abnormal scattering of α particles in the detailed structure of the nucleus and the forces arising from its constituent particles. We have also to take into consideration the enormous distorting forces in the nuclei which must come into play in these very intense collisions where the nuclei approach very closely to each other. In my original Paper† on the scattering of α particles by hydrogen nuclei, I drew attention to the importance of these distorting forces in altering the normal structure of nuclei, and thus the forces in their neighbourhood, and pointed out that at short distances the forces might change from those of repulsion to those of attraction.

In order to account for the defect of scattering in aluminium, Bieler worked out mathematically the alteration of the scattering to be expected if the α particle were acted on by a combination of forces—viz., the ordinary repulsive electrostatic forces between the nuclei, and an attractive force varying as the inverse fourth power. In this way, he was able to account approximately for his experimental results with aluminium. This question has been recently investigated in more detail by Debye and Hardmeier on a slightly different basis. They assume that as the α particle approaches closely to the nucleus there will be a polarisation of the nucleus in the same way as a charged particle in approaching a neutral atom is able to polarise its structure. This polarisation involves an attractive force on the α particle, varying as r^{-5}, where r is the distance from the centre of the nucleus. The forces are supposed to be proportional to the volume occupied by the nucleus. By reasonable assumptions as to the constants involved, they have been able to calculate the scattering of α particles of different velocity when the scattering angle is 135°. In this way, they obtain a theoretical scattering curve for aluminium, which resembles in general features the curve shown in Fig. 5. The defect in scattering is similar to that observed, and there is the same pronounced minimum in the scattering curve. It thus seems not unlikely that the main peculiarities of the scattering of α particles by aluminium and magnesium may be due to polarisation of the nucleus by the charged α particle. It is, however, not likely that the attractive forces which arise in this way can be expressed by a simple power law, for the distances involved are of the same order as the dimensions of the nucleus which may not have a symmetrical structure. It would, however, be of great interest to work out the simple theory in detail to fix the scattering of α particles of given velocity for all angles. Such a calculation would give valuable evidence as to the angular region in which the number of scattered α particles is in excess of the normal in order to account for the defect at large angles already discovered.

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* Since published Camb. Phil. Soc., 33, pt. 6, (1927).

† Phil. Mag., 37, p. 561 (1919.)

Reference should be made to another point of interest in this connection. Since in a close collision of the types considered there should be considerable movement of the nuclear constituents which may not be moving in quantized orbits, energy may be dissipated in some form as a result of the collision. If this were the case, the ordinary laws of momentum and energy would not be applicable to such collisions. So far no conclusive evidence of such an effect has been observed, except in cases where the nucleus suffers disintegration. The velocity of the scattered α particle, as far as our experiments have gone, corresponds to that calculated on the basis of an elastic collision. The most definite method of testing this point is a careful examination of the photographs of forked tracks due to collisions obtained in an expansion chamber. Blackett has examined a number of such photographs, and within the small experimental error finds no evidence of a loss of energy even in very close collisions of α particles with light elements. This question is an important one, and even more accurate data are required to give a definite answer. It appears, however, fairly certain that the loss of energy due to such impacts cannot be more than a per cent. or two of the total energy of the α particle. It thus appears probable that the greater part, if not all, of the energy which is lost by the α particle in distorting the nucleus in the first stage of the collision is returned to the α particle as it leaves the nucleus. It should, of course, be borne in mind that the colliding α particle, as well as the recoiling nucleus, both suffer some distortion, and in the case of collisions of α particles with light atoms the distortion may be of the same order of magnitude for both nuclei.

We have already discussed some of the striking pecularities in the scattering of α particles by the hydrogen and helium nuclei, and the mode of interpretation of the main features of the phenomena. There are, however, a number of other interesting points to which reference should be made. In the interpretation of the collisions between α particles and H nuclei it was supposed that the latter might be regarded as a point charge, and the "shape" of the α particle, or rather of the region where the forces are abnormal, was estimated. It is of interest to compare the results obtained with hydrogen and helium. As we have seen for very close collisions, the scattering is very abnormal in both cases, but becomes normal for sufficiently slow α particles where the closest distance of approach is of the order 4×10^{-13} cm. The general evidence indicates that, if anything, the scattering remains normal for an even closer distance of approach in the case of helium nuclei than with H nuclei. If the complexity of the forces were ascribed entirely to the α particle, while the H nucleus acted as a point charge, we should expect the scattering to become abnormal at about twice the distance for collisions of α particles with helium nuclei than for collisions with hydrogen nuclei. If anything, however, the shortest distance of approach appears to be even smaller for helium than for hydrogen. It is, of course, very difficult to estimate the effect of distortion, but it would appear that the hydrogen nucleus or proton must have dimensions comparable with those of the *α* particle. This is an unexpected result, for it is usually supposed that the proton is of dimensions much smaller than the electron. Since, however, we are referring to the dimensions of the region where the forces are abnormal, rather than that occupied by the actual structure of the nucleus, it would be more accurate to say that the region round the proton where the forces are abnormal is comparable in dimensions with the corresponding region for the *α* particle.

It is, of course, very difficult to offer any definite explanation of the origin of the forces which are brought to light by a study of collisions, for it must be remembered that the nuclei are both in motion during the collision, and both are in an abnormal state due to the distorting forces arising from their close approach to each other. It may be, however, that the abnormal scattering is due, not only to the modification of the electrostatic forces by distortion, but that magnetic forces become very important in such close collisions, and, indeed, may play a predominant part in determining the scattering for light elements. During the last few years it has been suggested that the negative electron has a definite magnetic moment associated with it, and, if this be the case, it is not unreasonable to suppose that the proton has a corresponding magnetic moment. Frenkel, who has discussed the effect of magnetic forces on the structure of the *α* particle, supposes that the proton has a magnetic moment only 1/1840 of the electron. Whatever view we may take on this question, it is clear that we may expect strong magnetic forces near the nucleus whether due to the intrinsic magnetic moment of its constituents or to their motion. While no doubt the particles constituting the nucleus may tend to adjust themselves to reduce the resulting magnetic moment to a minimum, there must always be strong local magnetic forces in the neighbourhood of the constituent parts of any nucleus.

In the case of the helium nucleus, which may be supposed to be made up of four protons and two electrons, it is not difficult to calculate from available data that the effect of these magnetic forces becomes relatively prominent for distances of the order of 4×10^{-13} cm.—a distance at which we know the scattering of *α* particles of hydrogen and helium nuclei become markedly abnormal. It is important to note, also, that since the deflecting force on a charged particle due to a magnetic field is proportional to the speed of the particle, we are able to understand in a general way why the abnormality in the scattering in hydrogen and helium increases so markedly with rise in velocity of the *α* particle.

It seems probable that the magnetic forces to be expected in the neighbourhood of a helium nucleus are of the right order of magnitude to account in a general way for the abnormal scattering observed in close collisions with an *α* particle. In order to explain the scattering of *α* particles by hydrogen nuclei it has been deduced that the region in which the forces due to the *α* particle are abnormal is spheroidal in shape, and that this spheroid must move with its short axis in the direction of the *α* particle. This indicates a definite orientation of the *α* particle which may be brought about during the collision by the turning couples due to the interaction of the magnetic fields due to the two nuclei. While, of course, such considerations are largely speculative in character, the general evidence does support the view that the region surrounding the nucleus must be the seat of intense magnetic forces which largely control the abnormal scattering which is observed.

RADIOACTIVE NUCLEI.

We shall first discuss a marked discrepancy between the dimensions of the nucleus of uranium when estimated from scattering and radioactive data. Rutherford and Chadwick found that the scattering of a film of uranium appeared to be normal—i.e., to correspond to a coulomb law of force up to a distance $3{\cdot}2 \times 10^{-12}$ cm. It should be pointed out that while the variation of scattering with velocity of the α particle appeared to be normal within the experimental error, yet, owing to the difficulty of estimating the weight of the uranium film with accuracy, the actual number of scattered α particles could not be compared with calculation with any certainty.

The α particle expelled from uranium I has the smallest initial energy known, corresponding to $6{\cdot}77 \times 10^{-6}$ erg, or $4{\cdot}25 \times 10^{6}$ electron-volts. Since the α particle must gain a part at least of its energy in escaping in the repulsive field, it is easy to calculate on a coulomb law of force that, even if the α particle has no initial velocity when it leaves the nuclear structure, it cannot originate from a point less than 7×10^{-12} cm. from the centre of the nucleus of charge 90.

While it is impossible that positively charged particles like the proton or α particle can remain in equilibrium under a coulomb law of repulsive force, the case is quite different if the particles are electrically neutral. A neutral particle can be held in equilibrium by the attractive forces either due to the polarization of the neutral particle by the electric field from the charged central nucleus, or due to magnetic forces arising from the nucleus or to a combination of both types of forces. Preliminary calculations on reasonable assumptions show that the attractive forces due to these causes are of the right order of magnitude to hold the particles in equilibrium when in circulation round the central nucleus.

We thus arrive at a general conception of nuclear structure in which the central charged nucleus is surrounded by a number of uncharged particles. In a Paper before the Franklin Institute in 1924 I put forward a suggestion that the central nucleus was a closely ordered arrangement of α particles and electrons in a semi-crystalline formation, and showed that certain simple arrangements were in fair accord with the charge and mass of some of the atoms. Whatever view we may take on this question, I am inclined to believe that the central nucleus of the heavier elements is a very compact structure, occupying a very small volume of radius of the order 1×10^{-12} cm. The neutral satellites circulating round this nucleus may extend to a distance large compared with the linear dimensions of the main nucleus.

If we take the uncharged α particle as a type of a nuetral satellite, it must consist of the helium nucleus which has gained two electrons. These electrons cannot occupy the same positions as in the ordinary helium atom in the free state, for in such a case they would at once be torn off by the intense electric fields due to the nucleus. They are probably much more closely bound to the nucleus, circulating in orbits which are only rendered possible by the distortion of the nuclear structure of the α particle by the intense electrical or magnetic fields from the central nucleus. Such a view seems not unreasonable, for undoubtedly all composite nuclei must suffer serious distortion under the enormous fields which are present in nuclear structures. In fields below a certain critical value, these electron orbits cannot exist, and consequently a neutral α particle in escaping from the nuclear structure would be robbed of its two electrons when the critical field is reached.

We can thus form the following picture of the emission of an α particle from a radioactive element. Occasionally one of the neutral α particles which are probably circulating in quantized orbits is for some cause displaced from its position of equilibrium and has sufficient energy of motion to escape from the attractive field of the central nucleus. When the field falls below the critical value, the neutralizing electrons are removed and fall back towards the nucleus. The α particle, which has now two positive charges, gains additional energy in passing through the repulsive electric field of the nucleus and emerges as a high-speed α particle. It is of interest to note that Fräulein Meitner from consideration of the successive modes of transformation of radioactive atoms concluded that some of the α particles in the nucleus must exist in the neutral state. On the present views all the α particles which escape from radioactive nuclei have their origin as neutral satellites.

We must now follow for a moment the fate of the electrons which are liberated from the neutral α particle. From the known change of charge in a radioactive transformation, it is clear that they must fall back towards the main nucleus, probably describing orbits under the complicated system of forces which must exist close to the nucleus in a region where the forces due to distortion may be all-important. Occasionally one of these electrons is given sufficient energy to escape entirely from the nucleus thus giving rise to a β ray transformation.

In our picture of the nucleus, we thus have a concentrated inner nucleus carrying a positive charge surrounded at a short distance by a number of electrons, and then at a distance a number of neutral satellites circulating round the system. I hope in a later Paper to give a fuller discussion of this type of nuclear structure which offers certain possibilities of interpretation of the wealth of radioactive data at our disposal.

This picture of a nucleus need not be confined only to the radioactive atoms, but is equally applicable to the ordinary atoms. We have so far only discussed the possibility of neutral satellites in the form of α particles of mass 4. It is quite possible, however, that other types of neutral satellites may be present of mass 2 or 3. The possibility of the existence of such nuclei has been drawn attention to by several writers as types of secondary units which play a part in the building-up of nuclear structures, but they have usually been supposed to exist as charged rather than neutral masses. Such secondary neutral units may be able to exist only in the powerful nuclear fields and thus would never be observed in the free state.

This view of nuclear structure at once offers a reasonable explanation of the existence of a number of isotopes of an element of given atomic number. When once the central nucleus is formed, a number of neutral satellites can be added which are kept in equilibrium by attractive forces. Aston has shown in some cases that a large number of isotopes can exist, thus indicating that a number of satellites may be added without disturbing the equilibrium of the nuclear system. He has drawn attention to the remarkable fact that in all cases the odd numbered heavier elements have either no isotopes or two isotopes differing in mass by two units, while even-numbered elements may have a whole group of isotopes. This striking distinction between the elements is paralleled by the observation that odd numbered elements which are disintegrated by the bombardment of α particles emit protons at a much higher average speed than the even-numbered elements. Harkins has also shown that even-numbered elements are much more abundant in nature than the odd-numbered elements.

This difference in isotopic properties between even and odd elements seems very fundamental in character. A possible interpretation may be given on the general view of nuclear structure that we have outlined. It is supposed that the central nucleus is ordinarily made up of helium nuclei carrying two charges arranged in an ordered way, and for an even-numbered element there must be present an even number of electrons.

These would tend to arrange themselves in equilibrium so that the resultant magnetic moment of the system is a minimum. If now, by the addition of a proton or removal of an electron, an odd-numbered element is formed, there may be no longer possible that balance between the magnetic moments as in the case of an even-numbered element. The effect of the resultant magnetic field of the nuclear system may make it impossible for neutral satellites to circulate in permanently stable orbits round the nucleus. If this view be correct, we should anticipate that the nuclear magnetic moment of odd elements should differ markedly from that of even elements. As far as I am aware, there is at present no definite evidence bearing on this point.

In this discussion of atomic constitution, we have been referring for the most part to the heavier elements where the nucleus is considered to be composed mainly of α particles and satellites and where it is to be anticipated that the whole number rule of atomic mass should hold within small limits. If the presence of neutral satellites in the nuclear system depends mainly on the electric charge of the nucleus, the addition of such satellites may be possible only when the nuclear charge exceeds a certain value. For this reason, the constitution of the lighter elements may differ markedly in general features from the heavier elements and the departures from the whole number rule of atomic mass may be more emphasized than for the heavier elements. An accurate determination of the atomic mass of the lighter elements such as Dr. Aston is undertaking is thus of great theoretical, as well as practical importance, as it may give us valuable information on the closeness of the binding of the component protons and electrons.

On account of the uncertainty of the constitution of the lighter elements, it is difficult to connect the disintegration of such elements by α particle bombardment with any special feature of their structure. The views given here are admittedly very speculative in character, but they may serve a useful purpose in suggesting possible lines of attack on this fundamental problem. Although the nucleus of a heavy atom is no doubt very complicated in structure, yet it may present certain simple general features which may be absent or difficult to detect in the lighter elements. For this reason, I am hopeful that we may yet be able by the study of radioactive data to throw light on some of the outstanding features of nuclear constitution of heavy elements.

DEMONSTRATION OF THE ASTROLABE AND SOME OTHER MEDIÆVAL NAVIGATIONAL AND SURVEYING INSTRUMENTS, ILLUSTRATED BY LANTERN SLIDES AND EXHIBITS.

By ALLAN FERGUSON, *M.A., D.Sc.*

OF the two co-ordinates which determine a navigator's position, latitude may be obtained from the height of the pole, or by measuring the meridian zenith distance of a body of known declination (usually the sun), and adding zenith distance and declination together. In the absence of clocks, longitude could be measured only by keeping " a perfect accounte and reckoning of the waye of the shippe."

Among the exhibits shown was a model of the cross-staff. This instrument, which was invented by Regiomontanus in the fifteenth century, comprises a rod with a backsight at one end, and a shorter transverse rod with fore-sights at each of its extremities. The shorter rod slides on the longer, which is graduated so as to give the angle subtended at the eye by two objects simultaneously sighted, such as two stars or the sun and horizon. In Tycho Brahe's form of the instrument the distance (a) between the foresights was adjustable, as well as the distance (b) of the transversal from the back-sight. Tycho reduced his readings to a value x by means of the proportion $a/b=x/1{,}200$, and looked up the angle corresponding to the value x in the tables of Gemma Frisius, which were, in effect, tables of tangents on a duodecimal basis.

The back staff, which obviated the necessity of looking directly at the sun, was also illustrated.

The astrolabe is made up of three parts. The *tablet* is a stereographic projection on to the plane of the equator from the S. pole of the azimuth and zenith distance circles corresponding to a point in a given latitude. The *rete,* which is pivoted above the tablet, and turns round the celestial pole, is a projection of the celestial sphere, and contains the projections of a few well-known stars and of the ecliptic, the circle of the ecliptic being marked with the sun's position on the celestial sphere for every day of the year. In metal astrolabes the rete is cut in an openwork pattern, on which the stars appear as points at the apices of metal spurs. The *alidade* is a sighting rule provided with pinhole sights by means of which the zenith distance of any celestial body may be measured.

The central problem of the astrolabe is to *set the rete so as to show the celestial bodies at any given instant with their correct altitudes and azimuths.* This is done by measuring the zenith distance of any known body and turning the rete till that body is over the appropriate zenith distance circle. The rete now shows the aspect of the heavens at the instant of observation, and this aspect includes the position of the sun (whether above or below the horizon). A 24-hour clock face is marked on the edge of the tablet, and the angle which the sun must turn through before reaching the meridian therefore gives the local time. Moreover, inspection of the azimuth circles on the tablet gives, at the instant of observation, the true bearing of every body marked on the rete. Hence a simultaneous compass bearing of any one of these bodies affords an easy means of correcting a compass. We are, in fact, dealing with three quantities—altitude, azimuth and time—and the astrolabe provides a means of solving problems in which one of these quantities being given, the others are required.

The astrolabe was an essential part of the equipment of the mediæval navigator who, *robur et aes triplex*, adventured forth equipped with astrolabe, cross-staff, compass, a table of the sun's declination, and possibly a chart or two of doubtful value.

Chaucer's treatise on the astrolabe (Bred and Mylke for Childeren), written in 1391 for his ten-year-old son Lewis, still remains the standard English work.

Among the instruments shown was a Persian astrolabe, the property of Mr. R. S. Whipple, provided with interchangeable tablets for use at different places, including Baghdad and Teheran.

XXIX.—A DUPLEX REVERSAL KEY WITH MERCURY CONTACTS.

By Edgar A. Griffiths, Government Physicist, Union of South Africa; and Ezer Griffiths, *D.Sc.*, *F.R.S.*, Physics Department, National Physical Laboratory.

Received April 26, 1927.

Abstract.

The key employs mercury contacts and is enclosed in a glass casing. The two upper fixed contacts consist of downwardly projecting copper rods, the two lower fixed contacts of cups containing mercury, and the two movable contacts of downwardly projecting copper rods (which enter the fixed cups in one position of the switch) surmounted by cups containing mercury (which receive the fixed rods in the other position of the switch). The connections are those of a reversing switch, and the middle contacts can be raised or lowered by turning a vertical shaft which is retained in either position by a jockey spring.

In many experiments it is desirable to have the means for reversing two or more circuits without introducing either variable contact resistances or uncertain thermal electromotive forces into the circuits.

The key illustrated in Figs. 1 and 2 was constructed about seven years ago, and has proved to be so convenient in a variety of tests that a brief description of it may be of value to others.

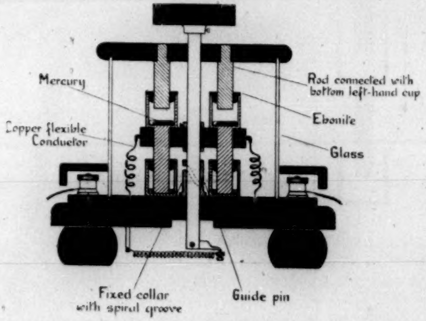
Mercury
Rod connected with bottom left-hand cup
Copper flexible Conductor
Ebonite
Glass
Fixed collar with spiral groove
Guide pin

Fig. 2.

To the base of the key four mercury cups are attached. Above these, and capable of displacement vertically, are four mercury cups with stout copper rods projecting beneath. Fixed to the top of the key are four similar copper rods. These various cups and rods are interconnected as shown in Fig. 3. By a partial turn of the knurled head the movable set of cups is brought up against the four fixed rods projecting from the top of the key. By a reverse turn the four copper rods projecting beneath the movable cups are brought into contact with the bottom of the cups fixed to the base.

The mechanism for translating the partial rotation of the knurled head into a vertical displacement of the cups will be understood from a study of Fig. 2. A guide pin attached to the shaft is constrained to travel along a spiral groove in a fixed collar and thereby the shaft is displaced longitudinally when the knurled head is twisted.

In both positions spring pressure is brought into action to press the contacts together. This is effected by fastening a spring to a radial arm fixed on the lower end of central shaft operating the key. The setting of the radial arm for the " up " and " down " positions respectively is shown in Fig. 4. It will be observed that the maximum extension of the spring corresponds to the midway position of the movable cups, so this is a position of neutral equilibrium and a small movement in either direction allows the spring to pull the contacts together, thus avoiding a variable mercury film resistance.

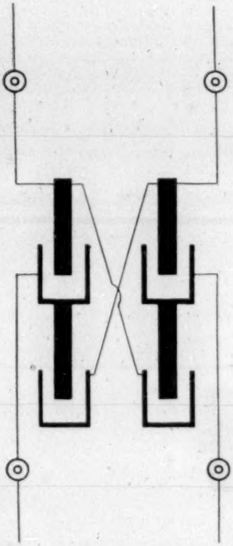

Fig. 3.

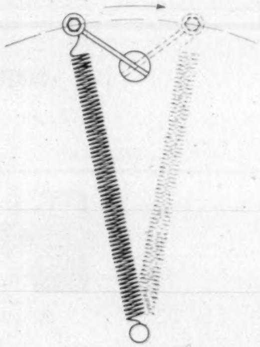

Fig. 4.

To eliminate, as far as possible, thermoelectric effects, the key is cased in a thick glass cylinder and the terminals are covered with an ebonite shield. All terminals, connecting straps, etc., are of copper.

The authors wish to thank Mr. F. H. Murfitt for his suggestions and skilful construction.

XXX.—THE MEASUREMENT OF THE INDUCTANCES OF FOUR-TERMINAL RESISTANCE STANDARDS.

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Received April 26, 1927.

ABSTRACT.

The measurement of the effective inductances of "four-terminal" resistances or "shunts" is discussed. The method of measurement developed in the Paper is an application of the Kelvin Double Bridge, used with alternating current, the phase angle adjustment being obtained by condensers shunting the ratio arms. The procedure to be adopted for eliminating or evaluating corrections due to "leads" is described, and the advantages of the bridge in being free from stray fields, practically independent of frequency, capable of use with almost any desired current strength, and, moreover, very easy to work, are pointed out. Results of measurements made by this method on a series of typical shunts are recorded.

FOUR-terminal resistance standards or "shunts" are widely used in precision alternating current measurements, their function being to provide a potential difference which is in phase with, and proportional to, the current flowing through them. Fig. 1 is a diagrammatic representation of such a shunt. The current I enters and leaves by the "current leads" C_1 and C_2. The potential difference measured is that existing between the "potential terminals" P_1 and P_2. In the ideal case, this potential difference is given by

$$V=RI,$$

where R is the resistance of the shunt. In actual practice, the potential difference V is never exactly in phase with the current I. We may regard it as consisting of two components, a large one, RI, in phase with the current, and a small one, $jwLI$, in quadrature with the current. Thus we may write

$$V=(R+jLw)I,$$

where j is the operator rotating through a right angle, and the frequency is $w/2\pi$. The quantity L in the equation is defined as the inductance of the shunt, while R is the resistance. The object of the present Paper is to describe a method for the measurement of the inductance L.

Two factors contribute to the quantity L—viz., the self-inductance of the shunt between the tapping points p_1 and p_2, and the mutual inductance between the potential leads l_1, l_2, and the whole circuit in which the current I flows. This mutual inductance component may oppose the self-inductance. In order that the inductance L may have a definite value for any particular shunt, it is obviously necessary that the current and potential circuits be arranged so that there can be no mutual inductance between the connecting leads which are attached to C_1 and C_2, and those attached to P_1 and P_2. Thus, the two current terminals must be near together, as also must the two potential terminals, and moreover these two pairs of terminals must not be near each other. In what follows it will be assumed that these conditions are satisfied. When using the shunts, the external leads connected to the potential terminals must be twisted together, and the leads to the current terminals must be similarly treated, since any open loop in the leads is likely to affect the mutual inductance between the potential and current circuits, and so have the effect of altering the value of L.

Previous Methods.

A number of methods of measurement of the self-inductances of standard shunts has already been described. A. Campbell has given a general account of these in Glazebrook's Dictionary.* At the National Physical Laboratory, two alternating-current potentiometer methods, developed by Mr. Campbell,† have been in use for some years. These methods make use of a mutual inductance as a standard of phase angle, and as it is generally necessary to work at telephonic frequency, errors due to impurity of the mutual inductance are liable to occur. The measurement is in any case a difficult one, since the circuits contain a number of inductive coils, and it is essential to make sure that the stray fields of these coils are not introducing an undesirable E.M.F. into any part of the circuit. Induction directly into the telephone detector is liable to be very troublesome. It was on this account that the author was led to consider the application of the Kelvin Double Bridge to this type of measurement. Arrangements of this bridge for alternating current measurements have been described by Sharp and Crawford ‡ and by Wenner,§ using inductances in series with the ratio arms for the inductance balance. Sharp and Crawford remark that stray field effects are apt to be very serious when this is done, and on that account preferred to use a separately excited dynamometer as detector, obtaining the resistance balance by in-phase excitation, and adjusting for zero deflection, and then deducing the inductance relations by observation of the deflection with quadrature excitation. The procedure adopted by Wenner, though capable of high accuracy appears to be somewhat exacting, since no less than three simultaneous double balances are required, and the stray field difficulty has to be attended to. In the method developed by the author, no inductive coils are used, and thus stray fields are reduced to a minimum. The quadrature balance is obtained by variable condensers shunting the ratio arms. It subsequently appeared that this same condenser adjustment had been used some time ago by Schering,‖ but as the development of the method is very different in the two cases, and the method used at the National Physical

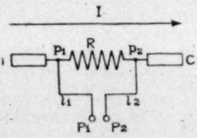

Fig. 1.

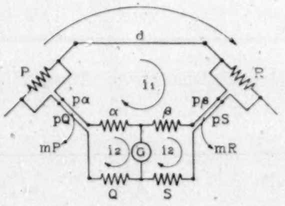

Fig. 2.

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* Dictionary of Applied Physics, Vol. 2, p. 441.

† A. Campbell, Proc. Phys. Soc., Vol. 29, p. 345 (1917).

‡ Sharp and Crawford, Trans. A.I.E.E., Vol. 29, p. 1540 (1910).

§ Wenner, Bull. Bur. Stds., Vol. 8, p. 559 (1912).

‖ Schering, E.T.Z., Vol. 34, p. 421 (1917).

Laboratory is particularly easy to work, and need require no very special apparatus, it is considered worth while to describe it in some detail. Other methods of measuring the inductances of shunts are described by Wenner, Weibel and Silsbee.* These, like Campbell's methods, are independent of standards of calculated inductance, but require special apparatus, and are not likely to be very quick in operation.

THEORY OF THE METHOD.

Fig. 2 is a diagrammatic representation of the bridge. P and R are four-terminal shunts to be compared for inductance. Q and S are the main ratio arms, and α and β the auxiliary ones. Let $P, R, Q, S, \alpha, \beta$ be the impedance operators of the various coils. The potential terminals of the coil P are connected to the ratio coils αQ by means of twinned leads, and there is a similar pair of leads connecting the coils β and S to the potential points of R. Let p_α p_β, p_Q, p_S be the impedance operators of these leads, and m_P, m_R their mutual operators; also let d be the impedance operator of the link connecting the current terminals of P and R. Let the mesh currents be i, i_1, i_2, as in Fig. 2. When the bridge is balanced the currents in the meshes αGQP, βRSG must be equal, as indicated (i_2).

Applying Kirchhoff's equations to the network we obtain, as conditions of balance the equations

$$(i+i_2)P+(i_2-i_1)(\alpha+p_\alpha+m_P)+i_2(Q+p_Q+m_P)=0, \quad . \quad . \quad . \quad . \quad (1)$$

$$(i+i_2)R+(i_2-i_1)(\beta+p_\beta+m_R)+i_2(S+p_S+m_R)=0, \quad . \quad . \quad . \quad . \quad (2)$$

$$(i+i_1)d-(i_2-i_1)(\alpha+p_\alpha+\beta+p_\beta)-i_2(m_P+m_R)=0. \quad . \quad . \quad . \quad . \quad (3)$$

Rewriting (1) and (2) we have

$$(i+i_1)P+(i_2-i_1)(P+\alpha+p_\alpha+m_P)+i_2(Q+p_Q+m_P)=0, \quad . \quad . \quad (4)$$

$$(i+i_1)R+(i_2-i_1)(R+\beta+p_\beta+m_R)+i_2(S+p_S+m_R)=0. \quad . \quad . \quad (5)$$

Now let the connector d be removed, and the bridge again balanced. The mesh currents are now i_3 and i_4 (Fig. 3), and the conditions of balance are

$$(i_3+i_4)(P+\alpha+p_\alpha+m_P)+i_4(Q+p_Q+m_P)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$(i_3+i_4)(R+\beta+p_\beta+m_R)+i_4(S+p_S+m_R)=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

whence

$$(P+\alpha+p_\alpha+m_P)/(R+\beta+p_\beta+m_R)=(Q+p_Q+m_P)/(S+p_S+m_R)=k \text{ (say)} \quad . \quad (8)$$

If now the bridge arms can be adjusted so that the bridge is balanced both when the link d is inserted, and when it is removed, then equations (3), (4), (5) and (8) must hold for all values of the current i. Now, evaluating (4) $-k\times(5)$, we find, on substituting (8),

$$(i+i_1)(P-kR)=0.$$

Thus, the condition for simultaneous balance is

$$P/R=k=(Q+p_Q+m_P)/(S+p_S+m_R)=Q'/S' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

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* **Wenner, Weibel and Silsbee, Bull. Bur. Stds., Vol. 12, p. 11 (1915).**

P, R, Q', S' are vector impedances. Let their phase angles be θ_P, θ_R, θ_Q', θ_S'. Then in order that equation (9) may hold, the relation between the phase angles must be

$$\theta_P - \theta_R = \theta_Q' - \theta_S'. \qquad (10)$$

If R_P and L_P are the resistance and effective inductance of coil P, we may write, in the usual notation,

$$P = R_P + jwL_P, \text{ and similarly}$$
$$R = R_R + jwL_R$$

The arms Q and S will usually consist of coils of resistance $R_Q R_S$ and inductances $L_Q L_S$, shunted by capacities $C_Q C_S$, in order to obtain the balance. Thus, to a first approximation

$$Q = R_Q + jw(L_Q - C_Q R_Q^2),$$
$$S = R_S + jw(L_S - C_S R_S^2).$$

We have also

$$p_Q = r_Q + jwl_Q,$$
$$p_S = r_S + jwl_S,$$

and since m_P and m_R represent practically pure mutual inductances

$$m_P = jwM_P,$$
$$m_R = jwM_R.$$

Q' and S' in equation (9) are evidently given by

$$Q' = R_Q + r_Q + jw(L_Q + l_Q + M_P - C_Q R_Q^2),$$
$$S' = R_S + r_S + jw(L_S + l_S + M_R - C_S R_S^2).$$

Now, $PRQ'S'$ represent impedances of very small phase angle. Thus, to a sufficient approximation for all practical purposes we may write

$$\theta_P = \tan\theta_P = L_P w / R_P = \tau_P w,$$

where τ_P is the time constant of P, and so on. Thus equation (10) may be written

$$L_P/R_P - L_R/R_R = (L_Q + l_Q + M_P)/(R_Q + r_Q) - (L_S + l_S + M_R)/(R_S + r_S)$$
$$- C_Q R_Q^2/(R_Q + r_Q) + C_S R_S^2/(R_S + r_S),$$

or, since the leads resistances r_Q and r_S are negligible compared with R_Q and R_S

$$L_P/R_P - L_R/R_R = L_Q/R_Q - L_S/R_S + C_S R_S - C_Q R_Q + \{(l_Q + M_P)/R_Q - (l_S + M_R)/R_S\} \quad (11)$$

This is the equation to be used for determining L_P/R_P. The quantities L_R/R_R, L_Q/R_Q, L_S/R_S, $C_S R_S$, $C_Q R_Q$, are usually known, while the term in brackets may be regarded as a correction term depending on the self and mutual inductances of the leads.

This correction term is usually very small, and may be neglected in practice, except for work of the highest precision. In such cases it may be evaluated as follows. Suppose that the conditions of simultaneous balance have been obtained as previously described. Let C_S be the reading of the condenser shunting the arm S. Now transfer the current supply leads from the points (1) (2) to the points (3) (4) (Fig. 3), and, the link d being removed, again balance by varying the arm S. Let C_S' be the new reading of the S condenser. The value of R_S will only need altering by an amount which is negligible, from the point of view of phase angle. The original condition of balance is given by (8). If the mesh currents i_3, i_4 become i_5 i_6 in the final balance, we have, as the equations for balance,

$$(i_5+i_6)\,(P+p_Q+p_\alpha+2m_P+\alpha)+i_6Q=0,$$

$$(i_5+i_6)\,(R+p_S+p_\beta+2m_R+\beta)+i_6S'=0,$$

giving $$(P+\alpha+p_Q+p_\alpha+2m_P)/(R+\beta+p_S+p_\beta+2m_R)=Q/S', \quad . \quad . \quad . \quad . \quad (12)$$

as against the original condition,

$$(P+\alpha+p_\alpha+m_P)/(R+\beta+p_\beta+m_R)=(Q+p_Q+m_P)/(S+p_S+m_R). \quad . \quad . \quad (8)$$

Now, each of these equations (8) and (12) involves a phase angle equation of the same form as (10). We may write down by inspection the difference of these two phase angle or time constant equations, remembering that the lead resistances are negligible compared with the coil resistances, as

$$(l_Q+M_P)/(R_P+R_\alpha)-(l_S+M_R)/(R_R+R_\beta)$$
$$=-(l_Q+M_P)/R_Q+(l_S+M_R)/R_S+(C_S'-C_S)R_S,$$

or, $$\frac{(l_Q+M_P)}{R_Q}\left(1+\frac{R_Q}{R_P+R_\alpha}\right)-\frac{(l_S+M_R)}{R_S}\left(1+\frac{R_S}{R_R+R_\beta}\right)=(C_S'-C_S)R_S.$$

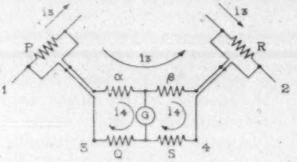

Fig. 3.

Now to a first approximation, the resistances obey the usual relation for the Kelvin Double Bridge, viz.,

$$R_P/R_R=R_Q/R_S=R_\alpha/R_\beta.$$

Hence we have

$$\{(l_Q+M_P)/R_Q-(l_S+M_R)/R_S\}$$
$$.\{1+R_Q/(R_P+R_\alpha)\}=(C_S'-C_S)R_S.$$

Thus, the correction term required is given by

$$\{(l_Q+M_P)/R_Q-(l_S+M_R)/R_S\}=(R_P+R_\alpha)\times(C_S'-C_S)R_S/(R_P+R_Q+R_\alpha) \quad . \quad (13)$$

In practice R_P is usually small compared with R_Q and R_α, which are equal. We then have the very simple relation

$$\text{Correction}=(C_S'-C_S)R_S/2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

PRACTICAL DETAILS AND PROCEDURE.

It is evident from equations (11) and (13) that we have a means of finding the difference of time constant of two four-terminal shunts P and R, in terms of the time constants of the coils Q and S, and the values of their resistances and the capacities shunting them, when the above conditions are fulfilled. For Q and we can use ordinary non-inductive ratio arms, of the order of 100 ohms and 1,000 ohms. Their time constants are likely to be of the order 10^{-8}, and the difference between them can be measured, if necessary, with an error of about 10^{-9}. Thus, since a standard shunt is not likely to have a time constant much less than 10^{-7}, the values of the time constants of Q and S are not usually of great importance.

The actual arrangement used is shown in Fig. 4. The two resistances P and R are connected in series to the current supply by means of their "current terminals." The go and return connecting leads are kept as close together as possible, so as to avoid any strong magnetic field. The ratio coils Q, S, α, β, are shielded coils of the type used in ordinary alternating current bridge work. Generally a ratio of 100 ohms : 1,000 ohms was used. These coils were placed some distance (say, 1 metre) away from the resistances P and R, so that the possibility of any undesirable stray field effects from the main circuit is further reduced. The potential terminals of P and R are connected to the ratio arms, by means of twinned flexible leads as shown. In some cases these leads were two or three metres long, and it is on this account that the correction for their self and mutual inductances was evaluated. Since it is necessary to vary the resistances of the ratio arms, small slide wire resistances were included in β and S or α and Q. In most of the experiments made, current of telephonic frequency was used and a telephone receiver was used as the detector G. This is convenient, since it allows the necessary sensitivity for time constant to be obtained with a smaller current than is required at lower frequencies. The bridge is, however, independent of frequency to a first approximation, and measurements were also made at a frequency of 50 ~, in which case a vibration galvanometer was used as detector. The phase angle adjustments were made by means of two variable air condensers, C_S and C_β, in Fig. 4. These had maximum values of about 0·002 μF. In some cases their ranges had to be extended by the use of fixed mica condensers in parallel with them. In order to locate definitely the potentials of the various coils with respect to earth, the outer current terminals of P and R were shunted by two resistances X and Y in series, their common point E being earth connected. The values of X and Y were chosen to satisfy approximately the relations $X/Y=P/R=Q/S=\alpha/\beta$. Thus, when the bridge is balanced, the common points of PR, $\alpha\beta$, QS will be approximately at the same potential as E. The detector G will, therefore, be at earth potential, and no undesirable earth capacity effects are likely to be introduced. The shields of the condensers C_β and C_S were connected to the galvanometer side of β and S respectively. The shields of the coils $\alpha\beta QS$ were earth connected. Under these conditions, the balance point of the bridge is very stable, and the bridge very easy to work. Reversing the connections to the current supply and the detector causes no change in the balance point.

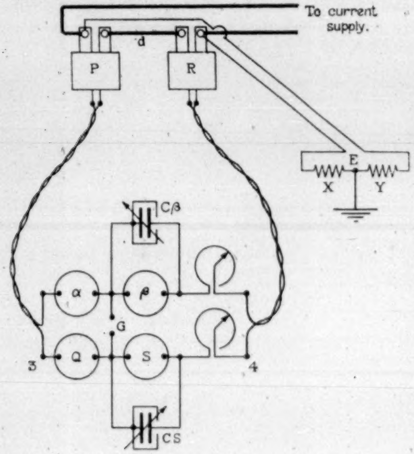

Fig. 4.

The procedure was as follows. The bridge was connected up as in Fig. 4, the link *d* being made as low in resistance as possible. Balance was obtained by varying the resistance and capacity of the *S* arm. The link *d* was then opened, and the bridge again balanced by varying the resistance and capacity of the β arm. The link *d* was then again joined up and balance once more obtained by varying the *S* arm. Usually the adjustment required at this stage was only very slight, and the bridge was then found to be balanced when the link was either closed or open. Current was now applied to the bridge at the points (3) and (4) of Fig. 4, the link *d* being open, and balance again obtained by varying the *S* arm. The amount of adjustment required for this was only very small. As explained above, it represents the effect of the self and mutual inductance of the leads running up to the potential terminals of *P* and *R*. If C_S is the reading of the *S* condenser for the first simultaneous balance, and C_S' is the final reading of this condenser, we have from equations (11) and (13)

$$L_P/R_P - L_R/R_R = L_Q/R_Q - L_S/R_S + C_S R_S - C_Q R_Q \\ +(R_P+R_a)\times(C_S'-C_S)R_S/(R_P+R_Q+R_a) \quad . \quad . \quad . \quad (15)$$

In general, it is not necessary to have condensers across both the *Q* and *S* arms, and the term $C_Q R_Q$ is thus usually zero. The quantity $(L_Q/R_Q - L_S/R_S)$ may be regarded as a small correction δ_{QS}, which is known, or which can be measured by any of the usual methods. Further, R_P is usually negligible compared with R_Q and R_a, which are equal. The working equation, therefore, becomes

$$L_P/R_P - L_R/R_R = C_S R_S + \delta_{QS} + \tfrac{1}{2}(C_S' - C_S)R_S, \\ = \tfrac{1}{2}(C_S + C_S') + \delta_{QS}.$$

The points (3) (4) (Fig. 4) should, of course, be brought as near together as possible, so that current can be brought to them through bifilar twisted leads, which will induce no stray E.M.F.s into the coils $\alpha\beta QS$. It is important to make the link *d* as low in resistance as possible, for the smaller this resistance compared with *P* and *R*, the less is the effect on the balance point of the values of α and β, and, therefore, the easier it is to obtain the condition of simultaneous balance.

THE PRIMARY STANDARD.

It will be observed that the bridge method can only give the difference of the time constants of two shunts. Thus, it is necessary to have at least one shunt of known time constant before values can be obtained for others. The ultimate standard used in these experiments was a 1 ohm four-terminal resistance, the effective inductance of which was calculable from its dimensions. This was designed by Mr. S. Butterworth, who has kindly supplied the following particulars. The resistance consists of a rectangular loop of Eureka wire C_1STUVC_2 (Fig. 5), C_1 and C_2 being the current terminals. Two brass contact pieces *X* and *Y* make contact with this loop at the potential points *S* and *V*. From the contact pieces *X* and *Y* potential leads proceed parallel to the sides of the main resistance loop, o the potential terminals P_1 and P_2. The various wires are located by passing through holes in the ebonite spacing pieces shown in Fig. 5. The current and potential terminals are as far apart as possible, so that the pairs of leads connecting them to the measuring circuit will not induce E.M.F.s into one another. In order to calculate the effective inductance, we have to evaluate (*a*) The self inductance of the rectangular loop *STUV*, between the potential points *S* and *V*.

(*b*) The mutual inductance between the potential leads *XW*, *YZ* and the sides of the loop *STUV*.

(*c*) The small correction terms due to the extensions C_1S, C_2V, WP_1, and ZP_2 of the leads.

If the dimensions are as shown in Fig. 5 then neglecting the effect of the leads (*c*) the inductance is given by the formula

$$L=4B\log_e\{Dd/R(D+d)\},$$

R being the radius of the wire. The actual values were $D=9{\cdot}68$ mm., $d=4{\cdot}64$ mm., $B=10{\cdot}0$ cm., $R=0{\cdot}154$ mm., giving $\mathrm{L}=130$ cm. $=0{\cdot}130\ \mu\mathrm{H}$. With the dimensions of the leads shown in Fig. 5 the correction due to these was found by calculation to be $-0{\cdot}006\mu\mathrm{H}$. Thus the final value for the inductance is $L=0{\cdot}124\mu\mathrm{H}$.

The time constants of 1 ohm resistance standards were determined by direct comparison with this calculated standard, using the method described above, with equal ratio arms, either 100 : 100 ohms, or 1,000 : 1,000 ohms. Standards of 0·1 ohm resistance were compared with a 1 ohm resistance using a ratio of 100 : 1,000 ohms; 0·01 ohm standards were compared with a 0·1 ohm standard using the same ratio, and so on down to 0·0005 ohm. Ratios other than 10 : 1 are sometimes used, but in general this is the most convenient. The accuracy in time constant obtainable on the shunts of very low resistance is of course somewhat less than that on shunts of 1 ohm and 0·1 ohm, since the errors of the various intermediate steps will be cumulative. The time constants of these low resistance shunts are, however, always higher than those of greater resistance, and it is not possible to use them with the same degree of precision, so that this point is not of great practical importance.

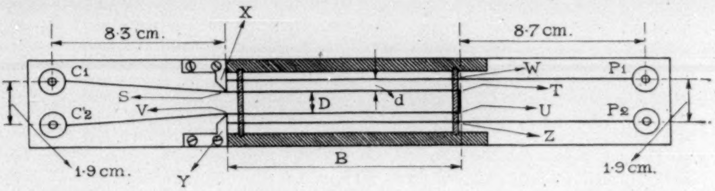

Fig. 5.

When no standard of calculable inductance is available, it is possible to proceed as follows. Take an ordinary 10 ohm resistance coil, and use it as a four terminal resistance, simply by using its terminals as both current and potential terminals. Its inductance as a four-terminal resistance may be taken as practically the same as its ordinary value as a two-terminal coil, which is readily measurable. There will, of course, be small errors due to the mutual inductance between the coil and the potential leads used, but if each pair is twisted together, and they are disposed so as to make this mutual inductance small, the effect on the time constant L/R will be negligible, owing to the comparatively large value of R. A 1 ohm shunt may thus be evaluated in terms of a 10 ohm coil used in this way, and results so obtained were found to agree with those obtained by comparison with the calculated standard with an error of less than 1×10^{-8} in time constant.

TYPICAL RESULTS.

The time constants and effective inductances of a number of four-terminal resistances of various types have been determined by this method. The resistances varied from 1·0 ohm and 0·1 ohm for coils such as are used in alternating current bridge and potentiometer work, to 0·0005 ohm and 0·001 ohm, for shunts capable of carrying one or two thousand amperes, such as are used for power measurements, and for testing current transformers. The results are given here, since they may be regarded as being fairly typical, and as indicating therefore the probable order of error involved in using apparatus of the particular type in question.

The measurements were nearly all made with current of frequency 800 cycles per second. The use of high frequency increases the value of the quadrature component of voltage, relatively to the in-phase component, and thus makes the measurement of this quadrature component in a sense easier. In making measurements with shunts of 1 and 0·1 ohms, a current of 0·25 ampere gave sufficient sensitivity. For the 0·001 and 0·0005 ohm shunts, a current of 5 amperes was required. The detector was an ordinary 120 ohm telephone receiver of the Bell type.

The heavy current resistance standards are used almost exclusively at power frequencies, and thus the value of the inductance at 50 cycles per second is the value actually required. It was found, however, that for all the resistances tested the inductance is sensibly independent of the frequency up to about 1,000 cycles per second. It is an advantage of the bridge that the balance point is practically independent of the frequency, and thus to find out whether the inductance varies appreciably with the frequency, it is only necessary to raise the frequency from, say, 500∼/sec. to 800∼/sec., and to note the change of balance point. In general the resistances are found to change slightly, but not so the inductances.

The possibility of the effective inductance changing with varying current strengths must also be considered. Any change in the inductance as the current strength is increased can only be due to one of two things, viz. :—

(*a*) A change in the distribution of the lines of flow of the current ;

(*b*) A change in the geometrical configuration of the shunt. Such changes will not occur in a well-designed shunt. Changes in the distribution of the lines of current flow will, of course, give rise also to changes of resistance, and these are easily detected. Bending and warping of the resistance alloy does occur in some cases, due to the heating effect of the current, but such shunts cannot be regarded as standards, from the point of view of phase angle, since the value of the phase angle will always be subject to considerable uncertainty. The inductance of a well-designed shunt must therefore be independent of the current strength. The method does, of course, present the advantage that it may be used with a most any desired value of current. It is merely necessary to obtain a shunt capable of use as a comparison standard, which will carry the maximum current required without distortion.

As an illustration of the order of the quantities involved in these measurements, the following observations, made on a bridge set up for the comparison of two shunts of resistances 0·1 and 1 ohm respectively, are given,

$R_P=0{\cdot}1$ ohm. $R_Q=100$ ohm. $R_\alpha=50$ ohms.
$R_R=1{\cdot}0$,, $R_S=1{,}000$,, $R_\beta=500$,,

The bridge arms S and β were provided with small slide wire adjustments of resistance, and were shunted with variable air condensers. When the first simultaneous balance was obtained, the reading of this condenser C_S was 1,330 $\mu\mu\mathrm{F}=1{,}330\times10^{-12}$ farad. The current supply leads were now transferred to the points (3) (4), Fig. 3, the link d removed, and balance again obtained by adjusting the arm S. The new reading of the condenser was $C_S'=1{,}342$ $\mu\mu$F. The readings of the various resistances R_QR_S, etc., were not noted in detail, since they were always within about 0·5 per cent. of their nominal values, and thus their actual values are of no consequence from the point of view of inductance measurement. Similarly, the reading of the condenser across the arm β is not required. The time constants of the ratio arms Q and S were known from two-terminal measurements to be $0{\cdot}6\times10^{-8}$, and $1{\cdot}3\times10^{-8}$ respectively, thus giving rise to the correction term $\delta_{QS}=L_Q/R_Q-L_S/R_S=-0{\cdot}7\times10^{-8}$. The time constant of the 1 ohm standard R, had been found by comparison with the calculated standard to be $L_R/R_R=1{\cdot}6\times10^{-8}$. Thus, from equation (14) we find

$$L_P/R_P-1{\cdot}6\times10^{-8}=-0{\cdot}7\times10^{-8}+133{\cdot}_0\times10^{-8}+\tfrac{1}{3}\times12\times10^{-9},$$

giving $L_P/R_P=134{\cdot}_3\times10^{-8}$, and therefore $L_P=0{\cdot}134\mu$H. The various correction terms are seen to be almost negligible. The values obtained for a series of typical shunts are given in Table I.

TABLE I.—*Effective Inductances of Typical Shunts.*

Resistance (ohms).	Type.	Maximum current (amperes).	Time constant (second).	Effective inductance (microhenry).
1·0	N.P.L.	0·5	$1{\cdot}6\times10^{-8}$	0·016
1·0	Drysdale-Tinsley.	3	$7{\cdot}5\times10^{-7}$	0·75
0·1	,,	22	$13{\cdot}4\times10^{-7}$	0·134
1·0	N.P.L. Box.	2	$2{\cdot}3_1\times10^{-7}$	0·231
0·1	,,	20	$2{\cdot}3\times10^{-7}$	0·023
0·04	,,	50	$2{\cdot}0\times10^{-7}$	$0{\cdot}008_0$
0·02	N.P.L. Tube.	100	$2{\cdot}9\times10^{-7}$	$0{\cdot}005_8$
0·01	,,	200	$4{\cdot}1\times10^{-7}$	$0{\cdot}004_1$
0·004	,,	500	$15{\cdot}5\times10^{-7}$	$0{\cdot}006_2$
0·002	,,	1,000	11×10^{-7}	$0{\cdot}002_2$
0·001	,,	2,000	32×10^{-7}	$0{\cdot}003_2$
0·005	Drysdale-Tinsley.	200	$2{\cdot}2\times10^{-7}$	$0{\cdot}001_1$
0·001	,,	700	14×10^{-7}	$0{\cdot}001_4$
0·0005	,,	1,000	25×10^{-7}	$0{\cdot}001_3$

The shunts described in Table I as being of N.P.L. Box Type are air-cooled shunts, consisting of a number of bifilar loops in parallel. As is to be expected, the time constant is practically independent of the actual resistance, since all the loops are exactly similar and the required resistance is obtained simply by putting an appropriate number in parallel. The " N.P.L. Tube " type of shunt is the Paterson-Rayner* water-cooled tubular shunt.

The inductances of some of these shunts can be calculated approximately from their dimensions—e.g., the Drysdale-Tinsley 0·001 ohm shunt consisted of two concentric cylinders, joined together at one end. The current is led in and out at the other end by " current leads," consisting of a pair of copper plates, parallel, and as near together as possible, so that they will produce practically no stray magnetic field, and the potential leads are very short, so that the mutual inductive effect associated with them is practically negligible. The mean diameter of the cylinder was 16 cm. $=2R$, say. Their length (l) was 15 cm., and the radial distance between them (t) was 0·35 cm. If a current I flows down the inner tube and up the outer one, then evidently the field is zero everywhere except in the annular space between the tubes. The mean field intensity in this region is $2I/R$, and thus the total flux is $2Il/R$. The inductance, or flux per unit current, is thus approximately given by $L=2lt/R=1{\cdot}3$ cm. $=0{\cdot}0013$ μH. The measured value was 0·0014 μH. A few calculated values given by Paterson and Rayner for their water-cooled shunts are given in Table II, together with the corresponding observed values.

TABLE II.—*Observed and Calculated Values of Inductance for Paterson-Rayner Shunts.*

Shunt.	Inductance.	
	Calculated (cm.).	Observed (cm.).
0·02 ohm	5·4	5·8
0·01 ,,	3·4	4·1
0·002 ,,	3·7	2·2
0·001 ,,	3·0	3·2

In these cases, the agreement between the two values is not perfect, although it would be near enough for many practical purposes. The differences are probably due partly to the fact that the conditions required by the formula used for the calculations are only imperfectly realised in the shunts (there is in practice a gap in the middle of the potential sheath), and partly to the fact that the insulation between the tube and sheath has been renewed since the calculations were made, and that its thickness is now almost certainly somewhat different from its original value. There is little doubt that the inductances of such shunts can now be measured with an accuracy greater than that obtainable by approximate calculations.

DISCUSSION.

Dr. A. RUSSELL congratulated the author on his obtaining definite results for a problem of great practical importance, which defied solution by the method of calculation. He inquired whether the effective inductance measured as described in the Paper included the shunt capacity inherent in the resistances and leads ?

Mr. R. P. FUGE : I would like to add my hearty congratulations to Mr. Hartshorn for his very able and clear Paper. I was going to ask a very similar question to the one that Dr. Russell asked, that is as regards the possibility of the separation of the capacities associated with the inductances, more especially as regards the capacities of the leads ? The author uses very long twisted leads, sometimes three metres long. I lately measured the capacity of twisted wire leads I was using, and similar twisted wire (leads) three metres long would have a capacity

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[*] C. C. Paterson and E. H. Rayner, Jour. I.E.E., Vol. 42 (1909) ; or, N.P.L. Collected Researches, Vol. 6, p. 99 (1910).

of, roughly, $100\mu\mu F$. I would like to ask the author what effect that capacity (in the leads) would have ?

Mr. G. W. BOWDLER pointed out that even a capacity of $100\mu\mu F$ would produce only a negligible time constant when shunted across a moderately small resistance.

Dr. D. OWEN said that while A.C. methods of measuring inductance were the most precise they involved the expenditure of much time and the use of expensive apparatus in order to ensure accuracy ; however, the results given in the present Paper conformed to the high standard expected of the National Physical Laboratory. In the measurement of small inductances it was important to eliminate the effect of adventitious capacities, and he noted the simple method, analogous to the Wagner earthing device, which was shown in Fig. 4 of the Paper. The time constant of one of the two 1 ohm resistances in Table I was 50 times smaller than that of the other, so that by careful design much could be done to eliminate reactance. In a standard the resistance and the time constant must both be known accurately. The largest of the time-constants shown in the table corresponded, however, to a voltage error of less than 1 part in 1,000 at electric supply frequencies, and so was negligible for many practical purposes.

The AUTHOR, in reply to the discussion, said that the shunt capacity of the standards considered in the Paper was negligible : a capacity of $1\mu\mu F$ shunting 1 ohm gives an effective inductance of $(-1\mu\mu H)$, so that even $100\mu\mu F$ would produce a negligible result. With higher resistances attention would have to be given to the length of the leads. The rough Wagner earth shown in the diagram sufficiently prevented errors due to stray earth capacities, but owing to the smallness of the resistances elaborate precautions in this connection were not called for.

XXXI.—MAGNETIC DISTURBANCE AND AURORA AS OBSERVED BY THE AUSTRALASIAN ANTARCTIC EXPEDITION AT CAPE DENISON IN 1912 AND 1913.

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Received May 10, 1927.

ABSTRACT.

The Paper summarises the results obtained as to the diurnal variation of magnetic disturbance during 1912 and 1913 at Cape Denison (67° 0′ S. lat., 142° 40′ E. long.), the base station of the Australasian Antarctic Expedition. The results are compared with corresponding results from Eskdalemuir and Cape Evans, the base station of the British Antarctic Expedition. The Paper also discusses the relations between magnetic disturbance and aurora at Cape Denison.

§ 1.

IN any comparison between aurora and magnetic disturbance we encounter the difficulty that neither phenomenon presents itself naturally as a numerical quantity. Aurora may be confined to a small part of the heavens, or may cover a large part of the sky. It may be only a little way above the horizon, or it may extend to the zenith. It may be nearly stationary, or in rapid motion. The motion may be steady, or it may be of the most confused, turbulent nature. The colour may be a uniform yellow or greenish white, varying slowly in intensity, or it may be brilliant red, green, lilac or other colour, continually altering. The brightness and visibility of aurora depend largely on the absence or presence of moonlight or twilight. A bright aurora may be visible under conditions when faint aurora is not. The presence or absence of cloud, haze, or even snowdrift, may be all important. Unless a continuous outlook is maintained—hardly a possibility with ordinary resources—the accidental occurrence of cloud at an observation hour may prevent an aurora from being seen. In the Antarctic, overcast conditions and blizzard, with drifting snow, sometimes extend over several successive days. When data from a number of years are available, the influence of moonlight on the apparent diurnal variation of aurora should be satisfactorily eliminated; but assurance that such has been the case can hardly be felt when only a few months' data are available. Cloud and other meteorological phenomena may have a sensible diurnal variation, and may react on the apparent diurnal variation of aurora.

§ 2.

While appreciating all these difficulties, I decided that for any comparison with magnetic phenomena an attempt must be made to assign numerical measures or characters indicative of greater or less auroral intensity. For the purposes of the investigation Sir Douglas Mawson sent me advanced proofs of the log of auroras observed at Cape Denison—the base station of the Australasian Antarctic Expedition—in 1912 and 1913. To each observation of aurora I assigned a character figure, on the scale 0·5, 1·0, 1·5 and 2·0, answering to what may be roughly described as very faint, faint, bright and very bright aurora. The judgment was based entirely on the descriptive language. No attention was paid to the altitude or type, except that glows and patches were assumed of inferior intensity as compared with arches and curtains. Some allowance was made for unfavourable atmospheric conditions when these were explicitly mentioned. It was assumed, for example, that aurora visible in twilight or bright moonlight could not be very faint. For comparison with the magnetic phenomena it was necessary to employ Greenwich time, and to have hourly auroral character figures. On nights of bright aurora there were often several observations in the course of an hour. The auroral character accepted for the hour was the highest assigned to any observation during the hour. Any hour on which one or more observations had failed to detect any aurora, under ordinarily favourable atmospheric conditions, was assigned the character 0·0. Hours during which no observation had been taken were disregarded.

The hours with character 0·0—i.e., the hours when no aurora was detected under conditions affording a reasonable chance of its being seen—formed $32\frac{1}{2}$ per cent. of the whole. This implies that $67\frac{1}{2}$ per cent. of the hours which I had accepted as affording a reasonable chance of faint aurora being seen showed aurora. When assigning character figures I had only the auroral log to guide me. In his published volume* the conclusion reached by Sir Douglas Mawson himself is as follows: " On 52 per cent. of all hours of moderate twilight and darkness, and including all moonlight hours, when the sky was clear for observation, auroral lights would be seen at least sometime within the hour." Sir Douglas used local time, and included four months which I omitted, as unprovided with corresponding magnetic data. But the difference between us seems mainly due to my omitting a number of complete nights of bright moonlight, or other unfavourable conditions when no aurora was recorded. It seemed unnecessary to repeat my calculations, as the omission of entire nights of bright moonlight could hardly affect the diurnal variation, or prejudice any other object which I had in view.

§ 3.

Since little attention has been given to the question of a numerical criterion of magnetic disturbance for shorter intervals, we naturally turn for guidance to suggestions made for the day.† For it we have in the first place the well-known international scheme: 0 (quiet), 1 (moderately disturbed), 2 (highly disturbed). Various criteria have been proposed involving the first power of the daily range of one or more of the magnetic elements. Use has actually been made of $\triangle D$ and $\triangle H$, the declination (D) and horizontal force (H) absolute daily ranges. A variant is Dr. Bauer's proposal, $H\triangle H$, or the more complete $H\triangle H+V\triangle V$, where V denotes vertical force. Then, based on Bidlingmaier's " magnetic activity," a quantity involving squares of departures from the normal, we have the sum of the squares of the absolute ranges of three rectangular components, or the sum of the squares of all the hourly ranges throughout the day. At a fixed station H and V, when used as multipliers of $\triangle H$ and $\triangle V$, may be treated as constants. Also it is fairly obvious that any reasoning which supports $H\triangle H$ as a criterion of disturbance must equally support $V\triangle V$. At a station near the magnetic equator $V\triangle V$ may be regarded as negligible compared with $H\triangle H$, but at Cape Denison, where V was roughly 20 times H, it is the other way about. The use of the squares of hourly ranges as a criterion for the day's disturbance suggests for the hour the squares of 5-minute, if not 2·5 minute, intervals. But the labour this would have entailed did not seem justified. There was also an obstacle to hourly D ranges. In view of the large D ranges encountered, the D magnetograph at Cape Denison was hypersensitive. In the course of a disturbed day use had to be made now of the one, now of the other, of the traces supplied by the two mirrors carried by the magnet. This would have added materially to the trouble of measuring hourly ranges during active disturbance.

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* Records of the Aurora Polaris, p. 144

† See Gerland's Beiträge zur Geophysik, Bd. XV, p. 14 (1926).

It seemed desirable, as a preliminary, to try criteria of three kinds, one employing the scale 0, 1, 2 applied to each hour, as first suggested by Bidlingmaier, a second kind employing the first power of the absolute range, and the third kind the second power. It seemed unnecessary to employ D, as it seemed unlikely to differ much from H, but it appeared desirable to consider H and V ranges separately. Their subsequent combination, if such appeared desirable, would present no difficulty.

§ 4.

A remark applicable to any criterion involving only the ranges during the included interval of time, whether a day or an hour, is that it cannot directly measure *disturbance*, inasmuch as it contains a contribution from the regular diurnal inequality, which does not vanish on the quietest of days. All the criterion can do is to wax and wane with disturbance, but for our present purpose that is all we want.

The problem presented by the hour has one difficulty, which, though not wholly special to it, is shared to only a minor extent by the day at an ordinary station. Owing to the annual variation in the daily changes, days at the beginning and end of a month will naturally, quite apart from disturbance, differ in their daily ranges. This difference is, however, comparatively small, at least in most months. Relatively considered, it is trifling compared with the differences in the contributions made by the regular diurnal variation to the different hours of the day.

At an English station, if we accepted the hourly range as our criterion, the resulting diurnal variation would hardly merit acceptance as a register of *disturbance*. But Cape Denison, fortunately for our immediate purpose, was not an ordinary station. In June, 1912, the month we intend to use for the preliminary inquiry, the average range within the hour was $24{\cdot}7\gamma$ in H and $29{\cdot}3\gamma$ in V, while the average hourly changes arising from the regular diurnal variation were respectively only $2{\cdot}4\gamma$ and $4{\cdot}4\gamma$. The intervals, moreover, between the highest and lowest values of the element within the hour were more nearly 30 than 60 minutes. Thus the influence of the regular diurnal inequality on the hourly range was even less than might appear at first sight. In neglecting it we introduce no larger uncertainty than we should do in assuming equally disturbed March days in England to have equal daily ranges.

Another point to be remembered is that naturally 30 days contribute to each hourly value. Thus, the parallel case is not the derivation of the disturbance of a single day from the range at a single station, but the derivation of the average disturbance of a month at a particular station from the ranges of 30 days, or the derivation of the disturbance during an individual day from the ranges at 30 stations.

§ 5.

The results of the preliminary investigation, confined to June, 1912—as being one of the chief auroral months—are given in Table I. The columns are numbered at the foot for facility of reference. Local time was really 9 hours 29 minutes fast on Greenwich time. Thus, the 60 minutes ending at 1 hour G.M.T. really centred at 9 hours 59 minutes L.M.T. This is given in the table as 10 hours, as near enough for practical purposes. It was convenient to have the means of seeing at a glance what were day and night hours. The absolute maximum and minimum in each column are distinguished by heavy type.

TABLE I.—*Results from Various Criteria of Disturbance at Cape Denison, June,* 1912.

Hour G.M.T. ending at	Hour L.M.T. centring at	Squares of hourly ranges.				Hourly ranges.		Character Percentages.		
		From 30 days.		From 29 days.		All days.		Mean.	Numbers of	
		H	*V*	*H*	*V*	*H*	*V*		2's	0's
Hour.	Hour.									
1	10	72	51	114	75	109	95	116	130	46
2	11	104	143	147	187	129	154	**135**	**193**	46
3	12	98	190	166	**278**	132	**175**	126	130	30
4	13	133	166	**223**	195	**146**	150	**135**	155	**0**
5	14	94	74	147	92	130	104	116	142	61
6	15	96	50	108	64	117	86	119	104	30
7	16	51	43	78	38	90	70	86	78	137
8	17	62	30	87	35	89	63	84	65	137
9	18	24	**14**	30	**14**	58	**45**	**57**	39	**213**
10	19	36	39	55	46	70	64	70	65	198
11	20	49	51	50	34	74	67	71	65	198
12	21	**366**	179	102	156	121	113	84	104	182
13	22	276	252	132	184	134	140	99	117	121
14	23	327	**293**	160	180	134	150	106	129	106
15	24	136	243	142	189	117	141	113	168	121
16	1	144	200	135	160	112	132	96	104	137
17	2	**13**	21	**22**	33	**48**	59	65	**0**	152
18	3	20	29	29	39	56	66	70	52	182
19	4	19	66	30	51	56	79	83	53	121
20	5	32	49	39	60	64	79	103	91	61
21	6	49	52	78	66	89	85	106	91	61
22	7	63	40	105	60	106	83	116	91	15
23	8	67	75	107	88	109	107	128	130	15
24	9	69	50	114	76	110	93	116	104	30
Column	number	1	2	3	4	5	6	7	8	9

Columns 1 and 2 take the square of the hourly range as the criterion of disturbance; they refer respectively to *H* and *V*. The figures are percentages of $1{,}530\gamma^2$ and of $2{,}178\gamma^2$, the respective means of the 30×24 absolute values. Columns 3 and 4 differ from columns 1 and 2 only in neglecting the one largest range in each hour of the day. The means from the 29×24 data remaining were reduced to $777\gamma^2$ for *H*, and $1{,}221\gamma^2$ for *V*. Columns 5 and 6 use the first power of the hourly range as a criterion, employing all days of the month. The figures are percentages of the means $24{\cdot}7\gamma$ and $29{\cdot}3\gamma$ chronicled above. Column 7 gives the mean of the character figures 0, 1 and 2 applied, treated as numerical quantities. The adjudication was made on the general appearance of the traces, especial y the *H* and *V* traces, no regard being paid to the range. The figures are percentages of 1·04, the mean of all the hourly character figures assigned. The numbers of 2's and 0's in columns 8 and 9 are also percentages. The 2's assigned during all hours of June numbered 185, the 0's numbered 158.

We should obviously have got smoother values if we had combined *H* and *V* ranges in one criterion, but Table I has the advantage of showing that the two elements are in good general agreement. A phenomenon, however, which makes even greater appeal to the eye is the difference between columns 1 and 2, on the one hand and columns 3 and 4 on the other. The exclusion of but 1 out of the 30 data available for each hour of the day has profoundly altered the diurnal variation. Columns 1 to 4 all show a conspicuous double oscillation, with maxima in the early (Greenwich) morning and afternoon respectively; but columns 1 and 2 put the principal maximum in the afternoon, while columns 3 and 4 put it in the morning. Somewhat curiously, the position of the principal minimum is not affected. Columns 1 and 3 agree in putting it in the hour ending at 17 hours G.M.T., while columns 2 and 4 put it in the hour ending at 9 hours. What appears as the principal minimum in *H* appears, however, as a secondary minimum in *V*, and conversely, and the apparent difference between the two minima is trifling. Thus the difference between the two elements is not important, and may be accidental.

§ 6.

Before discussing Table I further, it is desirable to see more clearly how the difference between the first and second pairs of columns is brought about. To this end we give in Table II the hourly ranges whose exclusion from the five hours of the early Greenwich afternoon is mainly responsible for the difference. The table shows for comparison the corresponding means from all the June days for the five hours in question.

TABLE II.—*Hourly Ranges on Special Dates, June,* 1912.

Hour G.M.T. ending at	Special dates.	Ranges on special dates.		Mean ranges from all days.	
		H	*V*	*H*	*V*
Hour.		γ	γ	γ	γ
12	1st	381	248	30	33
13	9th	311	315	33	41
14	8th	338	358	33	44
15	9th & 8th	175	303	29	41
16	9th	189	272	28	39

It will be seen that the first three *H* ranges in Table II are each from 9 to fully 12 times the average range, while the five *V* ranges are each from 7 to 9 times the average. These 8 hours are each the equivalent of from 50 to 150 average hours, when our criterion is the square of the hourly range, and the equivalent of from 7 to 12 average hours when our criterion is the first power of the range. As it so happened, the more outstanding June ranges fell in different (Greenwich) hours, but this may have been a pure accident. Also the hourly ranges in Table II, large as they are, were exceeded on other occasions. On July 6, 1912, *H* had ranges of 498γ and 500γ in two successive hours. Such a day would exert as much influence for the special hour concerned as 400 ordinary days, when our criterion is the square of the range, or as 20 ordinary days when our criterion is the first power. The shorter the period we are considering the greater the effect of such windfalls. The complete range 311γ in H in the hour ending at 13 hours on June 9 was the range for the whole day as well as for the hour, but the maximum and minimum occurred within 5 minutes of one another. If we took 5 minutes as our unit of time, instead of the hour, the influence of such an incident would be simply enormous.

It may appear at first sight that the possibility that the omission of one out of thirty observations may produce an enormous effect is sufficient to condemn the use of the square of the range as a criterion of disturbance. A little reflection, however, will show that this is not a rational conclusion. Take the simpler case of rainfall. A single hour of thunderstorm in a dry month may produce as much rain as the rest of the month. The inclusion of that hour will make an astonishing difference to the diurnal variation of the month. But that implies no fault in the way of measuring rain. For the purposes of the mechanical or electrical engineer, if he can store all that falls, the thunderstorm inch is just as good as any other inch. What it does mean, however, is that a really representative diurnal variation of rainfall requires a very long series of observations. Similarly, if we adopt the square of the hourly range as a criterion, we cannot be certain at a station like Cape Denison of getting a really representative diurnal variation for any month of the year, unless we have a large number of years' observations at our disposal. The same remark applies, though to a lesser extent, to the first power of the hourly range as a criterion.

In addition to these considerations there is the fact that the scale 0, 1, 2 is much more analogous than the other criteria to the auroral criterion adopted, and we shall accordingly, in what follows, give it the preference. It is well, however, to consider the number of 2's and 0's as well as the mean character figure. It is, for example, desirable to know whether a mean character figure 1·0 represents a dead level of mediocrity, or a mixed assemblage of 0's, 1's and 2's.

§ 7.

Returning to Table I, it will be seen that the mean character figure and the number of 2's agree with the results in columns 3 to 6 in placing the principal maximum of disturbance in the early (Greenwich) morning; but they also show a very substantial secondary maximum in the early (Greenwich) afternoon. The mean character figure and the number of 0's agree with the V range criteria in putting the principal minimum of disturbance in the hour ending at 9 hours G.M.T., while the number of 2's agrees with the H range criteria in putting the principal minimum in the hour ending at 17 hours G.M.T.

A good deal of accident must enter into the results from a single month, and in dealing so fully as we have done with June, 1912, our principal object has been to bring out the nature of the several criteria, the extent to which they agree, and the reason why in our subsequent work principal use is made of the scale 0, 1, 2.

§ 8.

The main object of Table III is the investigation of a point of great physical interest. Ordinary meteorological elements like temperature follow local time. That is also true of certain magnetic phenomena, as, for example, the diurnal inequality on the international quiet days. But what happens in the case of magnetic disturbance? We have magnetic storms simultaneously in progress everywhere, and "sudden commencements" occur apparently at the same time—to a minute, if not to a second—the whole world over. It is at least conceivable that the diurnal variation of magnetic disturbance should follow Greenwich (or universal) time everywhere. The results obtained from magnetic disturbance during 1911 and 1912 at Cape Evans*—the base station of the British Antarctic Expedition—showed an unexpected resemblance to those at Eskdalemuir, when Greenwich time was employed throughout. It thus became of the utmost importance to know how the diurnal variation of disturbance at Cape Denison compared with that at Cape Evans. The geographical positions of the two stations are: Cape Evans, 77° 38′ S., 166° 24′ E., Cape Denison, 67° 0′ S., 142° 40′ E. According to the calculations of Mr. E. N. Webb, the magnetic observer at Cape Denison, the approximate position of the South magnetic pole in 1912 was 71° 10′ S., 150° 45′ E. According to these figures the magnetic pole lay not far from the direct line between the two stations, its distance from Cape Denison being only about two-thirds of its distance from Cape Evans. The difference of longitude between the two stations, 23° 44′, means a difference of 1 hour 35 minutes in local time, midnight coming first at Cape Evans.

TABLE III.—*Diurnal Variation of Magnetic Disturbance at Cape Denison, Cape Evans, and Eskdalemuir.*

Hour G.M.T. ending at	Mean hourly character.			Percentage numbers.					
				2's			0's		
	C.D.	C.E.	E.	C.D.	C.E.	E.	C.D.	C.E.	E.
Hour									
1	1·34	1·05	0·65	202	104	167	29	66	95
2	**1.44**	1·02	0·57	**229**	111	123	**15**	83	104
3	1·43	0·97	0·49	222	91	92	16	89	118
4	1·40	0·90	0·45	215	82	89	22	118	124
5	1·26	0·89	0·41	167	74	61	31	115	129
6	1·07	0·87	0·40	107	80	48	56	128	129
7	0·82	0·84	0·43	52	71	41	101	129	124
8	0·69	0·81	0·45	38	**67**	48	133	137	118
9	**0.58**	0·78	0·39	**27**	71	54	161	166	133
10	0·59	0·80	**0.33**	42	90	**14**	168	168	**139**
11	**0.58**	**0.77**	0·46	45	82	20	176	**169**	114
12	0·59	0·80	0·59	56	80	51	**180**	151	92
13	0·70	0·83	0·65	56	80	65	143	146	80
14	0·77	0·89	0·66	68	91	85	127	120	81
15	0·70	0·89	0·72	60	79	129	145	113	75
16	0·60	0·94	**0.73**	45	80	119	168	83	**70**
17	0·59	0·96	**0.73**	34	75	146	164	81	74
18	0·62	1·03	0·72	39	91	129	157	64	75
19	0·73	1·15	0·70	47	123	129	127	50	79
20	0·85	1·20	0·68	69	141	143	101	41	85
21	0·98	1·27	0·70	103	160	**201**	83	**34**	88
22	1·16	**1.28**	0·67	136	**177**	160	43	38	90
23	1·25	1·21	0·62	152	165	126	27	50	95
24	1·32	1·14	0·67	189	142	160	27	57	89
Column	1	2	3	4	5	6	7	8	9

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* British Antarctic Expedition, 1910-1913. Terrestrial Magnetism, pp. 142 et seq.

Table III gives three sets of comparative results for Cape Denison (C.D.), Cape Evans (C.E.) and Eskdalemuir (E). These are based respectively on the mean character figure (columns 1 to 3), the number of 2's (columns 4 to 6) and the number of 0's (columns 7 to 9). The principal maximum and minimum in each case are in heavy type. The C.D. and E. results are from the same 15 months of 1912 and 1913. The C.E. results, based on 1911 and 1912, refer to the year as a whole. In the original comparison the data employed for Eskdalemuir synchronised with those used for Cape Evans, but they agreed in all essential features with those now given. The point that chiefly impressed me when comparing Cape Evans and Eskdalemuir was the approximate agreement between the times when 2's were most numerous and the times when 0's were most numerous.

There is one feature in which there is at least an approach to agreement between the three stations—viz., a slackening of disturbance in the later part of the (Greenwich) forenoon. The principal minima in the first three columns occur between 9 hours and 11 hours, the principal minima in columns 4 to 6 occur between 8 hours and 10 hours, and the principal maxima in columns 7 to 9 occur between 10 hours and 12 hours. There is, however, a prominent difference between the stations even in the matter of the minima of disturbance. The mean character figure at Cape Denison (column 1) shows a secondary minimum in the hour ending at 17 hours, which falls little short of the principal minimum. Column 4 agrees in putting a prominent secondary minimum at the same hour, and column 7 shows a prominent maximum only an hour earlier. Thus all the Cape Denison criteria agree in putting a very decided if secondary minimum of disturbance between 16 hours and 17 hours G.M.T. There is no trace of such a minimum at Eskdalemuir, nor is there in the Cape Evans columns 2 and 8. In column 5, however, there is a slight suggestion of a minimum.

Coming now to the maxima: At Cape Denison the three criteria agree in indicating a principal maximum of disturbance in the hour ending at 2 hours G.M.T. At Cape Evans columns 2 and 5 show a principal maximum in the hour ending at 22 hours G.M.T., and column 8 shows a principal minimum of 0's one hour earlier. Thus here again the criteria are in practical agreement, and put the principal maximum of disturbance some 4 hours earlier than at Cape Denison.

At Eskdalemuir the criteria are less accordant. As appears from column 6, the maximum number of 2's appears in the hour ending at 21 hours G.M.T.—i.e., only an hour earlier than at Cape Evans. According, however, to columns 3 and 9, the mean character figure has its principal maximum and the number of 0's its minimum at 16 hours or 17 hours G.M.T. This, in fact, is an excellent example of the advisability of inquiring how the mean character figure is made up. Its high value at Eskdalemuir at 16 hours and 17 hours is due not so much to the presence of active disturbance as to the absence of quiet conditions. It will be seen that the variations in the mean character figure at Eskdalemuir during the afternoon are not large. The tendency to increase in the number of 2's is attended by a tendency to increase in the number of 0's.

In addition to the principal maximum there is, according to columns 1 and 4, a quite decided secondary maximum of disturbance at Cape Denison in the hour ending at 14 hours G.M.T. There is no suggestion of a maximum at this hour at Eskdalemuir, nor is there in the case of the mean character figure at Cape Evans.

Column 5, however, is not unfavourable to a trifling secondary maximum in the number of 2's.

One or two years' observations near sunspot minimum cannot be relied on to supply results which are closely representative in every detail of all years. But the results obtained for Eskdalemuir from 1911 to 1912, and again from 1912 to 1913, derive support from results obtained for Kew by Mr. J. M. Stagg[*] for the 11 years 1913-23, following a similar procedure. For the 11 years as a whole he found the number of 2's to have its greatest and least values in the hours ending at 22 hours and 11 hours respectively—i.e., an hour later than in Table III. But for sunspot minimum years he found the maximum in the hour ending at 21 hours. His figures also agree in showing no certain secondary maximum or minimum in the diurnal variation. Kew is a much less naturally disturbed station than Eskdalemuir. Also during the years considered by Mr. Stagg it suffered increasingly from electric trains, but that ought not to prejudice the estimate of hours of character 2.

§ 9.

The outstanding result in Table III, the difference in the hours of the principal maximum of disturbance at the three stations, appeared so important that a further investigation was made on somewhat different lines. The results appear in Tables IV and V. Table IV is confined to the 7 complete months April to October and the first

TABLE IV.—*Diurnal Variations of Magnetic Disturbance at Cape Denison and Cape Evans, April to November, 1912.*

Cape Denison	0			1			2			0			1			2			Cape Evans
Cape Evans	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	Cape Denison
1 h. to 6 h.	88	12	0	47	51	2	4	67	29	15	79	6	1	46	53	0	8	92	1 h. to 6 h.
1 h. to 24 h.	64	35	1	27	65	8	3	54	43	57	41	2	18	58	24	1	28	71	1 h. to 24 h.
16 h. to 21 h.	52	47	1	10	76	14	1	46	53	81	18	1	30	58	12	3	44	53	16 h. to 21 h.
Column	1*a*	2*a*	3*a*	4*a*	5*a*	6*a*	7*a*	8*a*	9*a*	1*b*	2*b*	3*b*	4*b*	5*b*	6*b*	7*b*	8*b*	9*b*	Column

TABLE V.—*Diurnal Variations of Magnetic Disturbance at Cape Denison and Eskdalemuir, January to July, 1913.*

Cape Denison	0			1			2			0			1			2			Eskdalemuir
Eskdalemuir	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	Cape Denison
1 h. to 6 h.	86	14	0	65	32	3	33	55	12	18	62	20	5	46	49	0	30	70	1 h. to 6 h.
1 h. to 24 h.	71	29	0	46	49	5	25	57	18	46	46	8	21	57	22	3	41	56	1 h. to 24 h.
13 h. to 18 h.	58	41	1	27	67	6	4	67	29	18	82	0	35	53	12	8	46	46	13 h. to 18 h.
Column	1*a*	2*a*	3*a*	4*a*	5*a*	6*a*	7*a*	8*a*	9*a*	1*b*	2*b*	3*b*	4*b*	5*b*	6*b*	7*b*	8*b*	9*b*	Column

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[*] Meteorological Office Geophysical Memoirs, No. 32, p. 13.

part of November, 1912, for which absolutely synchronous data were available from Cape Denison and Cape Evans. In the first nine columns, 1*a* to 9*a*, the Cape Denison hourly character figures 0, 1, 2 are the guiding ones. It was found how often character 0 at Cape Denison was associated with characters 0, 1, 2 respectively at Cape Evans. Taking, for example, the whole 24 hours, there were 1,462 hours of character 0 at Cape Denison. Of these hours, 935 (or 64·0 per cent.) got character 0 at Cape Evans, 517 (or 35·4 per cent.) got character 1, and 10 (or 0·7 per cent.) got character 2. These percentages, 64, 35 and 1 to the nearest unit, appear in the first three columns, 1*a* to 3*a*, in the line devoted to the whole 24 hours. The other percentage figures in columns 1*a* to 9*a* were arrived at in a precisely similar way. For the second half of the table, columns 1*b* to 9*b*, the Cape Evans characters were treated as fundamental. Of the 1,653 hours awarded character 0 at Cape Evans, 935 (56·6 per cent.) obtained 0 at Cape Denison, 686 (41·5 per cent.) obtained 1, and 32 (1·9 per cent.) obtained 2. These percentages taken as 57, 41 and 2 appear in columns 1*b*, 2*b* and 3*b*. Out of the 5,252 hours employed in Table IV, 1,462 got 0's and 1,258 got 2's at Cape Denison, while 1,653 got 0's and 765 got 2's at Cape Evans. The Cape Denison magnetographs were on the whole decidedly the more sensitive, and with greater sensitiveness the tendency seems to be to give fewer 0's and more 2's. Again, at Cape Evans the standard was largely determined by 1911, which was a decidedly less quiet year than 1912. Thus the difference between the numbers of 0's and 2's at the two stations does not necessarily mean that Cape Denison was the more disturbed. As the totals of 0's and 2's are not the same at the two stations, we should not expect absolute identity between the corresponding figures in corresponding columns of Table IV, even if the diurnal variations of disturbance were identical, but we should expect parallelism between the changes shown as we pass from one column to the next of the same group, or from one line to the next. Now this is exactly what we do not get. For example, as we pass from line 1 to line 3 the figures in column 1*a* go down from 88 to 52, and the figures in column 2*a* go up from 12 to 47, while the figures in column 1*b* go up, and the figures in column 2*b* go down. In short, all the data in Table IV support the view that at Cape Denison the six hours ending at 6 hours G.M.T. were more disturbed, and the six hours ending at 21 hours G.M.T. were less disturbed than the average, whereas at Cape Evans it was exactly the other way about. Of course, the hours in Table IV were specially selected with a view to emphasising the difference between the two stations. In the first instance the 24 hours were considered individually, and then arranged in groups, first of 3, then of 6. The three hours ending at 21 hours G.M.T. were those in which Cape Evans *relatively considered* was most disturbed, and the three hours ending at 6 G.M.T. were those in which the pre-eminence of Cape Denison disturbance was greatest.

Table V, constructed on exactly similar lines, and also confined to seven months, January to July, 1913, compares Cape Denison and Eskdalemuir. In spite of the great disparity in the distribution of the character figures, 2's being scarce at Eskdalemuir, the evidence that the diurnal variation referred to Greenwich time is widely different at the two stations is overwhelming. The pre-eminence of Cape Denison is conspicuous in the same group of hours as when the comparison was between it and Cape Evans. But the hours when Eskdalemuir, *relatively considered*, comes to the front occur earlier in the Greenwich day than those in which Cape Evans was pre-eminent.

The fact that the diurnal variations of disturbance should differ more at Cape Denison and Cape Evans, stations differing by less than 11° of latitude and 24° of longitude, than they do at Cape Evans and Eskdalemuir, stations separated by nearly 133° of latitude and 170° of longitude, appears most surprising, at least at first sight. A possible explanation would be obvious if the 12-hour wave were dominant in disturbance; but at Eskdalemuir, especially, there is little trace of a 12-hour wave.

§ 10.

The fact that the hours of the day when disturbance is most active are not the same at two stations does not mean that conditions tend to be quiet at the one station when disturbed at the other. The reverse is the case. Out of the whole 5,252 hours on which Table IV is based there were only 32 when character 2 at Cape Denison was synchronous with character 0 at Cape Evans, and only 10 when character 0 at Cape Denison was synchronous with character 2 at Cape Evans. Disturbance was so very much more in evidence at Cape Denison than at Eskdalemuir that associations of character 2 at the former station with character 0 at the latter were not uncommon; but there were only 8 hours when character 0 at Cape Denison was synchronous with character 2 at Eskdalemuir. In the majority, moreover, of these exceptional cases the 2's and 0's were rather on the borderland of 1's. Supposing disturbance gradually to increase after a decidedly quiet time, we inevitably have a series of 0's followed by a series of 1's, and that by a series of 2's. But the hour at which the transition is made from 0 to 1, or from 1 to 2, is largely a matter of accident. That no doubt played some part in a good many of the exceptional cases just referred to, but on the whole they represented a very decided difference between the disturbance conditions at the two stations. This should be clear on inspection of Table VI, which gives the mean values of the ranges from the special occasions at the pairs of stations. The force equivalent is given for the *D* range at Cape Denison. The components E′, N′ at Cape Evans were directed about 7·5° south of east and east of north. There were no vertical force data from Eskdalemuir for 1913. Disturbance, as a rule, was so enormously greater at Cape Denison than at Eskdalemuir that it was almost a surprise to find that there were hours in which magnetic changes were actually greater at Eskdalemuir than at Cape Denison.

TABLE VI.—*Mean Hourly Ranges from Special Occasions.*

Occasions ...	32		10		8	
Character ...	2 at C.D., 0 at C.E.		0 at C.D., 2 at C.E.		0 at C.D., 2 at E.	
	At C.D.	At C.E.	At C.D.	At C.E.	At C.D.	At E.
	γ	γ	γ	γ	γ	γ
	D 32	E′ 14	*D* 24	E′ 52	*D* 15	N 28
	H 38	N′ 12	*H* 15	N′ 37	*H* 7	W 18
	V 38	*V* 7	*V* 14	*V* 24	*V* 10	...

§ 11.

Table VII is devoted to the question of the parallelism between the diurnal variations in aurora and in magnetic disturbance. Column 1 shows the distribution throughout the day of the 250 hours to which I had allotted auroral characters of 2·0 or 1·5, the significance of which has been already explained. Column 2 gives the distribution of 180 hours when aurora was in the zenith. Column 3 gives the mean hourly magnetic character figure at Cape Denison derived from the four winter months (W.) May to August, during which aurora was principally seen. Column 4 does the same for the summer months (S.), November, January and February, when no occurrence of aurora was noted.* Columns 5 and 6 give the distribution of magnetic character 2 for winter and summer, and columns 7 and 8 deal similarly with the 0's. The figures in columns 5 to 8 are percentages of the respective

TABLE VII.—*Diurnal Variation of Aurora and Magnetic Disturbance at Cape Denison.*

Time.		Auroral occurencies.		Magnetic disturbance.					
Hour G.M.T. ending at	Hour L.M.T. centring at	Bright aurora.	Zenithal aurora.	Mean character.		Percentages.			
						2's		0's	
				W	S	W	S	W	S
Hour.	Hour.								
1	10			1·02	1·82	139	219	44	0
2	11			**1.19**	1·80	**215**	213	**24**	0
3	12			1·15	**1.83**	184	**219**	26	0
4	13			1·11	1·82	177	216	34	0
5	14			1·04	1·61	160	161	45	8
6	15			0·84	1·51	101	137	79	16
7	16	1	3	0·59	1·25	52	73	124	25
8	17	6	3	0·50	1·02	56	43	150	90
9	18	10	4	**0.43**	0·93	38	33	160	139
10	19	11	1	0·47	0·91	52	39	155	172
11	20	19	2	0·50	**0.72**	56	**24**	151	262
12	21	25	4	0·50	0·73	83	33	**161**	**278**
13	22	**38**	2	0·64	0·79	90	39	124	254
14	23	37	6	0·72	0·90	108	49	111	205
15	24	19	3	0·71	0·85	115	43	117	221
16	1	19	2	0·58	0·80	76	30	138	221
17	2	7	5	0·49	0·86	**35**	30	145	180
18	3	7	11	0·49	0·91	45	33	146	156
19	4	10	24	0·59	1·01	63	33	127	82
20	5	12	32	0·70	1·19	76	73	105	57
21	6	20	**40**	0·80	1·37	111	112	91	33
22	7	7	30	0·92	1·60	118	161	62	0
23	8	2	8	1·02	1·67	118	179	38	0
24	9			1·02	1·77	132	207	41	0
Means				0·75	1·24				
Column	...	1	2	3	4	5	6	7	8

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* December *H* and *V* records at Cape Denison were lacking or unsatisfactory.

means. Principal maxima and minima in columns 3 to 8 are in heavy type. Among Sir Douglas Mawson's deductions were the following :—

"Our experience at Cape Denison very conclusively indicated that the daily period of apparently most intense auroræ was between 9 p.m. and 11.30 p.m." (i.e., between 21 hours and 23·5 hours L.M.T.) (l.c., p. 158), and again, speaking of zenithal aurora, "The most obvious feature is the extreme frequency of zenith aurora in the early morning hours immediately preceding daylight" (l.c., p. 168). Table VII is in excellent agreement with these statements. Sir Douglas counted as zenithal any aurora within 20° of the zenith, and he employed several months which are not included in Table VII owing to the lack of corresponding magnetic data. This explains his greater number (272) of zenithal occurrences. In addition to a principal maximum frequency of bright aurora about 13 hours G.M.T. (or 22½ hours L.M.T.), column 1 shows a secondary maximum at 6 hours L.M.T., synchronous with the maximum frequency of zenithal auroras in column 2.

Taking the magnetic data for winter (W.) in Table VII as the more appropriate, it will be seen that the principal maximum of bright aurora is nearly coincident with a secondary maximum in the mean magnetic character figure and the number of 2's, and with a secondary minimum in the number of 0's. Also the minimum at about 17 hours G.M.T. in the frequency of bright aurora is fairly coincident with the principal minimum in magnetic 2's, and with a secondary maximum in the number of magnetic 0's. There is thus a very sensible parallelism between some of the phenomena in the diurnal variations of aurora and magnetic disturbance. But what appears at first sight a complete disagreement is the occurrence of the principal maximum of magnetic disturbance at 11 hours L.M.T., an hour when aurora never was and never could be seen. A possible explanation, however, at once suggests itself. The figures in Table VII indicate at first sight a rapid decline after 6 hours L.M.T., both in bright and zenithal aurora. But this may well have been wholly due to the effects of increasing daylight. In the midwinter months, June and July, aurora was occasionally seen as early as 16 hours L.M.T., and as late as 8 hours L.M.T., but even in May and August 16 hours and 8 hours became impossible, and in the equinoctial months the limits were much narrower. In Sir Douglas Mawson's words, "The rapid decrease (of auroral occurrences) at the limits is . . . to be ascribed to increasing twilight obscuring weaker effects, and it seems probable that were the darker hours of longer duration the rise of the curve (showing mid-winter auroral frequency) would continue on either hand instead of reversing" (l.c., p. 166). If we assume that the overhead electrical phenomena, seen as aurora at night, may exist during the day though invisible, it is at least possible that the increase in frequency of the electrical storms did not really stop at 6 hours L.M.T., but went on right up to the time of the principal maximum of magnetic disturbance about 11 hours L.M.T.

The resuscitation of brilliant aurora in the early morning L.M.T. shown in column 1 of Table VII is probably underestimated, owing to the effects of daylight; but in Sir Douglas Mawson's opinion the brightest auroras did have their maximum at the time suggested by the table. "The auroræ so regularly appearing in the twilight hours of early morning . . . though never extremely brilliant to look at, must often have been intense in order to be at all apparent in twilight. But . . . it would appear that, after all, the later evening hours were *par excellence* the period of the day when auroræ of the greatest intensity were to be expected" (l.c., p. 159).

Before leaving Table VII it is well to compare the winter and summer magnetic results. The chief difference between the mean character results in columns 3 and 4 is that, absolutely considered, the summer values were much the higher. This was equally true of character figures for the day. The winter months, the principal months for aurora, were on the whole much the quietest months magnetically. The summer months, when no aurora was seen, were much the most disturbed months. In both respects the equinoctial months took an intermediate place. We see in both columns 3 and 4 a principal maximum in the late (local) forenoon, a secondary maximum an hour or two before midnight. As regards the number of 2's, winter and summer, columns 5 and 6, again agree in showing a principal maximum a little before (local) noon, and a secondary maximum a little before midnight; but this secondary maximum is less prominent in summer than in winter. In both seasons there are two minima almost equally conspicuous; they are separated by only about 8 hours in winter and less in summer.

As regards 0's, columns 7 and 8, the two seasons agree in placing the principal maximum at or a little before 21 hours L.M.T., near the time of a minimum in 2's. In winter there is a secondary and not very much inferior maximum between 2 hours and 3 hours L.M.T., near the time of the minimum in bright aurora. In summer not a single 0 was attached to any hour between 21 hours and 4 hours G.M.T. (6½ hours to 13½ hours L.M.T.). It is possible that if a very much higher standard of disturbance had been accepted for summer, so as to produce a large increase in the number of 0's, a double daily oscillation might have been clearly recognised in that season as well as in winter. There is a suggestion of a minimum at 23 hours L.M.T. in column 8, synchronous with the secondary maximum in column 6. However this may be, it seems indisputable that the main features of the diurnal variation of magnetic disturbance are the same for summer, the season of no auroras, as for winter, the season when auroras are at their maximum.

§ 12.

Table VIII compares the night and day phenomena in a different and more detailed way. It uses the results of twelve months, March to October, 1912, and April to July, 1913. Any (Greenwich) hour in which there was even one aurora seen in the course of a month was regarded as an hour when aurora, if existent,

TABLE VIII.—*Magnetic Disturbance, Night Hours and Day Hours, at Cape Denison.*

Month.	Hours per diem. Aurora		2's in hours when Aurora		Expectation of a 2 in hours when Aurora	
	Visible.	Invisible.	Visible.	Invisible.	Visible.	Invisible.
March, 1912	5	19	17	141	52	113
April, 1912 & 1913 ...	14	10	85	204	50	169
May, 1912 & 1913 ...	15	9	92	111	73	146
June, 1912 & 1913 ...	17	7	122	80	85	136
July, 1912 & 1913 ...	17	7	101	67	85	137
August, 1912	16	8	37	81	47	205
September, 1912 ...	12	12	16	98	28	171
October, 1912 ...	5	19	24	234	44	115
Totals			494	1,016		

could be seen during the whole month. In months like April and October the length of the day is altering somewhat rapidly, and failure to see the aurora in a particular hour of the day is not incompatible with the possibility of seeing it during that hour during a small part of the month. But for our present object meticulous accuracy is not called for.

In the expectation figures in Table VIII, 100 is taken to represent the chance which the average hour of the month has to get magnetic character 2. According to the table, during every month of the year when aurora was recorded, the part of the day when aurora was seen was much quieter than the part of the day when it was not seen. The greater prominence of magnetic disturbance in the daylight hours, though reduced, is still decided in June and July, which were the months when aurora was most pronounced. These two months included 14 out of the 28 occasions specified by Sir Douglas Mawson (l.c., p. 149) as being those when colour effects were most brilliant.

It is well, however, to bear in mind the fact that these conclusions depend on the magnetic character criterion which we have decided to employ. It is by no means certain that we should have reached the same conclusion for the winter months, or at least for June and July, if we had employed a criterion such as the square of the hourly range. In fact, if we did employ as our criterion of magnetic disturbance the sums of the squares of the hourly *H* and *V* ranges, our conclusion would be that in June, 1912, the night as a whole was almost equally disturbed with the day. There were only 13 hours during June which had *H* hourly ranges exceeding 100γ, and the range next largest to the 5 specified in Table II was only 140γ. Thus in June, whether accidentally or not, the night hours contained all the more outstanding disturbances in *H*, and the same was true to a nearly equal extent of *V*. Most of these outstanding occasions were certainly accompanied by brilliant aurora; but this is uncertain as regards the largest of the *H* ranges, which occurred on June 1. At Cape Denison the sky was overcast during the whole night. At the "Grottoes," the station occupied by the Australian Antarctic Expedition in Queen Mary Land (66° 20′ S., 95° 2′ E.), an aurora "with faint colouring" was reported on June 1, but the time of occurrence is not given. The Cape Evans record, which I was able to examine thanks to the courtesy of Captain C. S. Wright, stated that on the night in question there was clear strong moonlight and no aurora. Under the circumstances specified, however, faint aurora might have escaped detection. Also, as a matter of fact, simultaneous observations of aurora at Cape Evans and Cape Denison would seem to have been very rare, and this perhaps is not surprising, as aurora was seldom seen in the south at Cape Denison.

§ 13.

Table IX makes a more detailed comparison of aurora and magnetic disturbance

TABLE IX.—*Incidence of Auroral and Magnetic Characters at Cape Denison.*

Auroral character ...	0·0			0·5			1·0			1·5			2·0		
Magnetic character ...	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2
All auroral occurrences ...	293	157	31	120	65	13	252	267	59	56	98	37	1	8	26
Percentages	61	33	6	61	33	6	44	46	10	29	51	20	3	23	74
Zenithal aurora occurrences				8	5	2	39	59	10	10	29	8	0	2	8
Percentages				54	33	13	36	55	9	21	62	17	0	20	80

based on the auroral characters described in § 2. Table IX is constructed on similar lines to Tables IV and V. Take, for example, the hours of auroral character 1·0, which numbered in all 578. Of these 252, or 44 per cent., were hours of magnetic character 0 ; 267, or 46 per cent., were hours of magnetic character 1 ; and 59, or 10 per cent., were hours of magnetic character 2. Of the 578 hours there were 108 during which there was at least one observation of aurora in the zenith. Of these 108 hours, 39, or 36 per cent., had magnetic character 0 ; 59, or 55 per cent., had magnetic character 1 ; and, finally, 10, or 9 per cent., had magnetic character 2.

One of the most striking results in Table IX is that, if we may judge by the percentage figures, there is no certain difference, so far as magnetic disturbance is concerned, between hours when aurora was not seen and hours in which is was seen but was very faint.

As we pass up the auroral scale from 0·5 to 2·0 the percentage of hours of magnetic character 0 falls, and the percentage of hours of magnetic character 2 rises. Of the 35 hours awarded auroral character 2, only 1 had magnetic character 0, and 26 had magnetic character 2.

It may not be superfluous to state explicitly that the auroral characters were assigned at a much later date than the magnetic characters, and absolutely independently.

The hours of zenithal aurora in Table IX are the same as in Table VII, numbering in all 180. The distribution of magnetic characters in these 180 hours is not widely different from the distribution in the whole 1,002 hours of aurora included in the table. The percentage number of occurrences of character 0 is, however, slightly reduced in each auroral category. This difference would, of course, be somewhat amplified if we contrasted the hours of zenithal aurora with the 822 hours when aurora was seen only at lower altitudes. It should be noted, however, that on many of the 180 hours when aurora was seen in the zenith it was also seen at much lower altitudes, and it was sometimes brightest at the lower altitudes.

Unless the overhead position has some compensating disadvantages, the greater proximity to the station of an overhead aurora would naturally lead to its causing more magnetic disturbance than an aurora of equal intensity at a smaller altitude above the horizon. This consideration may well explain the slightly greater effectiveness of zenithal aurora suggested by Table IX. In view, however, of what has been said of the special tendency of zenithal aurora to occur in the hours near daybreak, the observed phenomenon may be partly due to the effects of twilight in leading to a slight underestimate of the auroral character.

§ 14.

Further evidence of the association of magnetic disturbance and bright aurora may be derived from some statistics as to the hourly magnetic ranges. There were only two months, June and September, 1912, for which measurements were made of the ranges of all hours of the day. In June there were 39 and in September 19 hours of bright aurora—i.e., with auroral characters 1·5 or 2·0. The mean ranges from the hours of bright aurora and from all hours, whether with or without aurora, were as follows :—

	Hours of bright aurora.		All days of the month.	
	H	*V*	*H*	*V*
June	54γ	77γ	26γ	33γ
September ...	42γ	46γ	17γ	22γ

In forming the all day means the influence of any diurnal variation was eliminated by omitting all hours of the 24 when aurora was not seen—i.e., all daylight hours—and weighting the others according to the number of times when aurora was actually seen. In both months the excess in the ranges for the hours of bright aurora is very large.

The measurement of magnetic ranges for the hours of bright aurora was not confined to June and September, 1912, but extended to all the hours of bright aurora included in Table IX for which the *D*, *H* and *V* records were all complete, and to a few additional hours of bright aurora in March, 1912. There were magnetic curves for part of that month, though they were not fully tabulated. The following were the mean values thus obtained for the hourly ranges :—

Year.	Number of hours.	*D*	*H*	*V*
1912 & 1913	227	37γ	38γ	43γ
1912	140	48γ	52γ	56γ
1913	87	20γ	17γ	21γ

As before, the changes in *D* are expressed in terms of the force equivalent. June and September may be taken as fairly representative of the months when aurora was seen in 1912. Thus we seem entitled to assume that in other months of that year, as well as in June and September, the magnetic ranges during hours of bright aurora were much above the average.

The comparatively small size of the hourly ranges on hours of bright aurora in 1913 may suggest that aurora exerted little if any effect on magnetic disturbance in that year. That, however, is probably not the true explanation. 1913, with a Wolfer's frequency of only 1·4, was a year of sunspot minimum, but 1912, with a sunspot frequency of 3·6, was nearly as quiet a year in Europe. In the Antarctic, however, whether we judge by the daily and hourly character figures, by the absolute daily ranges, or by the ranges of the regular diurnal inequalities, the magnetic changes regular and irregular were much smaller in 1913 than in 1912. For example, the ranges of the regular diurnal inequalities of *D*, *H* and *V* from all days of the winter season of 1913, using Greenwich days, were respectively only 53, 59 and 65 per cent. of the corresponding ranges for 1912. Thus a very large reduction was to be expected in the hourly ranges during hours of bright aurora in 1913, though hardly so large as that shown above.

There was a remarkable reduction in the amplitude of the magnetic changes at Cape Evans in 1912 as compared with 1911 ; and, as we now see, there was a very large reduction at Cape Denison in 1913—the year of sunspot minimum—as compared with 1912. It is obviously important to know whether this is accidental, or whether the 11-year period in magnetic phenomena has an exceptional development in the Antarctic (and Arctic). Comparative results from years near sunspot maximum would clearly possess much interest.

SUMMARY OF RESULTS.

§ 15.

Magnetographs were in operation at Cape Evans (77° 38′ S., 166° 24′ E.) during the greater part of 1911 and 1912, and at Cape Denison (67° 0′ S., 142° 40′ E.) from March, 1912, to August, 1913. The diurnal variations of magnetic disturbance at the two stations differ decidedly as regards the hour of maximum. Referred to

Greenwich or universal time, this is some 4 hours later at Cape Denison than at Cape Evans, a difference more than double the difference in local time. The time of occurrence of the principal maximum of disturbance at Cape Denison is roughly 2 hours G.M.T. the whole year round.

Active magnetic disturbance at Cape Denison is much less common in winter than in summer, and much less common by night than by day. Thus it is least common during the hours when aurora can be seen.

Very faint aurora seems to have no sensible effect on the occurrence of magnetic disturbance, but stronger aurora unquestionably has an effect, which increases with the intensity of the aurora.

Comparing the diurnal variation of aurora and magnetic disturbance at Cape Denison, it is found that the time of greatest frequency of bright aurora, 9.30 to 11 p.m. local time, synchronises at least approximately with the time of a secondary maximum of magnetic disturbance. During the next two or three hours there is a decline both in bright aurora and in magnetic disturbance, but in the early morning hours aurora begins to increase in frequency and to approach the zenith, and this seems to go on until aurora is interfered with by the approach of daylight. During this time magnetic disturbance is also increasing, and in its case the increase continues right up to the hour of maximum disturbance near local noon. This is at least consistent with the view that the overhead electrical storms which become visible at night as aurora continue in the day, and are, in fact, then most numerous. A fact also favourable to this view is that the diurnal variation in the frequency of active magnetic disturbance at Cape Denison exhibited the same general features in the midsummer months when no aurora was seen, as in the midwinter months when aurora was at its maximum.

At the same time, it must be allowed that near midwinter, 1912, there were a few comparatively short magnetic disturbances during the night, most, if not all, synchronous with specially brilliant aurora, which were much larger than any experienced at that season during the day. There was also an individuality about many of the minor night disturbances, most at least accompanied by aurora, which suggested the presence of some unique source of disturbance during a time varying from half an hour or less to two hours. This is, however, discounted, at least to some extent, by the fact that during the day, especially in summer, disturbance on a considerable scale was often so continuous that no short period disturbance could stand out, unless it were of altogether exceptional rapidity or amplitude. Also there were occasions during the day hours in summer when the rapid oscillations rivalled those occurring during the most brilliant winter aurora.

DISCUSSION.

Dr. G. C. **Simpson** said that all members of Scott's expedition would be grateful to Dr. Chree for his labours, without which the observations discussed would have been wasted. The observations on the aurora he would regard as qualitative rather than quantitative. The record taken hourly through the night had been very valuable in keeping the night watchman alert, but it was hardly to be expected that on leaving a warm hut the latter would spend time in a minute scrutiny of all parts of the sky; although on one occasion the speaker had been called out to see a display of such beauty that he had spent no less than two hours in watching it, in scanty attire, in a temperature of −40 or −50 degrees Fahrenheit. Again, it was difficult to write down a significant record at times when the aurora was constantly changing and moving; one might as well try to record observations on a firework display.

Capt. C. S. **Wright** said that he had looked forward to the Paper, because he had been anxious to see how the Cape Denison results fitted in with those from Cape Evans. He challenged the assumption that the early morning auroral maximum would have occurred later in the day but for the incidence of daylight, because at Cape Evans the corresponding maximum occurred earlier in local time, during darkness, and he felt sure that there the maximum was a real one. Then again the other maximum (corresponding to the midnight maximum of Cape Denison), which occurred at 7 p.m. in local time at Cape Evans, was quite different in character; in the curve it consisted of a "pimple" in a general minimum, whereas the early morning maximum lay on the top of a crest. Further, the aurora itself had different characters at these two times, being irregular and rapidly shifting during the earlier maximum, but more uniform in the early morning. Did the author endorse the view that the maximum of magnetic disturbance may occur at "magnetic local midnight," when the sun is immediately below the magnetic meridian?

The AUTHOR, in reply to the discussion, said that in favour of the view that the early morning auroral maximum should be placed later than the observed maximum was the fact that Sir Douglas Mawson's criterion showed an upward gradient right up to the incidence of daylight. As regards the effect of magnetic local time, he had suggested this as a factor some years ago, but a difficulty was that the magnetic axis is a mathematical fiction. The North and South magnetic poles are not antipodal, and though this fact might be of no account some 40,000 miles or more above the earth's surface, it was difficult to attach a definite meaning to magnetic time on the earth itself.

The CHAIRMAN, in thanking the author for his Paper, added that it had given great pleasure to Fellows present to welcome two members of Scott's expedition.

XXXII.—SERIES IN THE SPECTRUM OF TREBLY-IONISED TIN (Sn IV).

By K. R. Rao, *M.A.* (Madras University Research Scholar).

Abstract.

The Paper gives particulars of regularities observed in the spectrum of tin corresponding to the trebly-ionised stage of the element. Pairs of sharp and diffuse series are discovered which converge to a common limit with a sixteen-fold value of the Rydberg constant. The fundamental and super-fundamental series doublets are also indicated. Term values are evaluated from the related system of members by assuming a value for the term "*G*."

Great efforts have been made in the determination of the wavelengths of the lines of the tin spark spectrum. The plates were finally measured from spectra taken in the second order of a 5-ft. concave grating and an accuracy to about 0·05 Å.U. is claimed.

The experimental method of sorting out the lines due to this high stage of ionisation (Sn IV) is the same as that adopted previously by the writer in the case of Sn, Ga, In.

Introduction.

In accordance with the recent developments in the analysis of spectra it is to be expected that, while the neutral atom of tin will yield a spectrum derived from systems of terms of odd multiplicity, the spectra at successive stages of icnisation of the element will be constituted from terms of alternately even and odd multiplicity.

Although the number of lines in the tin arc spectrum is not large, the study of the spectrum from the series view point has not made great progress. Our knowledge is confined only to a table of frequency differences based on results obtained from absorption experiments,(1) the nature or the absolute value of the individual energy levels being yet undetermined.

Investigations for series relationships among the spark lines of tin are also meagre. No extended series in the spectra of the element in any of the ionised states have yet been detected, but in a recent paper(2) the present writer has set forth the leading members of the principal, diffuse (secondary) and fundamental series of a triplet system belonging to the spectrum of doubly-ionised tin (Sn III). Mention was also made of two pairs of the same frequency difference, and though no definite assignment* of the pairs was possible, they were found to form a Rydberg sequence with a $4R$ constant. This may be taken as evidence of the probable existence of doublets representing Sn II. It is the purpose of this Paper to present regularities in the spectrum of tin corresponding to the third-step ionisation of the atom, the system of series being characterised by a sixteen-fold value of the Rydberg constant.

Experimental.

As is usual in investigations of this nature, the spark spectrum of tin has been examined under different degrees of excitation. The sources included the spark between pure electrodes of tin in air, in vacuum and in an atmosphere of hydrogen at varying pressures, and also the arc in vacuum. The ordinary spark in air and the vacuum arc served well to eliminate the lines due to the lower stages of ionisation of the element. As in previous experiments, recourse was also had to the method of studying the spectrum when the inductance and capacity in the circuit were varied. In general, the lines of Sn IV are found to be best developed in the condensed spark in hydrogen at low pressures, while they are feeble and diffuse, or perhaps absent, in the condensed spark in air or in hydrogen at atmospheric pressure.

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* The magnitude of the separation (625) suggests that it may be the "second p" separation, while the limit and terms suggest the assignment $x - \varphi$ of the type found in Si II.

For preliminary observation of the spectra an ordinary constant deviation spectrograph and a plane grating spectrograph were used for the visible region, while a quartz spectograph was utilised for the ultra-violet. But the wavelengths of the lines were finally determined from spectrograms taken in the second order of a 5-ft. concave grating, recently purchased and mounted by Prof. Narayan, giving a dispersion of about 5·67 Å.U. per mm. In the region below about λ2,600 these values were also checked by measurements from plates taken with a Hilger quartz instrument giving a dispersion of about 7 Å.U. per mm. at λ2,450, the wavelength being calculated by the use of the simple Hartmann formula. The plates were measured with a comparator of the Cambridge Scientific Instrument Co., reading to 0·01 mm. Each plate was measured a large number of times by two different observers, and the average of the readings was taken. The arc lines of tin, for which the accurate measures of Arnold(3) have been adopted, served as standard lines, and for the region between λ2,200-2,000 the usual standards of copper(4) and aluminium spark lines were also utilised. In general, the wavelengths may be claimed to be accurate to about 0·05 Å.U. Great labour was bestowed in measuring up the plates, as the existing measures of many of these lines were far from being suitable for the present purpose. A detailed list of the spark lines of tin, giving the stage to which each line is believed to belong, will be presented in a separate communication.

The notation is the same as that used in the previous Papers. The subscript denotes the inner quantum number. The first term is denoted by " 1 *P*," the second by " 2 *P*," and so on in each case, and when only one term is found the numeral is omitted.

IDENTIFICATION OF THE SERIES.

The method of identification of the series in Sn IV was, in the first instance, by applying to the case of the spectra of silver-like atoms the relativity laws, the fecundity of the application of which to a long chain of atoms of the same electronic structure has been clearly brought out in a series of Papers by Bowen and Millikan. The chief members of the secondary series of the spectrum of In III, resembling those of Ag I, Cd II, have already been indicated in an earlier communication.(5) The location of these in the spectrum of Sn IV was therefore greatly facilitated. The regular and irregular doublet-law-progressions with atomic number, as shown below, indicate the value of the frequency of $2S_1 - 2P_2$ to be approximately 26,000 cm.$^{-1}$, and the doublet separation ($2P_1 - 2P_2$) to be about 2,200.

Element.	$2S_1 - 2P_2$ ν	Diff.	$2S_1 - 2P_2$ ν	Diff.
Ag I	(5,946)		(5,741)	
		6,447		5,978
Cd II	12,393		11,719	
		6,655		5,992
In III	19,048		17,711	
		6,843		6,003
Sn IV	[25,891]		23,714	

(Fourth power law) $(Z - s) = \sqrt[4]{\Delta\upsilon/0{\cdot}0135}$.

Atomic Number (Z).	Element.	$\Delta\upsilon$ ($2P_1 - 2P_2$)	$Z - s$	s (screening) constant).	Δs
47	Ag I	202·9	11·08	35·82	2·76
48	Cd II	672·82	14·94	33·06	1·80
49	In III	1,337·4	17·74	31·26	1·30
50	Sn IV	[2,177·4]	[20·04]	[29·96]	

Comparison of the separations with the squares of net nuclear charges of the four atoms Ag I to Sn IV gave about 2,190 cm^{-1} for the second "P" separation. Examination of the spectrum showed at once the strongly enhanced doublet $\lambda\lambda$3,861·26, 4,215·74 to fit in perfectly into the above progressions. The component on the short wavelength side is certainly stronger than the other, showing that the doublet is of a principal series. It will be of interest to observe, also, the variation of the doublet separation with the square of the atomic number of the elements C, Si, Sn of the same vertical group of the periodic table.

Atomic Number Z.	Element.	$\Delta\upsilon$ $1P_1 - 1P_2$	$\Delta\upsilon/Z^2$	$\Delta\upsilon$ $2P_1 - 2P_2$	$\Delta\upsilon/Z^2$
6	C IV	107·4	2·984		
14	Si IV	460·0	2·347	162	0·8264
50	Sn IV	6507·0	2·603	2177	0·8712

The progression of the member of the diffuse series indicated that the strong line λ2,848·51, which certainly belongs to Sn IV, may be $2P_2 - 2D_3$.

Variation of $2P_2 - 2D_3$.

Element.	υ	Diff.
Ag I	(5,715)	9,149
Cd II	14,864	9,924
In III	24,788	10,308
Sn IV	[35,096]	

But for a long time the examination of the quartz spectrograms failed to reveal its companion $2P_1 - 2D_2$, which, according to the intensity rule, must also be equally strong. However, the second order spectrum (Fig. 3-B), taken with the concave grating, distinctly showed the strong lines λ2,706·23, which, in behaviour, is identical with λ2,848·51. Its proximity to the intense arc line λ2,706·5 caused the initial difficulty. The pair and satellite given below were therefore taken as the required diffuse member.

λ I.A. (Int.).	ν (vac.).	Δν
2,875·73 (5)	34,763·6	332·2
2,848·51 (10)	35,095·8	2,177·21
2,706·23 (9)	36,940·8	

The detection of this member removed all doubt as to the certainty of the identification.

A further search among the enhanced lines belonging to Sn IV for pairs with the above separation 2,177·4 resulted in the finding of the two following:—

λ I.A.	Int.	ν	Δν
3,071·51	(8)	32,547·9	2,177·6
2,887·79	(10)	34,618·4	107·1
2,878·89	(4)	34,725·5	
2,659·85	(4)	37,585·0	2,178·0
2,514·14	(4)	39·763·0	

The first of these is accompanied by a satellite, and the disposition of the group shows that it must be an inverted diffuse member, while the second is taken to be a pair of the sharp series.

The consistency of the allocation of these lines is shown by calculating the formulæ for the two series. The brighter components of the pairs are used for the determination of the limits.

(1) Limit from the sharp series—

$$s_1(2)=2P_2-2S_1=-25{,}891{\cdot}4$$

$$s_1(3)=2P_2-3S_1=\ \ 37{,}585{\cdot}0$$

$$s_1(m)=130{,}191-\frac{16R}{[m+1{\cdot}35313]^2}$$

i.e., limit $2P_2=130{,}191$.

(2) Limit from the diffuse series—

$$d_3(2)=2P_2-1D_3=-34{,}618{\cdot}4$$

$$d_3(3)=2P_2-2D_3=\ \ 35{,}095{\cdot}8$$

$$d_3(m)=132{,}841-\frac{16R}{[m+1{\cdot}23722]^2}$$

limit $2P_2=132{,}841$.

The agreement between the limits is as close as can be expected from the use of the simple Rydberg formula.

The Primary Series.—The chief lines of the Sn IV spectrum constituting the primary series are to be expected in the extreme ultra-violet. But the observations of the spark spectrum of tin in the vacuum grating region are not satisfactory for a sure identification of these series of lines. From the formula calculated above from the sharp series there results for the first principal series doublet

$$2P_2=130{,}191=\frac{16R}{[3\cdot67136]^2}$$

$$2P_1=132{,}369=\frac{16R}{[3\cdot64111]^2}$$

——	ν	Δν
$1S_1-1P_2=$	71,030·4	5,684·8
$1S_1-1P_1=$	65,345·6	

Carroll(6) has already located from a consideration of the sequence of atoms Ag I to Sn IV the pair

——	ν	Δν
$1S_1-1P_2=$	76,063	6,507
$1S_1-1P_1=$	69,556	

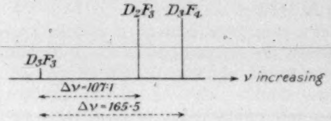
The agreement between the observed and calculated positions is not quite perfect, and this must probably be due to the approximate character of the formula employed. The position of the first diffuse series member can also be approximately determined through the observed inverted doublet, but the data of Lang(7) do not show the required pair and satellite with the appropriate separation. The vacuum grating measurements must be more exhaustively made before this member can be fixed correctly in the light of these results.

Fundamental Series.—The location of the members of the fundamental series is next considered. In the case of most spectra the first member of this series is very prominent, and characterised by a high intensity and diffuse nature of the lines. It is expected that the corresponding member in the spectrum of Sn IV must also have been strongly excited under the conditions studied. And the separation $\triangle\nu D_{3\,2}$ is known from the disposition of the inverted doublet $1D-2P$ to be equal to 107·1 cm.$^{-1}$ From the approximate value of the first "*D*" and "*F*" terms, as judged by the sequence of terms of Ag I, etc., and also from the general run of these corresponding members in this sequence,

——	ν
Ag I	5439
Cd II	18588
In III	33228
Sn IV	

it is seen that this member very probably occurs between λ2,100 → λ2,000.

Exposures of about an hour and a half with the large size Hilger quartz spectograph on Schumann plates brought out clearly the spectrum to the very limit of the instrument. The first order of the sharply focussed concave grating was also made use of for a careful examination of the spectrum in this region. The line λ2,040 is seen faintly on the concave grating plate, while the quartz spectrum extended up to λ2,072. The measurements of Lang begin from λ2,040 and extend towards the region of shorter wavelengths, so that it is believed that a knowledge of the spectrum in the entire region that is required for the identification of this member is available. As a result of examination of the spectrum down even from λ2,400, it is found that there is no prominent pair with the appropriate separation 107·1. This at first caused great anxiety, but it was afterwards discovered, as will be presently seen, that the difficulty is due to the larger energy difference between the first doublet "*F*" levels than that between the first doublet "*D*" levels. The strong pair of lines shown in the following table was prominently observed from the outset. That this pair belongs to the trebly-ionised stage is unquestionable, and it has been classified as due to Sn IV by Kimura and Nakamura.(8) The magnitude of the separation 165·5 shows that it cannot possibly be either a "*P*" or a "*D*" separation. The region λ2,220 is exactly where the first pair of the super-fundamental (*FG*) series may be expected. The corresponding members in the spectra of In III and Ga III have been found to be fairly intense, as indicated in a previous Paper. It is therefore supposed that the above pair constitutes the combination $(F_{43}-G)$ doublet.



λ I.A.	Int.	υ	Δυ
2,229·18	(8)	44,845·6	165·5
2,220·98	(8)	4,501·1	

If so, the first member of the fundamental series must consist of two strong lines with a separation (165·5 − 107·1) equal to 58·4, accompanied by a faint satellite, the structure being as indicated in the above diagram.

A search for this pair at once showed the following doublet, which agrees well with the above view. But the satellite could not be traced, perhaps owing to its faintness and the region in which it must appear. As for the second fundamental series, there is a pair λλ3,779·6, 3,732·90 with the separation $\triangle\nu$=331·1, or perhaps the pair and satellite 3,779·63, 3,764·49, 3,732·90, which may be assigned to be the first member. But the second (*FG*) combination line or pair has not been found, and much reliance cannot be placed on this combination, as the second "*F*" term value calculated from this is much too small.

λ I.A.	Int.	υ (vac.)	Δυ
2,084·27	(8)	47,963·0	58·1
2,081·75	(8)	48,021·1	

The consideration of the location of other combination pairs—e.g., of the third series—from among the fainter lines of the enhanced spectrum is deferred for the present till the spectrum of Sn III has been examined in greater detail. This and the study of the series representing Sn II are still in progress.

The Diffuse Term Separations.—A peculiarity noticed in the "P" separations exhibited in this spectrum deserves mention. Carroll(9) has called attention to a rule that $\triangle\nu/c^2$ is constant for a given sequence. The following table gives the first and second "D" term separations in silver-like atoms and the values of $\triangle\nu/c^2$.

	$(1D_2-1D_3)$	$\Delta\upsilon/c^2$	$2D_2-2D_3$	$\Delta\upsilon/c^2$
Ag I	20·3	20·3	10·6	10·6
Cd II	154	38·5	73·5	18·4
In III	283	31·4	181·4	20·2
Sn IV	107	6·7	332·2	20·8

It is seen that the second "D" term separation agrees with the rule, but the first separation falls much below the value suggested by the rule, and, in fact, very peculiarly below even the second "D" separation. It is significant here to point out that the "D" separations are recorded in Al III but not in Sn IV.

Term Values.—The series presented form a connected system of members and the term values are evaluated, as usual, through the frequencies of the individual lines, by assuming the value of the "G" term to be equal to 70,400 cm.$^{-1}$($\nu/c^2=$ 4,400). Owing to the lack of the first diffuse doublet, the correct evaluation of the largest term $1S_1$ of the spectrum, and therefore of the exact value of the fourth ionisation potential of tin, is not yet possible. Table I gives the values of the terms calculated, with the effective quantum number against each, obtained from the usual formula

$$\nu=\frac{16R}{(n-q)^2}$$

where n is the total quantum number and $(n-q)$ the effective quantum number.

TABLE I.

Term Designation.	Term Value.*	Effective Quantum number.
G	[70,400]	
F_4	115,246	3·9032
F_3	115,411	3·9004
$1D_3$	163,267	3·2793
$1D_2$	163,374	3·2783
$2D_3$	93,553	4·3322
$2D_2$	93,885	4·3245
$2P_2$	128,648	3·6943
$2P_1$	130,826	3·6634
$2S_1$	154,540	3·3707
$3S_1$	91,063	4·4419

* The terms are given correct to the nearest unit.

The term $2P_2=128,648$, determined thus, agrees well with the value 130,191 obtained as a limit from the sharp series.

Comparison of Terms.—In Table II a comparison is made of the series terms of the four atoms of similar electron structure. The terms, as usual, are divided by 1^2, 2^2, 3^2, 4^2.

TABLE II.

	$2S_1$	$3S_1$	$2P_2$	$1D_3$	$2D_3$	F_4
Ag I	18,540	9,209	12,596	12,331	6,881	6,892
Cd II	13,347	[7,300]	10,248	11,633	6,532	6,986
In III	11,228	6,500	9,111	10,821	6,357	7,128
Sn IV	9,659	5,692	8,041	10,204	5,847	7,203

Finally, for the sake of convenience, the series lines indicated here are collected in Table III.

TABLE III.

Classification.	λ I.A.	Int.	υ (vac.).	Δυ.
$2S_1-2P_2$	3,861·26	(10)	25,891·4	2,177·4
$2S_1-2P_1$	4,215·74	(7)	23,714·0	
$2P_2-3S_1$	2,659·85	(4)	37,585·0	2,178 1
$2P_1-3S_1$	2,514·14	(4)	39,763·0	
$1D_2-2P_1$	3,071·51	(8)	32,547·9	2,177·6
$1D_3-2P_2$	2,887·79	(10)	34,618·4	107·1
$1D_2-2P_2$	2,878·89	(5)	34,725·5	
$2P_2-2D_2$	2,875·73	(5)	34,763·6	332·2
$2P_2-2D_3$	2,848·51	(10)	35,095·8	2,177·2
$2P_1-2D_2$	2,706·23	(9)	36,940·8	
$1D_3-F_3$	...		[47,855·9]	107·1
$1D_2-F_3$	2,084·27	(8)	47,963·0	165·2
$1D_3-F_4$	2,081·75	(8)	48,021·1	
F_4-G	2,229·18	(9)	44,845·6	165·5
F_3-G	2,220·98	(9)	45,011·1	

In conclusion, I gladly take this opportunity of acknowledging my great indebtedness to Prof. A. L. Narayan, M.A., D.Sc., F.Inst.P., who has suggested this investigation, and given me valuable guidance throughout its course.

EXPLANATION OF PLATE.

Figs. 1, 2, 3 are portions of spectra taken of the condensed spark in hydrogen at low pressures between electrodes of pure tin.

Fig. 1.—Quartz spectrum. (*a*), (*b*), (*c*) are taken with increasing inductance.

Fig. 2.—Quartz spectrum in the ultra-violet below $\lambda 2400$, taken on a Schumann plate.

Fig. 3 (A).—First order concave grating spectrum.

Fig. 3 (B).—Second order concave grating spectrum, showing the line $\lambda 2706{\cdot}23$ distinctly resolved from the arc line $\lambda 2706{\cdot}5$.

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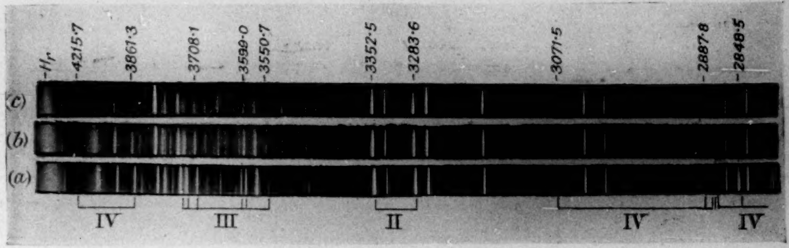

Hr
4215·7
3861·3
3708·1
3599·0
3550·7
3352·5
3283·6
3071·5
2887·8
2848·5
(c)
(b)
(a)
IV
III
II
IV
IV

FIG. 1.—QUARTZ SPECTROGRAM.

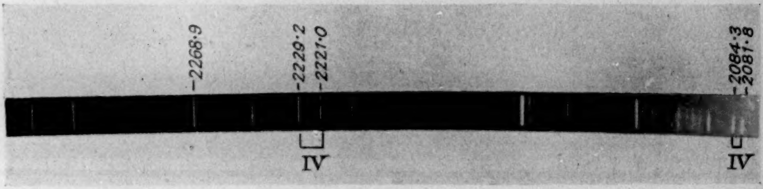
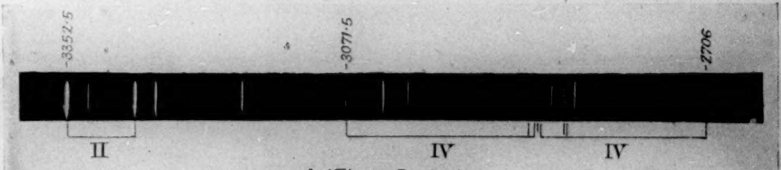
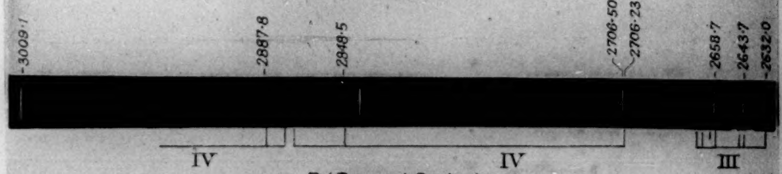

2268·9
2229·2
2221·0
2084·3
2081·8
IV
IV

FIG. 2.—QUARTZ SPECTROGRAM.


3352·5
3071·5
2706
II
IV
IV

A (First Order)


3009·1
2887·8
2848·5
2706·50
2706·23
2658·7
2643·7
2632·0
IV
IV
III

B (Second Order)

FIG. 3.—CONCAVE GRATING SPECTROGRAMS.

XXXIII.—THE LATENT HEAT OF EVAPORATION OF SULPHUR.

By J. H. AWBERY, *B.A.*, *B.Sc.*, Physics Department, The National Physical Laboratory.

Received March 31, 1927.

ABSTRACT.

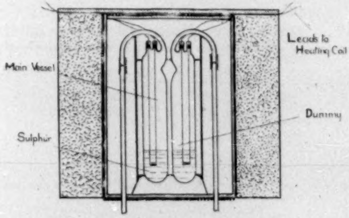
The latent heat of evaporation of sulphur was determined by finding the loss of weight of a vessel full of sulphur when energy was dissipated in it at a known rate. Heat losses were prevented by immersing the vessel in the vapour of sulphur; a method of correction for the heat loss by having a dummy vessel with a smaller heating coil, and using the differences in energy and in weight evaporated, for the two vessels, was tried but gave unsatisfactory results.

The value found for the latent heat was 79, the accuracy being estimated at 2 per cent.

THE principle of the method to be described is that of measuring the quantity of sulphur boiled off from a vessel, when a known amount of heat energy is supplied. If the loss of heat from the vessel can either be eliminated or estimated, the latent heat is given directly from the two quantities.

PRELIMINARY SERIES.

In some preliminary experiments efforts were made to eliminate the heat loss by using two similar vessels containing sulphur, and placing them symmetrically in an electric resistance furnace. A radiation shield was interposed between the vessels and the walls, as shown in Fig. 1, and the temperature in the sulphur was maintained as near to the boiling point as was possible, by adjustment of the furnace current. Thus the heat loss should be the same for each, and would be eliminated by taking differences.



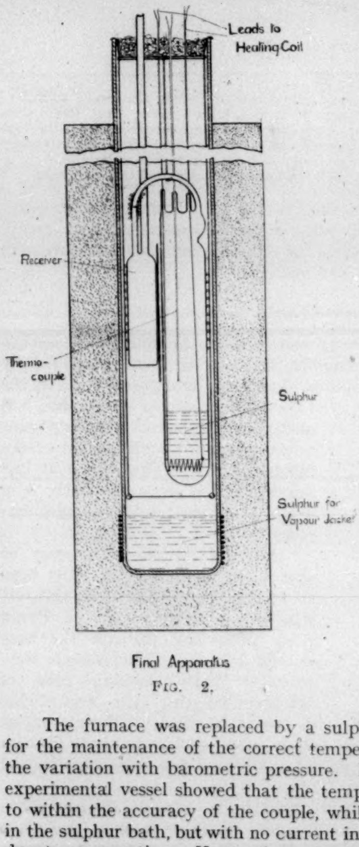
Apparatus for Preliminary Experiments.

FIG. 1.

Each of the vessels, as may be seen in the figure, had an outlet bent over to face downwards. To this was attached a straight tube of Pyrex glass, projecting through the base of the furnace. The vessels were provided with heating coils of different lengths, the two being connected in series and each provided with potential leads; thermocouples were also inserted into the vessels through narrow tubes, similar to those through which the leads for the heating current passed. All these fine tubes were packed with asbestos wool.

Two cases then arise. If the furnace temperature is initially slightly above the boiling point of sulphur, both vessels will slowly distil off sulphur, and when the current in the heating coils is switched on they will both continue to receive energy from the furnace walls at the same rate as before (since both are at the boiling point). The energy dissipated in the heating coils will be employed entirely in vapourising sulphur, so that the difference in the amounts volatilized should correspond to the difference in the energies dissipated in the respective coils.

The other case occurs if the furnace is initially a little below the boiling point. In this case the energy supplied has first to raise the tubes and their contents to this temperature, after which boiling commences, but a part of the energy is radiated all the time to the furnace walls. It is clear that, whatever the initial temperature, the method demands that the tubes should be similar and symmetrically placed, so that the one with the smaller heating coil either gains or loses heat to the same extent as the other. If this condition is fulfilled, the difference in energy should again correspond to the difference in weights evaporated.



Final Apparatus

FIG. 2.

The energy was measured by readings of current and potential drop, and the weight of sulphur determined by weighing the container before and after.

The results of a number of experiments, in which the conditions were widely varied, indicate the difficulty of attaining the conditions in which the heat losses are sufficiently equal to be eliminated in the differences. They gave, as the latent heat at the normal boiling point, the following figures (in calories per gram): 80·3, 70·5, 90·0, 63·7, 69·1, 66·0, 70·0; mean, 72·8.

SECOND SERIES.

The method was therefore considerably modified, and a new series of experiments undertaken in the apparatus of Fig. 2.

The furnace was replaced by a sulphur bath, which automatically provided for the maintenance of the correct temperature, eliminating even the difficulty of the variation with barometric pressure. Experiments with a thermocouple in the experimental vessel showed that the temperature of the boiling point was reached to within the accuracy of the couple, whilst prolonged experiments with the tubes in the sulphur bath, but with no current in the heating coil, showed no loss of weight due to evaporation. Hence the pair of vessels were no longer necessary, since there was no gain or loss of heat other than that due to the heating current. The series of experiments was therefore carried out with only one vessel, surrounded by a radiation shield and inserted in the sulphur vapour, as in Fig. 2.

Since the outlet would now be immersed in the vapour of sulphur, precautions had to be taken to prevent its ingress after the heating current in the coil was switched off. The means of avoiding this is indicated in Fig. 2. A bulb with two vertical outlet tubes was attached to the main vessel, one tube projecting out of the bath to serve as an air condenser. Thus the sulphur evaporated during an experiment was collected in this bulb, and a check on its weight was obtained by weighing both the main vessel and the collector before and after an experiment. The joint between the two was made by winding asbestos paper tightly over the two tubes, whilst fitted end to end, and binding it with asbestos string. It was found that this makes an excellent joint, if the asbestos paper is moistened immediately before use. A summary of the experiments is given below :—

Observation period. (Minutes.)	Heating current. (Amps.)	Latent heat at the boiling point (444·5°C.) (Cals. per gram.)
6	2·1	78·5
7	2·2	82·0
4	2·0	77·5
8	2·1	81·4
7	1·4	76·0
8	1·1	78·8
		——Mean 79·0

Taking into account the possible systematic errors, the accuracy of this result is likely to be of the order of 2 per cent.

COMPARISON WITH PREVIOUS RESULTS.

Only two determinations appear to have been made previously. One is due to Person, who worked under reduced pressure, and stated that the temperature at which he worked was 316°C. The result, obtained calorimetrically, was 362 calories per gram. Another estimate was made in 1914 by Beckmann and Liesche, from the rise of boiling point of solvents when sulphur was added, as 64·8, but this involves a knowledge of the molecular weight. These authors accept estimates from chemical data of the relative quantities of S_6 and of S_8 respectively, but appear to assume that the proportions of the two kinds of molecules is the same in the vapour and liquid phases.

ACKNOWLEDGMENTS.

I have to thank Dr. G. W. C. Kaye, Superintendent of the Physics Department, for the ready facilities he has given for carrying out this work. I am also indebted to Mr. A. R. Challoner for his assistance in taking many of the observations, and for constructing the apparatus.

DISCUSSION.

Capt. C. W. HUME : The author suggests the possible occurrence of small systematic errors. It is noticeable that the value of the specific heat rises quite definitely with the quantity of sulphur evaporated, if this be taken as proportional to the product of the " observation period " and the square of the current ; though there is a marked exception in the case where the current, or rate of evaporation, is exceptionally small. Perhaps these considerations may suggest some source of systematic error, and enable the most reliable value of the latent heat to be selected.

Dr. E. H. **Rayner** pointed out that a certain quantity of heat would be expended in filling the vessel with vapour which would not pass out.

Dr. **Ezer Griffiths**, in reply to the discussion, said that the chief systematic error would arise from the carrying over of liquid particles by the vapour. The kink in the walls of the vessel, just under the outlet, was intended to prevent this, but a large number of experiments would have to be made for a conclusive trial of its efficiency. It was presumed that the vessel was initially full of sulphur vapour, as it had been immersed in boiling sulphur for some time before the experiment began.

XXXIV.—THE REFRACTION AND DISPERSION OF GASEOUS CARBON TETRACHLORIDE.

By H. Lowery, *M.Sc.*, *F.Inst.P.*, Head of the Physics Department, Technical College, Huddersfield.

Received April 27, 1927.

Abstract.

The refractive index of gaseous carbon tetrachloride has been found for the green mercury line (λ5461), the result being expressed in connection with the density of the gas, that is, so as to show the refractivity of the gas by the same number of molecules as 1 c.c. of hydrogen contains at N.T.P.

Adopting the value 1·001799 for the refractive index under these conditions, the dispersion of the gas has been studied over the range λ4800 to λ6700, and is represented by the expression

$$\mu - 1 = \frac{13{\cdot}543 \times 10^{27}}{7831{\cdot}7 \times 10^{27} - \nu^2}$$

ν being the frequency of the light.

I. Introduction.

In continuation of an investigation on the refraction and dispersion of gaseous compounds, the refraction and dispersion of gaseous carbon tetrachloride have been studied.

Mascart* found the refractivity of gaseous carbon tetrachloride, relative to that of air, to be 6·05 for sodium light. No measurements of the dispersion appear to have been made.

II. Experimental.

The optical arrangements were the same as those in the work on gaseous carbon disulphide,† a Jamin interferometer being employed in connection with a Hilger constant deviation spectroscope as monochromatic illuminator.

To determine the refractive index of the gas, preliminary experiments were carried out, using a method employed by C. and M. Cuthbertson,‡ in which the substance under investigation is introduced (in the liquid state) into the refraction tube by means of a small capillary tube. Full details of the method are given in the Paper referred to.

In the case of gaseous carbon tetrachloride, however, it was found more convenient to work with apparatus similar to that employed in the experiments on carbon disulphide. The vapour pressure of carbon tetrachloride at 30°C. is 14 cms., and this pressure is sufficient to allow of the order of 250 interference fringes being counted under the conditions of the experiments.

A reservoir of pure liquid carbon tetrachloride was put into communication with the refraction tube by a connecting tube lagged with asbestos fibre, and surrounded with a coil of wire which could be heated by an electric current. Both refraction tubes were mounted between the interferometer plates in a tank and were kept throughout an experiment at a steady temperature of about 45°C. by warm water. The interferometer plates were screened from the heat of the tank of water by sheets of asbestos.

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* Comptes Rendus, LXXXVI, p. 1182 (1878).

† Proc. Phys. Soc., Vol. 38, p. 470 (1926).

‡ Phil. Trans., A, Vol. 213, p. 15 (1913).

Before an experiment was begun, the carbon tetrachloride was solidified in the reservoir by means of solid carbon dioxide, and the refraction tubes thoroughly exhausted. It was then gradually warmed up to about 30°C., the number of interference bands displaced in the telescope being counted as the gas entered the refraction tube. Since the density bulb, refraction tube and connecting tube (surrounded by the heating coil) were at a temperature considerably higher than that of the reservoir, no condensation could occur in them. The amount of vapour which passed into the refraction tube was determined by means of the density bulb, and the refractive index was calculated from the expression

$$\mu-1=\frac{n\lambda}{l}\left(\frac{\text{standard density}}{\text{density observed in the experiment}}\right)$$

where n is the number of interference bands counted, l the length of the refraction tubes, and λ the wavelength of the light employed.

Pure liquid carbon tetrachloride was supplied by Messrs. British Drug Houses, Ltd.

III. RESULTS.

(*a*) *Refractive Index of Gaseous Carbon Tetrachloride.*

Ten experiments were performed in order to determine the refractive index of gaseous carbon tetrachloride for the green mercury line (λ5461), with the following results :—

$(\mu-1)\times10^6$—1795, 1798, 1805, 1802, 1794,
1799, 1804, 1798, 1795, 1800.
Mean—1799 (λ5461).

These values were worked out from the expression above, so that they show the refractivity of gaseous carbon tetrachloride by the same number of molecules as 1 c.c. of hydrogen contains at N.T.P.

From the data C=12·000, H=1·008, Cl=35·457, and density of hydrogen= 0·08985 grams per litre at N.T.P., the standard density of carbon tetrachloride vapour is 6·856 grams per litre.

In the present work the *mean* density of carbon tetrachloride vapour is found to be 6·814 grams per litre, the reduction being effected by the expression

$$d=d_0\,.\,\frac{(t+273)}{273}\,.\,\frac{760}{p}$$

where d is the reduced density, d_0 the observed density at pressure p mms. of mercury and temperature t° C. The experimental density of carbon tetrachloride vapour is given in Watt's Dictionary of Chemistry as lying between 5·24 and 5·33 (air=1), that is, between 6·76 and 6·89 grams per litre.

(*b*) *Dispersion of Gaseous Carbon Tetrachloride.*

The dispersion was measured in six experiments, with the results shown in the accompanying table.

The second column contains the results actually obtained in the experiments. From these the constants in the Sellmeier dispersion formula (single term) have been obtained by the method of least squares, the resulting expression being

$$\mu - 1 = 13{\cdot}543 \times 10^{27} / (7831{\cdot}7 \times 10^{27} - \nu^2).$$

The values in the third column have been calculated from this expression.

Dispersion of Gaseous Carbon Tetrachloride.

λ in Å.U.	$(\mu - 1) \times 10^6$.		Difference.
	Observed.	Calculated.	
Li 6708	1774, 0	1774, 6	+6
Cd 6438	1778, 4	1778, 6	+2
Li 6104	1784, 6	1784, 3	−3
Hg 5770	1791, 8	1791, 1	−7
Hg 5461	1799, 0	1798, 6	−4
Cd 5086	1810, 1	1809, 6	−5
Cd 4800	1819, 6	1820, 0	+4

(*c*) *Comparison between Mascart's Work and the Present.*

For λ5893, the above expression gives $\mu = 1{\cdot}0017884$ as the refractive index expressed in terms of the density of the gas. To reduce this value to the corresponding value under N.T.P. conditions, it must be altered in the ratio

$$\text{(Experimental density at N.T.P.)}/\text{(Standard density)} = 6{\cdot}814/6{\cdot}856.$$

The reduced value so obtained is 1·001777 (for λ5893 and N.T.P. conditions). Mascart's value is 1·001768 if the refractive index of air be taken as 1·0002923 under the same conditions.

The author's thanks are due to Professor W. L. Bragg, F.R.S., and Mr. C. Cuthbertson, F.R.S., for the loan of apparatus and much helpful advice.

XXXV.—REGULARITIES IN THE SPECTRUM OF IONISED NEON.

By P. K. Kichlu, *M.Sc.*, Research Scholar, University of Allahabad.

Received May 17, 1927.

Abstract.

A number of lines lying between λ7282 and λ3142 occurs in the condensed discharge spectrum of the more volatile gases of the air. These have been attributed to singly ionised neon, and the present Paper comprises an analysis which confirms this view.

LIVEING* and Dewar in 1900 observed a large number of lines between λ7282 to λ3142 by passing a condensed electric discharge through tubes filled with the more volatile gases of air. They were unable to trace the source from which they were derived. Fourteen years later Merton† obtained most of these lines in tubes filled with pure neon. None of these, however, were included in Paschen's‡ analysis of the arc spectrum of neon. More recently L. and E. Bloch§ and Dejardin have obtained these lines by the method of electron-bombardment, and found that they come out strongly at 49 volts. On this basis, they ascribe these lines to singly ionised neon. The present analysis confirms the indentification of L. and E. Bloch and Dejardin beyond doubt and is in strict accord with Hund's theory of complicated spectra.

We may begin by writing down the terms which may be expected according to Hund's theory :—

Structure Diagram of $\mathrm{Ne.}^{+}$

$$\begin{tikzcd}
\mathrm{K} & & & \\
2 & \mathrm{L}_1 & \mathrm{L}_2 & \\
 & 2 & 5 & \\
 & & \mathrm{M}_1 \arrow[r] & \mathrm{M}_2 \arrow[r] \arrow[d] & \mathrm{M}_3 \arrow[d] \\
 & & & \mathrm{N}_1 \arrow[r] & \mathrm{N}_2
\end{tikzcd}$$

Table of Theoretical Terms.

Position of valence electrons.		Terms calculated.	Terms observed.
(*a*) ↓	$5L_2$	^2P	
(*b*) ↓	$4L_2M_1$	^4P ^2D ^2S $^2\bar{P}$	$^4\bar{P}$
(*c*) ↓	$4L_2M_2$	$^4\bar{D}$ ^4P $^4\bar{S}$ $^2\bar{D}$ ^2P $^2\bar{S}$	$^4\bar{D}$ ^4P $^4\bar{S}$
(*d*)	$4L_2M_3$	$^4\bar{F}$ ^4D $^4\bar{P}$ $^2\bar{F}$ ^2D $^2\bar{P}$	$^4\bar{F}$ ^4D $^4\bar{P}$
(*e*)	$4L_2N_1$	$^4\bar{P}$ ^2D ^2S $^2\bar{P}$	

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* Liveing and Dewar. Proc. Roy. Soc., A, Vol. 67, p. 467 (1900).

† T. R. Merton. Proc. Roy. Soc., A, Vol. 89, p. 447 (1914).

‡ F. Paschen. Ann. d. Phys., Vol. 60, p. 405 (1919).

§ L. and E. Bloch and Dejardin. Jour. de Phys., Vol. 7.

We shall prefix successively the small letters *a, b, c.* . . . to the various levels arising from different electronic configurations to distinguish those of the same type, e.g., all terms due to

$4L_2M_1$ will have the prefix " b,"
$4L_2M_2$ will have the prefix " c," etc.

The clue to the classification of the lines of ionised neon was furnished by the $c^4P-d^4\overline{P}$ multiplet forming a distinct and isolated group of lines between λλ 2799 and 2756. The deepest terms so far found correspond to $b^4\overline{P}$ obtained from the combination $4L_2M_1$. These account for some of the strongest lines present in the spectrum, and occurring as reversals in L. and E. Bloch and Dejardin's list. Obviously the lines arising from a^2P, if they are present at all, would be situated in the far ultraviolet and lie outside the region investigated.

The Multiplets.

	$b\,^4\overline{P}_1$	298·8	$b\,^4\overline{P}_2$	518·0	$b\,^4\overline{P}_3$
$c\,^4P_1$ 182·5	26650·2 (4)		26949·0 (7*R*)		
$c\,^4P_2$ 222·7	26467·6 (8*R*)		26766·6 (7)		27284·5 (9*R*)
$c\,^4P_3$			26543·4 (8*R*)		27061·8 (10*R*)
$c\,^4\overline{D}_1$ 144·2	29891·8 (5)		30190·8 (4)		
$c\,^4\overline{D}_2$ 249·7	29747·8 (5)		30046·5 (5)		30564·9 (2)
$c\,^4\overline{D}_3$ 337·3			29796·9 (7)		30315·1 (8)
$c\,^4\overline{D}_4$					29977·4 (10*R*)
$c\,^4\overline{S}_2$	33005·7 (4)		33304·5 (7)		33822·7 (7)

The lines

(1) 3713·07
26924·2 (10*R*)

(2) 3694·19
27061·8 (10*R*)

(3) 3664·05
27284·5 (9*R*)

have been listed by Paschen as arc lines. They are without doubt spark lines, and represent some of the fundamental lines of the ionized element. Their intensity is considerably enhanced in the condensed discharge.

The multiplets obtained between the terms of the combinations $4L_2M_2$ and $4L_2M_3$ are shown below :—

	$c\ {}^4P_1$	182·5	$c\ {}^4P_2$	222·7	$c{}^4P_3$	$c{}^4\bar{S}_2$
$d\ {}^4\bar{P}_1$ 304·8	36082·3 (1)		36264·8 (3)			29727·7 (1)
$d\ {}^4\bar{P}_2$ 377·0	35777·6 (3)		35960·0 (2)		36182·3 (2)	29422·0 (1)
$d\ {}^4\bar{P}_3$			35582·9 (4)		35805·5 (4)	29044·6 (1)
$d\ {}^4D_1$ 98·0	32825·2 (3)		* ?			
$d\ {}^4D_2$ 106·0	32726·9 (*5dv*)		32909·5 (3)		33131·9 (3)	26370·5 (0)
$d\ {}^4D_3$ 80·8			32803·6 (6)		33025·7 (4)	26264·7 (1)
$d\ {}^4D_4$					32944·9 (5)	
$d\ {}^4\bar{F}_2$ 145·2	34574·0 (0)		34755·4 (*3dr*)		34979·0 (1)	28217·5 (7)
$d{}^4\bar{F}_3$ 325·4			34611·2 (1)		34833·9 (1)	28072·3 (4)
$d\ {}^4\bar{F}_4$ 528·1					34508·0 (1)	
$d\ {}^4\bar{F}_5$						

* The line here is evidently masked by the strong line 33005·4 (4).

	$c\ {}^4\bar{D}_1$	144·2	$c\ {}^4\bar{D}_2$	249·7	$c\ {}^4\bar{D}_3$	337·3	$c\ {}^4\bar{D}_4$
$d\ {}^4\bar{P}_1$ 304·8	32840·3 (2)		32984·8 (2)				
$d\ {}^4\bar{P}_2$ 377·0	32535·5 (1)		32679·3 (3)		32929·1 (2)		
$d\ {}^4\bar{P}_3$			?		32553·3 (2)		32889·3 (4)

	$c\,{}^4\bar{D}_1$	144·2	$c\,{}^4\bar{D}_2$	249·7	$c\,{}^4\bar{D}_3$	337·3	$c\,{}^4\bar{D}_4$
$d\,{}^4D_1$ 98·0	29583·0 (0)		29727·7 (1)				
$d\,{}^4D_2$ 106·0	29485·2 (0)		29629·2 (2)		29878·9 (1)		
$d\,{}^4D_3$ 80·8			29523·2 (1)		29772·1 (2)		30109·3 (1)
$d\,{}^4D_4$					[29691]*		30028·8 (4)
$d\,{}^4\bar{F}_2$ 145·2	31330·5 (2)		31475·7 (3)		31725·6 (2)		
$d\,{}^4\bar{F}_3$ 325·4			31330·5 (2)		31579·7 (2)		31917·2 (2)
$d\,{}^4\bar{F}_4$ 528·1					31254·8 (6)		31592·2 (3*dr*)
$d\,{}^4\bar{F}_5$							31064·1 (8)

In addition to the quartet terms given before we also expect a number of doublet terms of slightly smaller value than the corresponding quartet terms of any particular combination. Though the existence of a number of other terms has definitely been established it has not yet been found possible to identify their natures unambiguously. In any case, no intercombination lines connecting quartet systems with doublets have been discovered. Pending further analysis these other terms have been designated variously as α, β, γ, ... in the list of classified Ne^+ lines which follows :—

Table of Classified Ne+ *Lines.*

λ	I	ν (Vac.)	Combination	λ	I	ν (Vac.)	Combination
2756·68	3	36264·8	$c^4P_2-d^4\bar{P}_1$	2876·41	3*dr*	34755·4	$c^4P_2-d^4\bar{F}_2$
2762·97	3	36182·3	$c^4P_3-d^4\bar{P}_2$	2888·39	1	34611·2	$c^4P_2-d^4\bar{F}_3$
2770·63	1	36082·3	$c^4P_1-d^4\bar{P}_1$	2891·50	0	34574·0	$c^4P_1-d^4\bar{F}_2$
2780·05	2	35960·0	$c^4P_2-d^4\bar{P}_2$	2897·03	1	34508·0	$c^4P_3-d^4\bar{F}_4$
2792·04	4	35805·5	$c^4P_3-d^4\bar{P}_3$	2910·11	2	34352·9	$c^4P_2-\gamma$
2794·22	3	35777·6	$c^4P_1-d^4\bar{P}_2$	2916·22	1	34280·9	$c^4P_3-\alpha$
2809·51	4	35582·9	$c^4P_2-d^4\bar{P}_3$	2925·66	2	34170·3	$c^4P_1-\gamma$
2858·02	1	34979·0	$c^4P_3-d^4\bar{F}_2$	2933·71	1	34076·6	$c^4P_3-\beta$
2869·93	1	34833·9	$c^4P_3-d^4\bar{F}_4$	2935·29	0	34058·2	$c^4P_2-\alpha$

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* There is a line 29689·3(6) which falls very near the line required here, but its intensity is so strong that it is probable that it comes from some other source and masks the line $c^4\bar{D}_3-d\,{}^4D_4$. The line 31330·5 occurs twice as $c\,{}^4\bar{D}_1-d\,{}^4\bar{F}_2$ and $c\,{}^4\bar{D}_2-d\,{}^4\bar{F}_3$, because $c\,{}^4\bar{D}_1-c\,{}^4\bar{D}_2=d\,{}^4\bar{F}_2-d\,{}^4\bar{F}_3=145$.

Table of Classified Ne+ *Lines*—(continued).

λ	I	ν (Vac.)	Combination	λ	I	ν (Vac.)	Combination
2951·15	1	33875·2	$c^4P_1-\alpha$	3334·89	10*R*	29977·4	$b^4\bar{P}_3-c^4\bar{D}_4$
2953·10	0	33852·8	$c^4P_2-\beta$	3344·44	5	29891·8	$b^4\bar{P}_1-c^4\bar{D}_1$
2955·77	7	33822·3	$b^4\bar{P}_3-c^4\bar{S}_2$	3345·88	1	29878·9	$c^4\bar{D}_3-d^4D_2$
3001·72	7	33304·5	$b^4\bar{P}_2-c^4\bar{S}_2$	3355·09	7	29796·9	$b^4\bar{P}_2-c^4\bar{D}_3$
3017·36	3	33131·9	$c^4P_3-d^4D_2$	3357·89	2	29772·1	$c^4\bar{D}_3-d^4D_3$
3027·07	4	33025·7	$c^4P_3-d^4D_3$	3360·63	5	29747·8	$b^4P_1-c^4D_2$
3028·90	4	33005·7	$b^4\bar{P}_1-c^4\bar{S}_2$	3362·90	1	29727·7	$c^4\bar{D}_2-d_4D_1$
3030·82	2	32984·8	$c^4\bar{D}_2-d^4\bar{P}_1$	3367·25	6	29689·3	$c^4\bar{D}_3-d^4D_4$
3034·49	5	32944·9	$c^4P_3-d^4D_4$	3374·08	2	29629·2	$c^4\bar{D}_2-d^4D_2$
3035·95	2	32929·1	$c^4\bar{D}_3-d^4\bar{P}_2$	3379·34	0	29583·0	$c^4D_1-d^4D_1$
3037·75	3	32909·5	$c^4P_2-d^4D_2$	3386·20	1	29523·2	$c^4\bar{D}_1-d^4D_3$
3039·62	4	32889·3	$c^4\bar{D}_4-d^4\bar{P}_3$	3390·56	1	29485·2	$c^4\bar{D}_1-d^4D_2$
3044·10	2	32840·9	$c^4\bar{D}_1-d^4\bar{P}_1$	3397·84	1	29422·0	$c^4\bar{S}_2-d^4\bar{P}_2$
3045·56	3	32825·2	$c^4P_1-d^4D_1$	3441·99	1	29044·6	$c^4\bar{S}_2-d^4\bar{P}_3$
3047·60	6	32803·2	$c^4P_2-d^4D_3$	3542·89	7	28217·5	$c^4\bar{S}_2-d^4F_2$
3054·70	5*dv*	32726·9	$c^4P_1-d^4D_2$	3551·52	1	28149·0	$c^4P_2-\delta$
3059·15	3	32679·3	$c^4\bar{D}_2-d^4\bar{P}_2$	3561·21	4	28072·3	$c^4\bar{S}_2-d^4\bar{F}_3$
3070·99	2	32553·3	$c^4\bar{D}_3-d^4\bar{P}_3$	3574·65	5	27966·8	$c^4P_1-\delta$
3072·68	1	32535·5	$c^4\bar{D}_1-d^4\bar{P}_2$	3594·15	2	27815·1	$c^4\bar{S}_2-\gamma$
3132·20	2	31917·2	$c^4\bar{D}_4-d^4\bar{F}_3$	3632·74	1	27519·6	$c^4\bar{S}_2-\alpha$
3151·12	2	31725·6	$c^4\bar{D}_1-d^4\bar{F}_2$	3659·91	2	27315·3	$c^4\bar{S}_2-\beta$
3164·42	3*dv*	31592·2	$c^4\bar{D}_4-d^4\bar{F}_2$	3664·05	9*R*	27284·5	$b^4\bar{P}_3-c^4P_2$
3165·68	2	31579·7	$c^4\bar{D}_3-d^4\bar{F}_3$	3694·19	10*R*	27061·8	$b^4\bar{P}_3-c^4P_3$
3176·14	3	31475·7	$c^4\bar{D}_2-d^4\bar{F}_2$	3709·66	7*R*	26949·0	$b^4\bar{P}_2-c^4P_1$
3190·86	2	31330·5	$c^4\bar{D}_1-d^4\bar{F}_2$	3734·94	7	26766·6	$b^4\bar{P}_2-c^4P_2$
3198·58	6	31254·8	$c^4D_3-d^4\bar{F}_4$	3751·25	4	26650·2	$b^4\bar{P}_1-c^4P_1$
3218·22	8	31064·1	$c^4\bar{D}_4-d^4\bar{F}_5$	3766·28	8*R*	26543·9	$b^4\bar{P}_2-c^4P_3$
3232·38	3	30928·0	$c^4\bar{D}_1-\gamma$	3777·14	8*R*	26467·6	$b^4\bar{P}_1-c^4P_2$
3243·42	2	30822·8	$c^4\bar{D}_3-\beta$	3791·05	0	26370·5	$c^4\bar{S}_2-d^4D_2$
3248·16	3*dr*	30777·8	$c^4\bar{D}_2-\alpha$	3806·32	1	26264·7	$c^4\bar{S}_2-d^4D_3$
3263·40	2	30634·1	$c^4D_1-\alpha$	4224·57	1	23664·4	$\nu-c^4P_1$
3269·84	3	30573·7	$c^4D_2-\beta$	4257·25	1	23482·8	$\nu-c^4P_2$
3270·79	2	30564·9	$b^4\bar{P}_3-c^4\bar{D}_2$	4298·00	0	23260·1	$\nu-c^4P_3$
3297·74	8	30315·1	$b^4\bar{P}_3-c^4\bar{D}_3$	4377·95	2	22835·3	$\mu-c^4P_1$
3311·32	4	30190·8	$b^4P_2-c^4D_1$	4413·16	4	22653·1	$\mu-c^4P_2$
3320·28	1	30109·3	$c^4\bar{D}_4-d^4D_3$	4457·04	4	22430·1	$\mu-c^3P_3$
3327·22	5	30046·5	$b^4P_2-c^4\bar{D}_2$	4569·02	5	21880·4	$\theta-c^4P_2$
3329·18	4	30028·8	$c^4D_4-d^4D_4$	4616·04	3	21657·5	$\theta-c^4P_3$

Further investigation is proceeding.

The author wishes to express his best thanks to Prof. M. N. Saha for his interest and guidance during the progress of the work outlined in this Paper.

Note added during correction :—After this Paper has been through the press, doublet terms have been identified. They will form the subject of a further communication.

XXXVI.—NEWTON'S LAW FOR THE EMISSION OF HEAT IN CARBON DIOXIDE.

By SYBIL MARSHALL, *B.Sc., A.R.C.Sc., D.I.C.*

Received May 10, 1927.

ABSTRACT.

This Paper is an extension of the investigation, started by the author and Mr. Vick, of the justification of employing Newton's Law of Cooling when estimating the external heat loss from the surface of a solid in a gaseous medium. In this case, the loss in a vertical system, containing in turn CO_2, oxygen and nitrogen, was examined, the apparatus consisting of compensated platinum wires mounted along the axes of ice-water cooled glass tubes and connected to the terminals of a Callendar-Griffiths bridge. The results show that, for tubes between 1 cm. and 3 cm. diameter, the law is applicable only as far as 10°C. for pressures between 1 atmosphere and 6 cm. mercury. The deviation above this temperature depends on the width of tube and the density of the gas, being greater for CO_2 than for oxygen or nitrogen.

INTRODUCTION.

IN May, 1925, the author and Mr. Vick presented a Paper to this Society on " The Control Conditions under which Newton's Law is Valid for the Emission of Heat from Electrically-Heated Wires."(1) The experimental results were shown in the form of curves obtained by plotting the total wattage loss, C^2R_θ, against the temperature, θ, of the wire, keeping the pressure constant for each curve. A horizontal system of tubes containing air was used.

For investigating the heat transmission through carbon dioxide, the author had built a new system of tubes, arranged similarly to those described in the above Paper, but vertically disposed, in order to minimise the very large convective loss one would expect in this heavy triatomic gas. Another advantage that the vertical system had over the horizontal was that the ice and water thermostat(2) designed for this apparatus proved more efficient than the type used for the horizontal tubes.

APPARATUS.

The apparatus consisted of two long and two short pieces of 4 mil platinum wire cut from the same specimen. Each of these was mounted along the axis of a lead glass tube by means of thicker platinum leading wires, one of each pair of leads being a spring to keep the fine wire taut. One long wire *A* and one short one *B* were enclosed in separate pieces of tubing of about 3 cms. diameter, and connected across the main and compensator terminals respectively of a Callendar-Griffiths bridge.(1), (3) The other long and short wires were mounted along 1·2 cm. bore tubing and connected similarly to another bridge, the wide and narrow systems being electrically independent, but in gaseous connection with each other. Both were connected to a pump and a CO_2 generating apparatus(2) and immersed in an ice and water thermostat.

Fig. 1 shows the vertical type of apparatus already described and used in the following investigation. The external leading wires are omitted from the diagram to avoid confusion. The electrical connections of one of the two systems are shown in Fig. 2. *A* and *B* represent the main and compensator wires, respectively, inserted in the arms of the Callendar-Griffiths bridge, each in series with a standard ohm, *S.R.* *P* is a potentiometer circuit connected across the terminals of the standard ohm in series with the main wire, and *E* is a series of rheostats in the main battery circuit, by means of which the current supplied by this circuit could be varied.

EXPERIMENT.

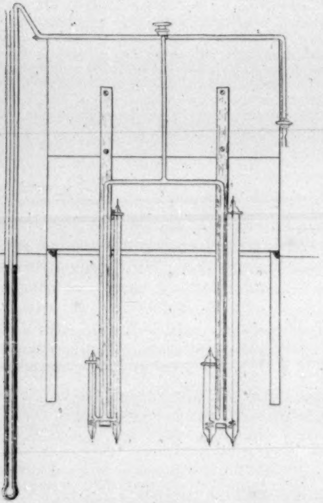

FIG. 1.—TUBES.

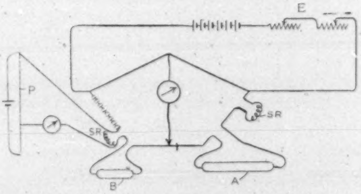

FIG. 2.—DIAGRAM OF CONNECTIONS.

The temperature range of the experiment extended between 0° pt. and 65° pt., the upper limit being the temperature corresponding to the maximum heating effect for which the bridge coils were designed. Starting with atmospheric pressure, and first using the wide system of tubes, the bridge was set at the highest temperature, the current in the battery circuit adjusted until a balance was obtained, and the current through the wires measured by means of the potentiometer across the terminals of the standard ohm in series with the main wire.(3) The wattage loss, $\frac{C^2R_\theta}{100}$, could then be calculated for this temperature θ; C being the current in amperes and R_θ the resistance indicated by the bridge in cms. bridge wire, where 100 cms. bridge wire is equal to 1 ohm. The temperature of the wire was then dropped in a series of steps of 5° pt. and the current measured for each, keeping the pressure constant. A curve could then be plotted showing the relation between the heat lost by the wire and its temperature. The procedure was repeated using various pressures below atmospheric, and a set of constant pressure curves was obtained for this system of tubes. A similar set of readings was carried out for the tubes with the smaller diameter. The following results were obtained :—

Pressure. Cms.	Hg.	I. 76·30		II. 62·60		III. 43·95		IV. 24·92		V. 6·15		VI. 0·20		VII. 0·0	
θ	R′	C	C^2R	C	C^2R_θ	C	C^2R_θ	C	C^2R_θ	C	C^2R_θ	C	C^2R_θ	C	C^2R
			WIDE	TUBES.											
69·3	321·6											0·2731	23·98		
64·2	316·6	0·3122	30·82	0·3066	29·77					0·2810	25·00	0·2638	22·02	0·1488	7·00
59·1	311·6	0·2980	27·85	0·2938	26·90	0·2852	25·35	0·2740	24·40	0·2700	22·72	0·2540	20·10	0·1439	6·45
54·0	306·6	0·2851	24·90	0·2803	24·10	0·2724	22·75	0·2624	21·12	0·2584	20·47	0·2436	18·33	0·1386	5·88
48·9	301·6	0·2707	22·10	0·2656	21·28	0·2589	20·21	0·2498	18·80	0·2462	18·30	0·2326	16·32	0·1328	5·32
43·8	296·6	0·2550	19·43	0·2508	18·67	0·2447	17·78	0·2362	16·55	0·2334	16·17	0·2210	14·50	0·1264	4·74
38·6	291·6	0·2384	16·58	0·2350	16·10	0·2297	15·38	0·2223	14·41	0·2190	13·98	0·2085	12·68	0·1200	4·20
33·5	286·6	0·2217	14·10	0·2185	13·69	0·2138	13·09	0·2074	12·32	0·2054	12·09	0·1945	10·80	0·1127	3·67
28·4	281·6	0·2038	11·70	0·2008	11·37	0·1963	10·86	0·1911	10·28	0·1894	10·10	0·1802	9·15	0·1047	3·09
33·3	276·6	0·1852	8·92	0·1809	8·99	0·1768	8·64	0·1731	8·29	0·1717	8·15	0·1636	7·40	0·0953	2·51
18·2	271·6	0·1612	6·80	0·1593	6·89	0·1550	6·53	0·1530	6·36	0·1520	6·27	0·1451	5·72	0·0850	1·96
13·1	266·6	0·1358	4·92	0·1342	4·80	0·1310	4·58	0·1290	4·49	0·1291	4·46	0·1235	4·06	0·0720	1·38
8·0	261·6	0·1037	2·79			0·1020	2·72	0·1017	2·71	0·1010	2·67	0·0968	2·45	0·0569	0·85
			NARROW	TUBES.											
64·2	310·75	0·3136	30·60	0·3008	29·45							0·2817	24·63	0·1499	6·99
59·1	305·75	0·3003	27·63	0·2999	27·50	0·3038	28·20	0·2980	27·14			0·2714	22·51	0·1446	6·39
54·0	301·0	0·2858	24·60	0·2860	24·65	0·2880	24·98	0·2836	24·21	0·2810	23·77	0·2604	20·42	0·1392	5·83
48·9	296·0	0·2742	22·28	0·2705	21·68	0·2734	22·13	0·2694	21·50	0·2667	21·08	0·2487	18·30	0·1334	5·27
43·8	291·0	0·2580	19·39	0·2567	19·18	0·2560	19·10	0·2553	18·97	0·2539	19·05	0·2362	16·25	0·1274	4·72
38·6	286·0	0·2433	16·97	0·2420	16·75	0·2411	16·65	0·2406	16·57	0·2392	16·38	0·2234	14·28	0·1207	4·17
33·5	281·25	0·2282	14·64	0·2290	14·77	0·2260	14·38	0·2284	14·68	0·2238	14·10	0·2090	12·29	0·1136	3·63
28·4	276·25	0·2096	12·15	0·2089	12·07	0·2077	11·93	0·2072	11·86	0·2063	11·75	0·1935	10·35	0·1056	3·08
33·3	271·25	0·1903	9·83	0·1896	9·75	0·1887	9·65	0·1883	9·62	0·1874	9·52	0·1763	8·44	0·0964	2·52
18·2	266·5	0·1678	7·49	0·1679	7·51	0·1672	7·45	0·1668	7·42	0·1660	7·34	0·1564	6·52	0·0859	1·97
13·1	261·5	0·1424	5·30	0·1428	5·33	0·1420	5·28	0·1415	5·23	0·1408	5·18	0·1333	4·65	0·0735	1·41
8·0	256·5	0·1082	3·00			0·1107	3·15	0·1106	3·15	0·1101	3·11	0·1044	2·81	0·0576	0·86

DISCUSSION OF CURVES.

It will be seen from the curves in Fig. 3 that for CO_2, even in a vertical system, convection was present to a considerable degree at temperatures above 10° pt. for tubes of about 1·5 cm. radius. This is shown by the fact that the curves corresponding to various pressures do not coincide above this temperature, as would be the case if conductivity, which is independent of pressure, alone were operative. For tubes of the above diameter, all curves corresponding to pressures down to 6 cm. mercury merge into the same straight line between 0° pt. and 10° pt., showing that for this region alone is Newton's Law obeyed. Here convection is absent, all the heat being carried by conduction, if we neglect radiation which is less than a half of one per cent. of the total heat loss. The dotted continuation of this straight line represents the variation with temperature of the heat lost by conduction over the whole temperature range of the experiment, and the difference between the ordinates of the total heat loss curve and those of the straight line give the heat lost by convection at any given temperature.* At the higher temperatures the curves diverge increasingly from the dotted line, the greater the pressure, the greater being the resulting increase of divergence with temperature.

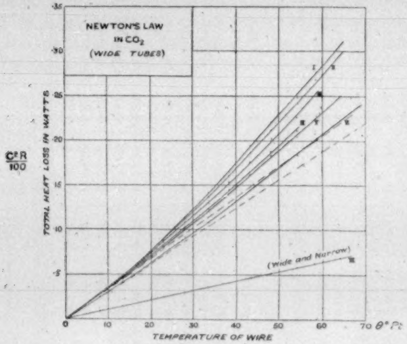

FIG. 3.

For the narrower tubes, whose radius was less than half that of the wide tubes, there was less convection. The curves are very much closer together, and they almost coincide with the same straight line up to a temperature of 20° pt. Above this

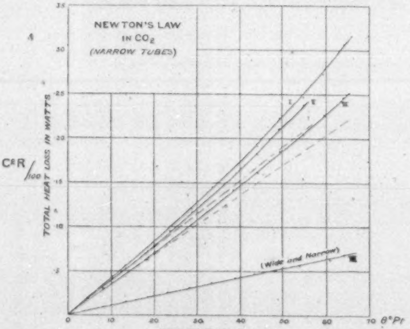

FIG. 4.

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* That the heat lost by conduction is represented by the tangent at the origin to the total heat loss curve, is indicated by the fact that the slope of this tangent is independent of the pressure over the same range as that over which the conductivity is independent of pressure, and that both diminish as the pressure is lowered when the latter is less than 6 cm. mercury.

temperature their divergence from one another increases very slightly, but there is a marked increase in their common divergence from their original direction. This tendency, which is also shown in the wide system, suggests that, even for a pressure of 6 cm. mercury, Newton's Law is valid only up to a temperature of 20° pt. for these tubes, on account of convection always being present at higher temperatures. Therefore, if one wishes to make use of the above law in estimating heat loss from surfaces in an undisturbed atmosphere of CO_2, these must be vertical, the distance from the hot surface to the cold not exceeding ½ cm., and the limiting temperature difference is 20° pt.

HEAT LOSS AT LOW PRESSURES.

When the pressure was reduced to 2 mm. mercury, the curve for each system tended to a straight line whose slope was less than that of the straight line to which the curves corresponding to the higher pressures approached,* and this straight line was different for each system. This change in the slope of the line, showing the variation of conductivity with temperature, is due to the fact that, at pressures where the mean free path of the molecules becomes comparable with the dimenssion of the tubes, conductivity no longer obeys Fourier's Law and ceases to be independent of pressure. At the above pressure the curve for the narrow tube is practically a straight line up to 50° pt., but for the wide the curvature becomes appreciable at 40° pt.

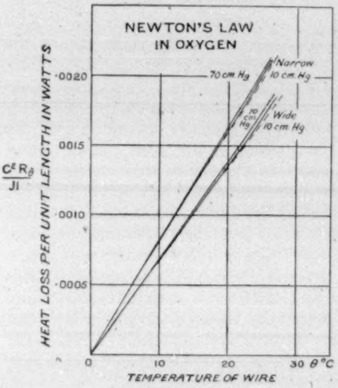

FIG. 5

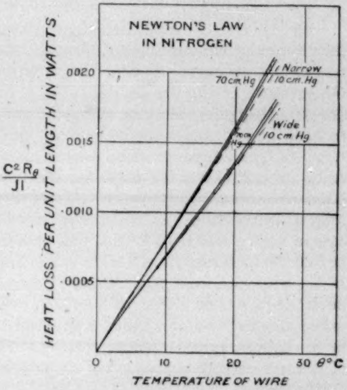

FIG. 6.

The only instance in which Newton's Law was obeyed over the whole temperature range and for both widths of tube was when the tubes were evacuated to the degree obtainable with a Gaede mercury pump. In this case the wattage/temperature curves for both tubes became one and the same straight line, whose slope was approximately one-third of the slopes of the two lines to which the two sets of curves tended for pressures above 6 cm. mercury, even though the curves for all pressures down to 2 mm. mercury tended to a different line for each set of tubes. Therefore, at this low pressure the heat transmission is independent of the distance between the hot and cold surfaces, showing that it is due to radiation and radiometer action⁽⁴⁾ alone. Newton's Law is never obeyed over an indefinite temperature range under the ordinary conditions necessary for conduction to take place according to Fourier's Law, where convection is bound to creep in as the temperature is raised and thus introduce a departure from the straight line law, but at low pressures where radiometer action occurs the law is obeyed and then only.

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* That the heat lost by conduction is represented by the tangent at the origin to the total heat loss curve, is indicated by the fact that the slope of this tangent is independent of the pressure over the same range as that over which the conductivity is independent of pressure, and that both diminish as the pressure is lowered when the latter is less than 6 cm. mercury.

NEWTON'S LAW FOR OTHER GASES IN A VERTICAL SYSTEM.

Dr. H. Gregory and the author having just completed their determinations of the thermal conductivities of Oxygen and Nitrogen, the latter made use of the results to draw curves showing the variation of the wattage loss per cm. of wire with the pressure of the gas in the tubes at constant temperature. From these were drawn wattage/temperature curves (Figs. 5 and 6) at constant pressure which, up to 10°C. and for pressures between 10 cm. and 70 cm. mercury, coincided with the same straight line whose slope depended on the diameter of the tubes, but was found to be approximately the same for the two gases on account of their thermal conductivities being nearly equal. Above this temperature a slight deviation from the original direction set in, which was more marked the higher the pressure. In the case of both gases at 25°C. this was between 3 and 4 per cent. of the total heat loss at that temperature for the wide tubes, and only 2 per cent. for the narrow. Below 10°C., therefore, one may disregard convective loss, which is represented by this deviation, for both widths of tube, and even up to 25°C., it is so small that one may safely assume Newton's Law for the heat transmission through Oxygen or Nitrogen in a vertical system, whose hot and cold surfaces are separated by a layer of gas not exceeding about half a centimetre at a pressure not appreciably above atmospheric.

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XXXVII.—SOME EXPERIMENTS WITH SOUND WAVES OF HIGH FREQUENCY.

By S. R. Humby, *M.C., M.A.*, The College, Winchester.

Received March 2, 1927.

(*Communicated by* Prof. E. V. Appleton.)

Abstract.

The interference and diffraction effects of sound waves of frequencies above 4,000 per second are studied quantitatively, using as detectors both sensitive flames and the ear.

A telephone energised by oscillatory currents produced by a thermionic tube circuit is used as a source of sound. This source has distinct advantages over other high pitch sources such as the Rayleigh bird call and other forms of whistle. Many of the experiments can be done with the sensitive flame fed by gas at ordinary pressures.

Two forms of sensitive flame are employed, one responding to points of maximum air density variation, and the other to maximum motion of the air.

The experiments on interference show results analogous to the optical interference experiments of Lloyd, Fresnel and Newton, and very close agreement is obtained between measured and calculated positions of points on the hyperboloid fringe systems. An experiment is described in which results analogous to certain Heaviside layer effects are obtained.

Diffraction near various obstacles and a sound zone plate are studied.

Some observations are recorded on the fatigue of the ear and binaural audition.

THE Papers of the late Lord Rayleigh contain details of numerous experiments made with high frequency sound waves and in these experiments the acoustic analogies to many well-known experiments in optics were examined.

The present Paper gives details of a repetition and extension of these experiments in which use was made of a different type of source of vibration.

Rayleigh used as his source a " bird call " or whistle of very high pitch. Experiments with this instrument for use in lectures showed that it is not an easy piece of apparatus for anyone whose laboratory equipment does not include a blower by which steady air pressure can be obtained.

It was therefore decided to test the possibilities of a telephone receiver placed in the plate circuit of a three electrode vacuum tube which was producing oscillatory currents of the desired frequency. This arrangement has the great advantage that its frequency is regular and is under control. Also the intensity of the sound can be varied by means of a resistance box used to shunt the telephone earpiece.

A coil of 1,500 turns of insulated wire was wound with mean diameter about 3 inches. This formed the inductance of the circuit which was completed as shown in Fig. 1.

A variable condenser of about 0·01 microfared maximum capacity was used to tune the telephone note.

The circuit used gives frequencies between 4,000 and 16,000.

A sensitive flame was used as a detector aided occasionally by the ear.

The results obtained were in most cases very definite, and this form of telephone whistle is recommended to anyone who wishes to perform with the minimum of trouble the very beautiful experiments on interference. These can be done for frequencies below 12,000 without the use of a high pressure flame, so that they are within the power of anyone who uses the valve circuit.

In making the sensitive flame jets it is best to draw out several from fairly wide bore (1 cm.) glass tube and to test each on the ordinary gas pressure. If the flame roars, its pressure can be reduced by the use of a screwcock until it is just silent. If the jet is of any use, it will now respond to an *S* sound.

An opening 0·1 cm. diameter is about right, and the jets can be adjusted by means of an ordinary glass file.

TWO FORMS OF SENSITIVE FLAME.

It was found that the flame could, with sounds of certain frequencies, be adjusted to one of two sensitive states. As one of these appears to be unknown to many experimenters, it may be of interest if details are given.

The more usual form of sensitive flame is obtained when the flame is just not roaring. Then, when the source of high pitch sound is switched on, the long silent flame drops to form a short noisy one, of which the behaviour has been described in detail in Tyndall's Sound.* Measurements of the positions for which the flame roared and was silent when stationary waves had been produced in front of a reflecting surface, showed that† the flame is most affected at a loop or antinode. On the other hand, it was found that in some cases, if the flame is first adjusted until it is rustling but not roaring loudly when there is no sound, then the switching on of the whistle causes the flame to become quiet and longer than before. When this type of flame was used to explore the stationary waves in front of a plane mirror the positions of least movement of the flame were such that this flame was silent at a loop of the stationary wave system. The coal gas pressure used for both of these flames was about 8·6 cms. of water.

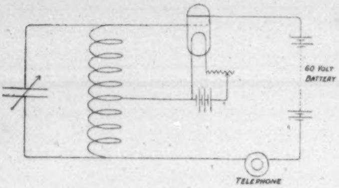

FIG. 1

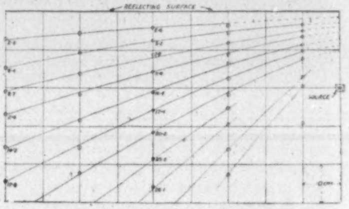

FIG. 2.—REFLECTION AT A PLANE MIRROR. WAVE LENGTH 2·9 CMS. PATH DIFFERENCES MEASURED IN CMS. ARE SHOWN.

Two sets of measurements are given later.

Measurements of the exact positions of interference and of diffraction fringes were made easier by mounting the stand carrying the sensitive flame on a small trolley fitted with rubber wheels. When this was done, the flame could be moved from place to place very smoothly so that it did not roar owing to mechanical vibration.

In the experiments, a high pressure flame was rarely used, because the gas reservoir available only maintained the flame at pressures of 25 cms. of water for about 5 minutes, and this made the location of a field of fringes a very tedious business. Most of the measurements were therefore made with the reservoir connected to the ordinary supply mains. These supplied the gas at about 10 cms. of water which, by choice of a suitable jet, gave very good results for sounds up to a frequency of 11,000. The wavelength so obtained (3 cms.) is short enough to show quite sharply defined shadows.

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* Pp. 232 to 240.

† As Rayleigh states, Phil. Mag., page 153 (1879).

The range of frequencies to which a jet responds is often very small—in one case 5 separate responses were measured between 4,260 and 4,640.

The sensitiveness, as other experimenters have pointed out, varies very much for disturbances of the same frequency coming from different directions.

With jets on which measurements were made, it was found that the frequencies of response were unaffected by small changes of gas pressure, although the sensitiveness fell off rapidly as the pressure was lowered from the critical stage at which continuous roaring begins.

Measurements were made to see if any simple relation exists between the various frequencies to which a given jet responds. The following are the records for 4 different jets showing the frequencies to which each responded :—

(1) 11000, 7610, (7100), 6600, (6000), 4450, 4000.
(2) 7610, 6470, (6000), 4390.
(3) (7400), (6530), 4640, 4520, 4400, 4350, 4260.
(4) 10600, (10000), 7800, 7100, (6470), 4550.

The frequencies in brackets gave much weaker effects on the flames.

EXPERIMENTS ON THE REFLECTION OF SOUND.

Reflection of the higher pitch sounds, particularly of those just audible to the average person, can be shown very well across a large lecture room by using the ordinary metal mirrors supplied for experiments in sound and in the radiation of heat. These give a very sharply defined beam of sound which can be focussed by another mirror on a sensitive flame at considerable distances. Very useful cylindrical mirrors of large aperture can be made at negligible cost from a large sheet of thin three-ply wood bent to a convenient shape by means of two pieces of cord.

Thin three-ply wood was also used to make a lecture table " whispering gallery " by which the creeping of the sound waves along the surface can be shown and the way in which the disturbance is cut off by an obstacle placed against the wall but is unaffected by obstacles some inches away from the face of the gallery.*

In this experiment use can often be made of the relative insensitiveness of the flame to sounds coming from certain directions.†

INTERFERENCE EFFECTS DUE TO STATIONARY WAVES FORMED BY REFLECTION.

The positions of the nodes of stationary waves formed in front of any reflecting surface are very clearly marked when the telephone whistle is used. The measurement of consecutive positions of minimum flame disturbance in front of a plane mirror gives the most convenient way of determining the wavelength and the frequency of the sound. Two sets of such measurements illustrate the difference between the two types of sensitive flame already mentioned.

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* Raleigh, Proc. Roy. Inst., Jan. (1904) ; Bragg, World of Sound, p. 84.

† Bragg, p. 26.

Using the ordinary type of flame the positions of least disturbance in front of a plane mirror were

2·1, 4·1, 6·3, 8·3, 10·9, 12·6, 14·7 cms.

These are in the ratio 1, 2, 3, 4, 5, 6, 7.

If the loops had been points of least disturbance the series would have been 1, 3, 5, 7.

With the other sort of flame a set of readings for least disturbance were 2·2, 3·6, 5·2, 6·6, 8·1, 9·5 cms. Here the ratios are 3, 5, 7, 9, 11, 13, showing that the positions of least disturbance of the flame corresponded to the position of loops of the stationary wave system.

It is interesting to hear the variations of loudness of sound throughout a room when the note is switched on. The room is found to be filled with stationary nodes and loops, for, as the ear is moved, the sound wells out and diminishes in a fascinating way. This is best tested at first by holding a pad over one ear; the other ear will then be able to pick up more easily the stationary fringes in front of any wall. It was found that with the one-valve circuit used and 60 to 200 volts high tension the stationary waves could be located by a sensitive flame at distances up to 15 feet from the source and for several feet from the particular reflecting surface, especially when beams of sound were obtained by the use of large curved mirrors. An additional valve amplifying circuit gave, of course, a great increase of intensity of sound.

"LLOYD'S MIRROR FRINGES" AND "CIRCULAR FRINGES."

Measurements were made of the interference system formed by placing the source in front of a large plane surface.

The points of maximum and minimum disturbance of the flame were very sharply defined.

A scale diagram is shown for wavelengths of 2·9 cms. (Fig. 2). It will be noted how well the measured path differences to various points agree with the theoretical values.

AN ACOUSTIC ANALOGY TO CERTAIN "HEAVISIDE LAYER" EXPERIMENTS.

Professor Appleton pointed out to me that the interference experiments with short sound waves should give an illustration of certain effects which were recently made use of in an investigation of the reflection of wireless waves from the Heaviside layer.* In the experiment wireless waves of constant amplitude transmitted from Bournemouth, or from Teddington, were received near Peterborough. When the wavelengths of the transmitted signals were continuously varied the amplitude of the received signals passed through a series of maxima and minima owing to interference between the ground waves and those reflected from the Heaviside layer, of which the height could then be found. The analogous effect with sound waves can be well shown if a sensitive flame jet is chosen so that the sensitiveness is fairly uniform over a range of frequencies.

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* E. V. Appleton and M. A. F. Barnett, "On Wireless Interference Phenomena between Ground Waves and Waves deviated by the Upper Atmosphere." Proc. Roy. Soc. A, Vol. 113. (1926).

The flame was placed one or two feet from a vertical wall and the whistle, frequency 5000, was placed about an equal distance from the wall and six feet from the flame. When the capacity of the circuit was slowly and continuously increased by 0·001 microfarad the flame recorded 16 maxima and minima. The tuning for the points where the flame was long and silent was extremely sharp so that very steady variation of capacity was required to make the changes of intensity clearly visible.

FRINGES FROM TWO SOURCES.

Two telephones in the same circuit give rise to a family of hyperboloid fringes and again the positions are very clearly defined. As the two sources are usually of different sensitiveness the best results are obtained by using a high adjustable resistance to shunt the windings of the louder earpiece.

A scale diagram is given with measured path differences to certain points (Fig. 3).

This arrangement of two telephones forms an impressive demonstration since the sensitive flame can be shown roaring violently under the influence of one source placed a few inches from it. Then, as the other telephone is brought up, the flame passes through alternate states of disturbance and quietude, and it can finally be left motionless and undisturbed while the two telephones are fixed an inch or two away on either side and the whole room is filled with the whistling sound.

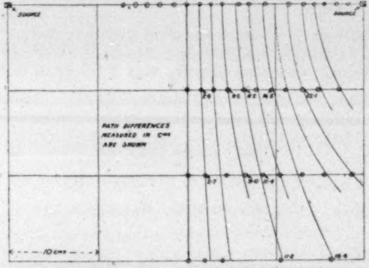

FIG. 3.—INTERFERENCE FRINGES DUE TO TWO SOURCES 40 CMS. APART. WAVELENGTH 2·8 CMS.

DIFFRACTION EFFECTS.

It is found that the ordinary diffraction effects can be shown well over a large frequency range, using telephone whistles. With a slit 20 cms. wide by 43 cms. high, and a source 40 cms. away, a central maximum and several side maxima could be measured at 40 cms. beyond the screen (Fig. 4).

The diffraction of the waves round an obstacle (a long rectangular board 23 cms. wide) gave a sharply defined " bright spot " on the line joining the source to the centre of the obstacle and a number of minima outside the geometrical shadow. In these experiments 3 cms. waves were used.

In a similar way the zone of disturbance at the centre of the shadow of a circular disc can be shown. A glass disc 33 cms. diameter, cut for a Wimshurst electrostatic machine, was used as the obstacle. With the source 50 cms. from the disc the central spot and two other zones could be detected within the geometrical shadow and one outside it. The wavelength was 4·4 cms.

THE ZONE PLATE.

A zone plate was cut out of thick cardboard with the inner circle of radius 7·7 cms. and the other rings with radii in the ratios of the square roots of 1, 2, 3, 4, 5, etc. Using waves of 4·4 cms. with the source 30 cms. and the flame 17 cms. from the zone plate it was easy to show that the disturbance was very strong with the central zone out and was much less with the second zone also removed. It was definitely greater again when the third zone was removed and still more vigorous with zones 1 and 3 out and with 2 replaced. The removal of other alternate zones seemed to have no marked effect presumably because of the obliquity of diffraction from them. The length along the axis over which the disturbance seemed to be a maximum was about 1 cm. The distance of the source and flame enable us to calculate the first focal length of the zone plate.

Thus, $$1/(-17)-1/30=1/f,$$

and $$f=-11 \text{ cms.}$$

The simple theory gives the focal length as—

$$f=-\frac{(\text{Radius of inner ring})^2}{\text{wavelength}}$$

$$=-\frac{(7\cdot 7)^2}{4\cdot 4}\doteqdot -13 \text{ cms.}$$

With shorter waves we get, as we might expect, a closer agreement between the observed and the calculated values. When the wavelength was 2·95 cms. and the source was distant 30 cms. the point of maximum disturbance was 57 cms. beyond the zone plate. This gives $f=-19\cdot 7$ cms., and the calculated value for this wavelength is 20·1 cms.

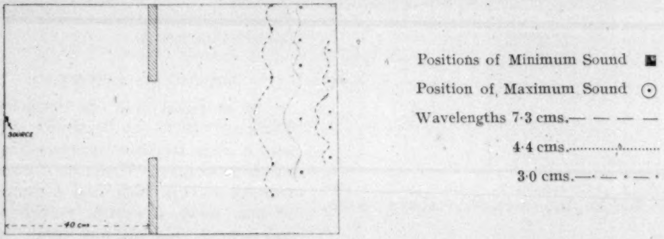
source
40 cm
Positions of Minimum Sound ■
Position of Maximum Sound ⊙
Wavelengths 7·3 cms.— — — —
4·4 cms.............
3·0 cms.—·—·—·

FIG. 4.—DIFFRACTION THROUGH A RECTANGULAR APERTURE.

DIFFRACTION AT A STRAIGHT EDGE.

Diffraction effects could be detected round the edge of a large wooden screen. The gradual and progressive weakening of the effect on the flame as it passes into the geometrical shadow is well shown and the presence of variation of intensity outside the shadow.

THE DIFFRACTION GRATING.

An attempt was made to construct a diffraction grating by means of cardboard strips fastened across a rectangular opening in a large screen. There were nine openings, each 3·4 cms., with the strips 1·7 cms. wide. The source was placed at the focus of a metal mirror 80 cms. from the grating. The frequency used was

11,600. With the flame 70 cms. behind the grating a set of interference fringes were found. They were spaced almost regularly at intervals of 9·2 cms. These positions were all confirmed by using a long rubber tube with its other end placed to the ear. No appreciable difference of intensity was noted among these positions. The use of the tube is not very comfortable or convenient. Only in a few cases can the unaided ear be used satisfactorily in quantitative observations, as usually the presence of the head causes too great a disturbance of the field of sound. When the ear is used as a detector it hears most sound where the density variations are greatest.* Therefore, its maxima coincide with points of quietude when use is made of the ordinary type of sensitive flame, and to places of maximum disturbance of the other form.

FATIGUE OF THE EAR.

Lord Rayleigh† records that for sounds of very high pitch the ear becomes insensitive after three or four seconds, and that a momentary screening of the ear by the hand allows the ear partially to regain its power of hearing the whistle. I cannot detect in my own case any variation whatever in the loudness of the sound heard.

When I can hear the sound (up to about 14,000) I find no appreciable fatigue even over long intervals. The head must, however, be kept quite still, because a movement through one centimetre may take the ear from a point of sound to one of silence. Experiments with a number of observers showed that while some noted no fatigue, others stated that the audibility diminished rapidly, but became temporarily greater again on moving the head.

BINAURAL AUDITION.

In accordance with a suggestion made by Professor E. V. Appleton, I have adjusted two high-frequency oscillators to give beats with one telephone earpiece in each circuit. This made possible a repetition at a high frequency of one of the experiments performed by H. Banister on the effect of binaural phase differences on the localisation of tones.‡

In an early experiment with frequencies about 5,500 it was possible to adjust the beats to keep steady with frequencies down to three per second. Further adjustment then brought the two notes into unison. The notes remained in unison for a certain range of condenser variation in either circuit, this range being less when the degree of coupling between the two circuits was decreased. With weaker coupling between the two circuits the beats could be brought as slow as one in two seconds before the circuits "locked" into unison. It has been shown that the variation of capacity from one side of the unison range to the other is accompanied by changes in the relative phases of the two oscillations, which are in phase for one position, and have a phase difference of 180° at the other.§

Visual evidence of this phase change was obtained by arranging two telephone receivers, one in each circuit, about 3 ft. apart with a sensitive flame somewhere in their neighbourhood. When the telephones were producing sounds of slightly different pitch the disturbances of the flame kept time with the audible beats. The two circuits were just brought into unison so that the beats ceased and the flame was moved to a position either of maximum or of minimum disturbance in the interference system produced. When the flame was quiet a slow change of the capacity of either circuit caused the flame to commence to roar as the condenser reading approached that which corresponded to the other limit of the unison adjustment, showing that the vibrations of the two telephone diaphragms had altered in phase through 180°. The use of a tube leading to the ear showed a corresponding change at any point from a maximum to a minimum of sound.

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* Rayleigh, Sound, Vol. 2, p. 77.

† Phil. Mag., p. 344 (1882).

‡ British Journal of Psychology, 15, p. 281, Jan. (1925); *see* also Phil. Mag., Vol. 2, July (1926) and August (1926).

§ E. V. Appleton, Proc. Camb. Phil. Soc., 21, p. 231 (1922), and H. Banister, *loc. cit.*

It is hoped in a further Paper to describe other experiments in connection with the binaural effect at both high and low frequencies.

Attempts are also being made to obtain visible sound fringes with a view to the possibility of photographing the effects so far obtained.

The author desires to thank Professor Appleton for the above suggestions and for his encouragement, and Mr. Abley, of the College Laboratory, for his assistance in the construction of the necessary apparatus.

DISCUSSION.

Dr. D. Owen: I should like to congratulate the author on an interesting series of experiments. The thermionic oscillator places at our disposal a steady source of sound of constant frequency, and the diaphragm of a telephone serves as a sufficiently small vibrating source to permit of a very beautiful and instructive series of experiments illustrating interference phenomena. The Paper contains two novel points. In the first place the discovery that under certain conditions a low-pressure sensitive flame indicates the points of maximum pressure variation in a stationary system of waves, in contrast with the usual high-pressure sensitive flame which indicates the points of maximum particle displacement. In the second place the conclusion is drawn that the low-pressure flames are sensitive only to certain frequencies. These are observations which merit fuller investigation. In regard to the second it should not be overlooked that with a given telephone diaphragm actuated by a valve oscillator whose frequency is gradually varied, the intensity of the emitted sound rises and falls very markedly as the frequencies of natural vibration of the diaphragm are in turn passed through. Before the author's conclusion that the flame itself is selective can be accepted this factor must obviously be eliminated. I should be interested to know whether the author had succeeded with this type of sound source in actuating Kundt's tube. Such a means would be a welcome substitute for the rod excited by stroking.

Prof. A. O. Rankine suggested that the Doppler effect might be demonstrated by a change in beat frequency when one of two simultaneously sounding telephones is moved towards or away from the flame.

This experiment was accordingly tried with success.

Dr. J. H. Vincent: The author has added greatly to his interesting and important Paper by the successful demonstration which followed it. Has Mr. Humby tried different types of telephone receivers? I have found that an instrument of the reed type has the useful property of acting as a source of high frequency sound when in the alternating magnetic field of a valve oscillator, even when unprovided with leads. As one can alter the intensity of the sound emitted by the telephone by changing its orientation with respect to the oscillator coil, the intensity can be adjusted to equality with an independent source; under these conditions beats can be obtained of surprising distinctness. I was interested to notice that the author in one of his experiments employed the automatic synchronisation of two valve oscillators which were first shown to the Physical Society in 1920.[*] This phenomenon provides a simple means of producing a high-frequency rotating magnetic field which perhaps would be useful in studying the gyromagnetic effect.

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[*] Vincent, Proc. Phys. Soc., p. 84 (1920).

Mr. W. E. **Benton** said that the author might find it convenient to control the gas pressure by means of a governor of the kind used by gas companies; this would keep the output pressure constant to about 2 per cent. for a 100 per cent. increase in the input pressure. An extremely delicate means for measuring small variations in amplitude was afforded by the pin-hole probe of Prof. Carl Barus. Owing to the flange which is formed in piercing a pinhole in a sheet of metal, the hole acts as a rectifier of sound waves, and when sound falls on it a difference in pressure is set up between the two sides of the sheet. This difference can be measured with a precision of 10^{-6} mm. of water by means of the interference manometer, in which a glass plate floats on the mercury in each limb of a manometer U-tube, relative changes in level of the glass plates being measured by means of an interferometer. This arrangement has been employed in the study of the amplitude of vibration in organ pipes.

Prof. O. W. **Richardson** said that Dr. Vincent's arrangement for producing a high-frequency rotating field had been used by Mr. Fisher, of King's College, in experiments on the gyro-magnetic effect.

Author's reply: I have attempted to use the telephone whistle on a Kundt's tube, but so far the power available had not been sufficient to disturb the lycopodium powder. With regard to the possibility of apparent flame selectivity being due to the variation of amplitude of the motion of the telephone diaphragm, I feel that the figures given in the Paper show that variation of output cannot be the chief cause of selective response. The figures are for different flame jets with the same telephone as transmitter, and there were no frequencies common to all the jets. There should be if variation of output were the most important factor in determining the frequency at which response of the flame was obtained.

Prof. Rankine's suggestion for the demonstration of the Doppler effect by a change in the observed beat frequency when one of the oscillators is moved relative to the flame, gives an experiment similar to that in which one of two slightly mis-tuned tuning-forks is moved relative to the other. In either experiment the change of beat frequency can be heard, and can be rendered visible by the motion of a suitably chosen sensitive flame.

Dr. Vincent's observation of the effect of high-frequency induction effects on telephone receivers is of interest. I have only used about half a dozen different receivers, and all were of the ordinary diaphragm type. I also have noticed the surprising distinctness of the beats obtained when the two independent sources are carefully adjusted to equality of output by the variation of a high resistance placed in parallel with the louder telephone whistle.

A gas governor of the type referred to by Mr. W. E. Benton should make it easier to carry out a series of measurements of stationary interference fringes. I had to be content with an improvised "gasometer" connected direct to the supply mains. For demonstrations, however, it was sufficient to use the gas direct from the mains, provided the jets were adjusted for the pressures which were available.

XXXVIII.—SOME ADDITIONAL REFINEMENTS FOR PRECISION BALANCES.

By J. J. Manley, *M.A.* (Fellow of Magdalen College, Oxford).

Received March 16, 1927.

ABSTRACT.

This Paper deals with certain small errors attendant upon the use of the rider of a precision balance, and shows how such errors may be rendered negligible. Attention is also drawn to some disadvantages resulting from the employment of velvet-covered pan arrestors and a plan is described for eliminating them.

In former Papers* certain devices for ensuring high precision in weighing have been described. In this Paper is given an account of some additional refinements whereby the attainment of still greater precision is rendered possible.

The need for the new refinements was brought into prominence by the requirements of a recent problem for the solution of which it was necessary, amongst other things, to determine with considerable accuracy masses of less than 0·3 mg. To improve upon the refinements already in use was more than a little difficult; hence attention was centred chiefly upon the rider and the manner of using it.

The scale for the rider is usually engraved upon the beam itself: but in some cases it is cut upon an attached thin strip of metal. The upper surface of a balance beam is often flat and less frequently bevelled. A bevelled beam is either graduated or more generally serrated. In the case of the flat top, the position of the rider is ascertained by a scale cut upon the face of the beam: and, provided due care be observed, the plan is for most purposes sufficiently accurate. It may, however, be noted that we here assume that the crown of the placed rider is a horizontal right line running strictly perpendicular to the length of the beam. In general the crown is neither perfectly straight nor placed with the theoretical accuracy supposed. Errors arising from these two defects are likely to be more definitely marked in a beam that is flat than in one that is serrated. In the case of the latter they become vanishingly small when the bevel is acute and the serrations shallow and geometrically correct. But when a beam having a flat top is used and a series of rider settings of one and the same indicated value carried out, we find that successive resting-point determinations, instead of being invariable, differ very appreciably. Hence for high accuracy we must, under these circumstances, adjust the rider and determine the corresponding resting-point a number of times and accept the mean value as the best available.

Another and not readily suspected source of error arises from a want of strictly accurate spacing of the scale divisions or the serrations, as the case may be. In illustration of this the results obtained by calibrating the beams of two high-grade balances with micrometers may be quoted. One beam was flat and the other serrated. In the first case the extreme difference in the spacing was 8·2 per cent.

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* Phil. Trans, Series A, Vol. 210, p. 387, and Vol. 212, p. 227. Also Proc. Roy. Soc. A, Vol. 86, p. 591. Article "Balance" Thorpe's Dict. of Chem. New Edition.

and in the second 0·8 per cent. Expressed as possible variations in weight, these errors are respectively ·085 and ·008 mg. These several errors were minimized in the following way.

First, to each end of the beam and at the back of it was attached by means of screws, as shown in Fig. 1 *a*, an L-shaped plate of aluminium. Next, two riders, having the form shown in Fig. 1, *b*, were made and their flat ends perforated. One was of gold and the other of aluminium; the first weighed 10 and the second 1 mg. After the riders had been adjusted, a platinum wire 0·1 mm. in diameter was drawn through each; the two wires were then passed through holes pierced in the L-shaped plates, drawn taut and secured as depicted in Fig. 1.

For moving the riders it was necessary to construct and substitute another apparatus for that commonly used; an end-on view of this is seen in Fig. 2. In the figure, *A* is a stout brass plate drilled for the reception of the tube used for operating it, and ω_1, ω_2, are two platinum wires 0·2 mm. thick, fixed and bent as shown. The relative positions of the two riders r_1, r_2, upon their wires are also shown. The movements of the stops s_1, s_2, about the centre *c*, are limited by a glass surface *f*, *g*. This limitation guards the stretched wires against undue pressure from ω_1 and ω_2. By suitable horizontal and rotational movements of the plate *A*, the wires ω_1, ω_2, can be brought to bear against either face of the corresponding rider: and then by pushing or pulling, as the case may be, the rider can be moved to any point of the supporting wire.

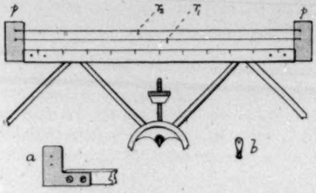

FIG. 1.

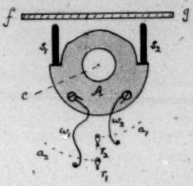

FIG. 2.

When the changes just described had been completed, it was no longer possible to stabilize the beam. This difficulty was overcome by removing one-half of the gravity bob from its normal position above the centre of the beam to the upper portion of the pointer upon which a fine thread had been cut for it. Thus the balance now possessed two gravity bobs, the one above and the other below the central knife-edge: by adjusting these any required sensitivity could again be obtained. We now consider the methods used for accurately determining any position occupied by a rider.

As it was obviously impossible to locate a rider with any degree of precision by using the beam scale alone, a large reading-telescope mounted upon a travelling platform *P*, as shown in Fig. 3, was employed. The platform was provided with three contacts, which consisted of two brass balls at the back and a wheel, seen in front. The balls rested within a lubricated and rigidly fixed V-groove and the wheel upon a flat surface. The telescope was set up at right angles to the balance, and therefore when the handle *H* was turned the screw *s* moved the platform in a direction parallel to the sighted beam. The traverse of the telescope was somewhat greater than the length of the beam: hence any part of the scale could be brought under observation.

When using this apparatus in connection with weighing, the simplest plan is first to move the 10 mg. rider until equilibrium is approximated, and then to obtain a sufficiently close approach to equality by moving the 1 mg. rider. The resting-point is then determined, the beam arrested and the telescope moved along until its fine vertical cross-thread coincides with, or rather covers, the thin edge of the sighted rider, the position of which is then noted. As will be seen, the position of the riders are in this way determined only with the usual degree of precision. The system does, however, offer two advantages. First, as the rider cannot be dropped, possible damage and loss are eliminated. Secondly, the position of the rider is defined with great exactness, and its contact is equivalent to a fine line drawn at right angles to the supporting wire. If by chance the eye of the rider is larger than it should be, the rider will at times rest slantwise; but in that case it can at once be correctly orientated by lightly tapping the stretched wire with the rider apparatus. The plan introduced for measuring the position of a rider with increased accuracy may now be described.

On referring to Fig. 3, it may be seen that a tangent screw *t* by which the telescope may be slightly rotated about its vertical axis is equipped with a head *k* and an index *i*. The index is more clearly seen in Fig. 4, wherein is also shown a metal arc *a* graduated in mm. The distance from the balance to the telescope was 225 cms. Under these circumstances, the coincidence of the cross-thread was transferred from one minor division of the beam to the next, when the index was moved over an arc of 42 mm. The accuracy with which the position of a rider was determinable had thus been increased from an *estimated* 1/10 to a *measured* 1/42 of a minor division of the beam.

The arms of the balance under discussion were each divided into 50 parts, and so for the two riders each division corresponded to the respective values of 1/5 and 1/50 mg. Hence, by the device of the tangent screw, the accuracy with which any positional value could be measured was respectively $1/5 \times 42$ and $1/50 \times 42$ mg. The advantage thus gained was utilised by means of the following optical method.

First the central knife-edge block was drilled and one end of a fine aluminium tube inserted and secured. Into the other end of the tube, which was centred in a small opening cut in the beam case,* was thrust a wire supporting a flat palladium mirror. The mirror, which was adjusted vertically, faced a window of parallel-plane glass set slantwise† in one end of the balance case. Opposite the window, and at a distance of 125 cm. from it, was a powerfully illuminated and perpendicular mm. scale. For viewing the image of this scale, a second reading telescope was directed towards the mirror and fixed at a distance therefrom of 3 M. As the effective length of the pointer of light was 6 M, and that of the ordinary pointer 30 cm., the accuracy with which a deflection of the beam could be read was thus increased twentyfold.

When the additional arrangements had been completed and the balance adjusted to a high sensitivity *S*, it was found that the resting-point changed by 300 units‡

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* Phil. Trans. A. Vol. 210, p. 407.

† Multiple images were thus avoided.

‡ The unit was 1/10 mm.

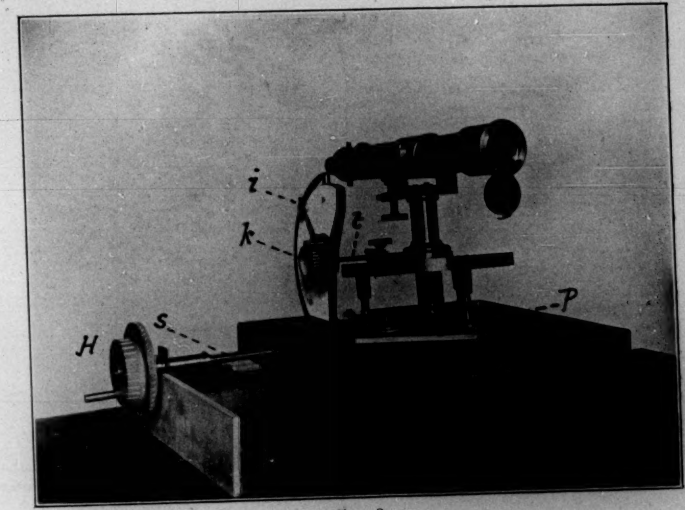

FIG. 3.

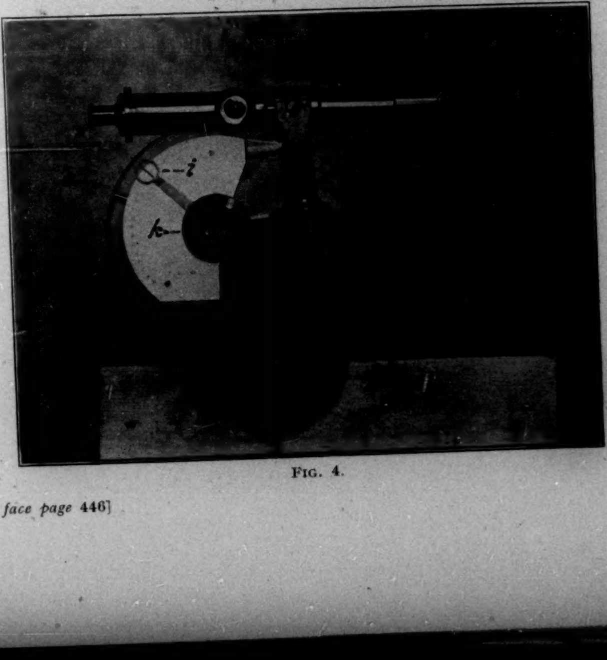

FIG. 4.

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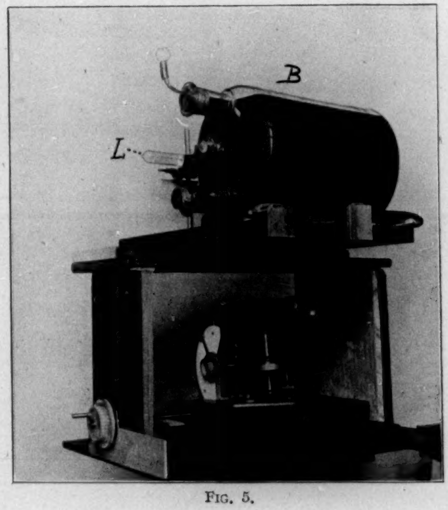

FIG. 5.

when the weight in one pan was changed by 4/50 mg.; whence, $S=375$ per 0·1 mg.—that is to say, a deflection of 0·1 mm. corresponds to $2{\cdot}7\times10^{-7}$ grm. It has already been shown (*vide supra*), that the positional value of a rider was in the one case ascertainable to 5×10^{-6} and in the other to 5×10^{-7} grm. Hence, the accuracy with which the resting-point of the balance could be determined was now in excess of that attainable when setting a rider and amply sufficient for our purpose.

It will be understood that the actual process of weighing is not quite so simple as that outlined above. In describing the method used for measuring the position of a rider it was tacitly assumed that the scale engraving upon the beam was free from error; and as this was not so, it was necessary to calibrate it. The calibration was carried out in the following way.

First the index i, Fig. 3, was set near the centre of its scale and its position a noted. Next the platform P was moved along by means of the screw s until the vertical cross-thread of the telescope coincided with the zero line on the beam. The index was then turned so as to transfer the cross-thread from the zero line to the next, and its new position β read. The first division of the beam was thus measured in terms of the scale a, Fig. 4, its indicated value being $\beta - a$ mm. The first division having been calibrated, the index was returned to its original position a and the telescope screwed along a second time, and on this occasion until it sighted the line marking division 1; then, on using the index as at first, the relative length of division 2 was determined. Proceeding thus, the whole scale was calibrated, and from the data obtained a table was prepared showing the weight values of the two riders for each of the 50 scale divisions upon the beam. The weight values for fractions of a scale division were found by interpolation. The plan adopted for guarding and lighting the beam must now be described.

Apparently the utmost accuracy in weighing can be attained only when the beam is enclosed and guarded against radiant energy.* Under ordinary circumstances the balance is readily affected and variations in its resting-point occur continuously. The beam of my own balance is protected by a small chamber which completely envelops it, also the glass panels of the case have been displaced by others of aluminium. The balance-room has but one window, and that is darkened during the operations of weighing, and whilst the balance is being loaded a water-cooled lamp is used. The powerful lamp employed for illuminating the vertical scale viewed by the second telescope (*vide supra*), is placed near the ceiling and screened by sheets of aluminium. Its light is thrown upon the scale by a system of lenses and a mirror.

When the resting-point has been determined and the beam arrested, an aluminium shutter covering the front is dropped and the tubular lamp L, Fig. 5, lit. The bottle B, filled with weak potassium chromate solution, constitutes a cylindrical lens; and this not only forms a bright band of light along the entire length of the uncovered beam, but also absorbs heat rays emitted by L. The potassium chromate renders the light largely monochromatic, and so the scale upon the beam when viewed through the telescope appears remarkably sharp. A final precaution which contributed to success in the determination of small masses may now be dealt with.

In some balances the pan arrestors have their tops covered with velvet, the use of which may be detrimental in two ways. First, I find that the applied velvet usually exudes a substance which in time forms a film upon the bottom of the pan. It is improbable that such films are either equal for the two pans or permanent; hence, in the case of such a high-grade balance as that under discussion, variations in the resting-point must almost inevitably ensue. The correctness of this view may be verified by those possessing first-class sets of gilt or platinized weights. The weights when lifted from their velvet-lined cells, are usually somewhat dull, especially when they have not been used for some considerable time. The removal of the dullness by silk results in an appreciable decrease in value. That the observed loss is not due to the removal of a part of the weight itself may be demonstrated by repeating the experiment with a number of weights. Usually they are constant after the first rubbing.

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* Phil Trans. A, Vol. 210, p. 405.

Secondly, the pile of the velvet gradually flattens, and in consequence of this the arrestors and their pans tend to adhere. As the adherence is rarely the same in each case, a defect is manifested when the beam is brought into use, by the non-release of one of the pans. When the pan is that containing the object, the fractional weights placed in the other may for a time exceed those actually required for weighing. The errors and inconveniences arising from the above-named sources were, in the present instance, avoided in the following way.

First, the velvet coverings were removed, the heads of the arrestors turned flat and three small holes drilled through each. These holes were just within the rim and separated from each other by an angular distance of 120°. Finally, a short gold wire, the upper end of which terminated in a sphere* was placed in each hole and its lower portion bent against the under surface of the arrestor and thus secured. Armed with their triple contacts of gold, the arrestors have proved highly efficient.

Contacts of gold were chosen for the simple reason that the pans were gilded, hence the loss and gain of the engaging points and surfaces mutually and automatically compensate each other. Had the pans been platinized, the arresting spheres and their wires would have been of platinum.

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* The spheres were formed by fusing one end of each wire, which was 0·5 mm. thick, in the flame of a blowpipe.

XXXIX.—ELECTRICAL SEPARATION BETWEEN IDENTICAL SOLID SURFACES.

By P. E. Shaw, *M.A.*, *D.Sc.*, *F.Inst.P.*, ~~University College, Nottingham.~~

Received February 20, 1927.

ABSTRACT.

(1) When one insulating solid is rubbed along and perpendicular to an identical one they acquire charges, $-^{ve}$ and $+^{ve}$, respectively.

(2) Continued rubbing so affects the surfaces of "rubber" and "rubbed" that their properties change, with reversal of sign to $+^{ve}$ and $-^{ve}$ respectively.

(3) The effect 2 is permanent, unless the surfaces be restored by heat treatment.

(4) Oblique impact of two identical solids generates on them unequal charges. The net charge is, in general, $-^{ve}$.

IN the old standard compilation of Tribo-electricity* mention is made of charges arising when apparently identical solids are rubbed on one another. One of these cases was observed by Faraday, using like feathers. But no systematic investigation has been accorded to these unexpected results in the long intervening period. It will be shown below that, when suitable care is taken, identical solids do charge one another by friction or impact in a *regular* way, and that at least for the many insulating bodies tried there is the same law of electrical separation for all. But if care be not taken the results will be found to be fortuitous.

The materials used are ebonite, celluloid, caoutchouc, sealing wax, shellac, amberite, sulphur, charcoal, paraffin wax, glass, vitreous silica, mica.

The most convenient form in the case of ebonite, celluloid, etc., is a rod, say, 10 cm. long and 1 cm. square section. Mica in thin sheet is wrapped round a glass tube and such material as shellac is melted on a glass rod.

RUBBING AT ORDINARY TEMPERATURE.

Ebonite.—In order to prepare the surfaces so that they may be identical they are thoroughly, but lightly, scraped with a keen razor edge and are then boiled in water till they are soft enough to bend easily. After drying and cooling the following effects are observed:—

(1) Placing the rods across one another, one (A) is rubbed down the other (B). We then find A is charged $-^{ve}$, B $+^{ve}$. Discharge the rods by the side of, *not in*, a flame. Rub B down A. We find B $-^{ve}$, A $+^{ve}$. This action occurs in all places, on both sides. Thus the surfaces are now identical in behaviour. The "rubber" in all cases being $-^{ve}$, the rubbed $+^{ve}$.

When the surfaces behave alike in this way, we call them "standard."

There is a real distinction between "rubber" and "rubbed," since a much smaller area of the former than of the latter takes part in the rub; and consequently, of the two, the rubber attains the higher temperature and is softer and more likely to yield and be strained under the great tangential forces applied in friction.

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* "Reibungs electricitat," by Riess, publ. 1855.

(2) Continued rubbing of the standard rods brings about a change of effect, the rubber gradually loses its $-^{ve}$, and the rubbed its $+^{ve}$; in time they become neutral, and finally the signs are reversed with rubber $+^{ve}$, rubbed $-^{ve}$.

When in the neutral state, either rod may become $-^{ve}$, the other being $+^{ve}$ according to the places rubbing, since the permanent effect of rubbing will not be uniformly distributed. A light rub may give a $-^{ve}$ charge, a heavy one a $+^{ve}$, to the rubber. A curious effect is sometimes seen in the neutral state; a direct rub giving $+^{ve}$, a reverse rub $-^{ve}$, to the rubber.

(3) Prolonged rubbing results in the rubber always showing $+^{ve}$, the rubbed $-^{ve}$, and this condition is permanent, showing no change as time goes on. Let us call the new state of both rods *strained*, in contrast with the initial "standard" or *unstrained* state.

Without observing their electrical action it is easy to tell whether two surfaces are unstrained by their soft feel when rubbed together. After much rubbing the surfaces feel hard and "gritty" on rubbing.

(4) On scraping and boiling the rods, as in (1), they return to their pristine standard state. Thus we have a reversible cycle. A warning is necessary: the rods should not be rubbed after boiling until they are cold, since when hot the material is so soft that strain may be imposed very rapidly and steps (1) and (2) may run into one another.

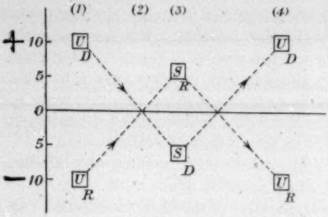

FIG. 1.

Sign and approximate amount of charge on surfaces unstrained (U) in (1), becoming strained (S) in (3), and by heat treatment reverting to unstrained in (4). R denotes rubber; D rubbed; (2) is the neutral state.

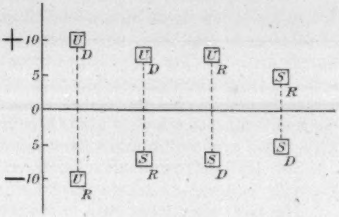

FIG. 2.

Sign and approximate charge of unstrained (U) or strained (S) surfaces, rubbed two at a time, depending on position as rubber (R) or rubbed (D).

The effects in the above experiments are shown graphically in Fig. 1, where the headings refer to the above four experiments.

The simple apparatus used to measure the charges has been described already.* A Hankel electroscope whose gold leaf is observed by a microscope has its parallel plates joined to a battery which maintains them at +50 volts and −50 volts respectively. The leaf is connected to an inductor which has the form of a brass tube about 8 cm. long and 8 cm. diameter. Into this inductor the charge to be measured is thrust. Since the charges are considerable it is best to raise the leaf clear of the parallel plates. Roughly speaking, one E.S.U. charge causes a divergence of 10 scale divs. in the microscope. In order further to indicate the strength of charges obtained in these experiments, it may be stated that when two standard ebonite rods are rubbed they give rise to charges one-tenth to one-twentieth of those obtained by rubbing an ebonite rod on warm flannel.

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* Proc. Roy. Soc., A., 111 (1926).

(5) A further experiment which contributes to the elucidation of these effects is as follows: Two surfaces are well rubbed together till they are strained, as shown in experiment (3). Two other surfaces are left unstrained. Rubbing them together in pairs we have U/U, S/U, U/S and S/S (*see* Fig. 2). The first and last of these four cases we have in experiments (1) and (3). The other two cases are new. If the figure be examined we find we can interpret all cases by the following rules:—

(*a*) The unstrained surface has a $+^{ve}$, and the strained a $-^{ve}$ tendency.

(*b*) The rubbed surface dominates the situation and decides the sign of the charges.

Having observed the behaviour of ebonite it may be stated at once that the other materials mentioned behave in the same way. All, except glass and vitreous silica (whose surfaces are relatively hard, and which show no charges) obey the rule that " rubber " is $+^{ve}$ when unstrained, $-^{ve}$ when strained. All of them on rubbing acquire strain with reversal of charge, which strain is final, but can be removed by heat treatment.

IMPACT EXPERIMENTS.

Dissimilar materials when struck together in normal impact develop opposite charges.* But, on trial, it is found that identical bodies also charge one another. A glancing blow, containing tangential as well as normal stresses, is best for the purpose. The result of a violent blow between identical specimens of our insulating materials is to give a net $-^{ve}$ charge almost invariably, whether the surfaces be standard or strained. To quote a few examples found in the case of ebonite, the charges on the two rods in scale units were $-13/+7$, $-27/+12$, $0/-14$, $-7/-4$. Prolonged blows in general increase the net $-^{ve}$ charge. In the case of caoutchouc, celluloid, shellac and sealing wax the surfaces if very strained show sometimes net $+^{ve}$, but the general rule for these and all the other materials is that net $-^{ve}$ arises. To quote one of the exceptions: Two standard celluloid surfaces on impact give $-20/+16$. On repeating the blows both $-^{ve}$ and positive are reduced, and finally when both surfaces are strained the charges formed are $+11/-5$, i.e., there is net $+^{ve}$.

DISCUSSION OF RESULTS.

From these experiments we learn that strain plays an important part in Tribo-electric effects, and it is clearly impossible to solve the general problems of the genesis of charges in Tribo-electricity unless this new factor is given due weight.

We reduce by one the variables in the equation by using, as in these experiments, only one kind of material.

Analogous with the behavior of insulators in acquiring strain when pressed is that of metals whose surfaces are unstrained (crystalline) after fusion and become strained (vitreous) by pressure.† We have a hint of the difference in physical characteristics in these two states of a metal, since a crystalline and a vitreous specimen show the Peltier effect at their interface.*

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* *See* Richards, Phy. Rev., p. 290 (1920).

† *See* Sir G. Beilby's " Aggregation and Flow of Solids."

The net total $-^{ve}$ charge found in impact experiments in Section (4) is most readily interpreted by supposing that $+^{ve}$ ions escape to the air when the impacting solids separate.

A suggestion may be made. Electrical snowstorms and sandstorms have ever been a mystery. But since, as in these experiments, like particles in impact may attain one charge, and the air an opposite charge, the repeated and violent blows between the particles of snow or sand may gradually accumulate a large charge on the particles and an opposite one in the air. We should then have a state of affairs such as arises from a different cause in an ordinary thunderstorm when the water drops are $+^{ve}$ and the $-^{ve}$ ions are dissipated in the air.

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* *See* Beilby, loc. cit.

XL.—DIFFRACTION OF LIGHT BY A TRANSPARENT LAMINA.

By Prof. C. V. Raman, *F.R.S.*, and I. Ramakrishna Rao, *M.A.*

Abstract.

The present Paper embodies an attempt to consider the problem of diffraction of light by a *very thin* plate of transparent material, bounded by a straight edge, with greater exactness than is attained in the usual elementary treatment on the Fresnel-Huygen principle. The method adopted, though not completely rigorous, bases itself on the electromagnetic theory of light, and seeks to express the disturbance in the field in the form of functions which are solutions of the equations of wave-propagation. The formulæ obtained indicate that the light diffracted by the edge should exhibit colour and polarization effects varying in a remarkable manner with the thickness of the plate and the direction of observation. Effects having the general character of those indicated by the theory have actually been observed in experiment. The theory, however, requires modification in the case of thicker laminæ, where further complications arise which are not here taken account of.

I. Introduction.

THE problem of diffraction by a plane transparent lamina arises in considering the theory of such phenomena as the colours exhibited by the "striæ" or laminar boundaries in mica,[*] and the colours of mixed plates.[†] It has been observed that the edges of thin laminæ diffract light through large angles, and that the light thus diffracted exhibits colour and polarization effects which are in some respects analogous to those discovered by Gouy[‡] with metallic edges. The elementary treatment of laminar diffraction usually given is thus inadequate. In the present Paper an attempt is made to place the theory of diffraction by laminar boundaries on a more satisfactory basis. The case of a thick lamina is too complicated to offer hope of an exact solution. In the case of a thin lamina, however, certain simplifications are possible, as we shall see presently, which enable the problem to be dealt with successfully.

II. Theory.

We shall confine ourselves to the case in which the light is incident on the lamina in a plane perpendicular to its edge. The region round the lamina can then be divided into three parts: The first consists of the region in which the light transmitted through the lamina alone appears. The second is the region in which only the incident waves appear. The third is the region in which the incident and reflected waves are superposed.

The middle surface of the lamina may be taken as the plane $\varphi=0$, and its edge as the *Z*-axis in a system of cylindrical co-ordinates. We may represent the incident wave by the expression

$$e^{i.2\pi t/T}\, e^{i\kappa\rho\cos(\varphi-\varphi_0)} \quad \dots\dots\dots\dots \quad (1)$$

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[*] C. V. Raman and P. N. Ghosh, Nature, Vol. 102, p. 205 (1918).

[†] C. V. Raman and B. N. Banerji, Phil. Mag., Vol. 41, p. 338 (1921).

[‡] G. Gouy, Annales de Chimie et de Physique, Vol. 8, p. 145 (1886).

φ_0 being the angle between the plane of the lamina and the incident rays. The train of waves reflected from the lamina may be represented by the expression

$$-e^{i.2\pi t/T}(A_s+iB_s)\,e^{i\kappa\rho\cos(\varphi+\varphi_0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

or by

$$e^{i.2\pi t/T}(A_p+iB_p)\,e^{i\kappa\rho\cos(\varphi+\varphi_0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

and the train of waves transmitted through the lamina by the expression

$$e^{i.2\pi t/T}(C_s+iD_s)\,e^{i\kappa\rho\cos(\varphi-\varphi_0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

or by

$$e^{i.2\pi t/T}(C_p+iD_p)\,e^{i\kappa\rho\cos(\varphi+\varphi_0)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

The expressions (2) and (4) refer to the case (which we shall refer to as the $\|^l$ case), in which the planes of incidence and polarization are coincident. The expressions (3) and (5) refer to the case (which we shall refer to as the $\perp^r$ case) in which they are mutually perpendicular. The multiplying factors (A_s-iB_s), etc., are those given by the well-known theory of the colours of thin plates,* and may be deduced directly from the electromagnetic theory so as to satisfy the boundary conditions on either face of the lamina.

Now the disturbances represented by expressions (1) to (5) do not extend throughout the whole field, but are confined to the particular regions of the field already indicated. To find the solution of the diffraction problem, we seek a function which satisfies the equations of wave-propagation, and which, while being valid throughout the whole field outside the substance of the lamina, represents a disturbance approaching asymptotically to the values given by those expressions at a sufficient distance from the origin in the respective parts of the field to which they refer.

Such an expression is

$$u=F(\rho,\ \varphi,\ \varphi_0)-(A_s+iB_s)F(\rho,\ \varphi,\ -\varphi_0)+(C_s+iD_s)F(\rho,\ \varphi,\ \varphi_0-2\pi) \quad . \; . \quad (6)$$

or

$$u=F(\rho,\ \varphi,\ \varphi_0)+(A_p-iB_p)F(\rho,\ \varphi-\varphi_0)+(C_p-iD_p)F(\rho,\ \varphi,\ \varphi_0-2\pi) \quad . \; . \quad (7)$$

in which $F(\rho,\ \varphi,\ \varphi_0)$ is the well-known solution of the wave equation due to Sommerfeld.

$$F(\rho,\ \varphi,\ \varphi_0)=e^{i.2\pi t/T}\left(\frac{i}{\pi}\right)^{\frac{1}{2}}e^{i\kappa\rho\cos(\varphi-\varphi_0)}\int_{-\infty} e^{-i\lambda^2}\,d\lambda \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

where

$$T=\sqrt{2k\rho}\cos\tfrac{1}{2}(\varphi-\varphi_0) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

$F(\rho,\ \varphi,\ -\varphi_0)$ and $F(\rho,\ \varphi,\ \varphi_0-2\pi)$ are obtained by writing $-\varphi_0$ and $(\varphi_0-2\pi)$ respectively for φ_0 in (8) and (9).

In the $\|^l$ case u represents the electric force parallel to the edge of the screen and in the $\perp^r$ case it represents the magnetic force in the same direction. It can readily be verified that the asymptotic values of (6) and (7) are those given by (1),

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* **Drude's Theory of Optics, English Translation, p. 302.**

(2), (3), (4) and (5), in the parts of the field to which they refer. When (9) is positive, the asymptotic expansion of (8) is

$$F(\rho,\ \varphi,\ \varphi_0)\sim e^{i.2\pi t/\mathrm{T}}\left[e^{i\kappa\rho\cos(\varphi-\varphi_0)}+\frac{i^{3/2}e^{-i\kappa\rho}}{4\pi\sqrt{\rho/\lambda}\cos\frac{1}{2}(\varphi-\varphi_0)}\right]\quad .\ .\ .\ .\ (10)$$

while, if (9) be negative, there is a similar expansion in which the first term is left out The light diffracted by the edge may accordingly be written thus

$$E_z=e^{i.2\pi t/\mathrm{T}}\cdot\frac{i^{3/2}e^{-i\kappa\rho}}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1-(C_s+iD_s)}{\cos\frac{1}{2}(\varphi-\varphi_0)}-\frac{A_s+iB_s}{\cos\frac{1}{2}(\varphi+\varphi_0)}\right]\quad .\ .\ .\ .\ .\ (11)$$

$$H_z=e^{i.2\pi t/\mathrm{T}}\cdot\frac{i^{3/2}e^{-i\kappa\rho}}{4\pi\sqrt{\rho/\lambda}}\left[\frac{1-(C_p+iD_p)}{\cos\frac{1}{2}(\varphi-\varphi_0)}+\frac{A_p+iB_p}{\cos\frac{1}{2}(\varphi+\varphi_0)}\right]\quad .\ .\ .\ .\ .\ (12)$$

We may write the expression within the square brackets in (11) and (12) respectively in the form (F_s+iG_s) and (F_p+iG_p). $(F_s^2+G_s^2)$ and $(F_p^2+G_p^2)$ are measures of the intensity of the components in the radiation diffracted from the edge, and their ratio indicates its state of polarisation when the incident light is unpolarised. $\delta_s=G_s/F_s$ and $\delta_p=G_p/F_p$ give the phases of the components, and $(\delta_p-\delta_s)/\lambda$ is a measure of the ellipticity of the diffracted radiation when the incident light is polarized in any arbitrary azimuth.

III. NORMAL INCIDENCE: VERY THIN LAMINA.

In the case of normal incidence, we have $\varphi_0=\pi/2$, and further

$$A_s+iB_s=A_p+iB_p=\frac{i(\mu^2-1)\sin p}{i(\mu^2+1)\sin p+2\mu\cos p}\quad .\ .\ .\ .\ .\ (13)$$

$$C_s+iD_s=C_p+iD_p=\frac{2\mu(\cos q+i\sin q)}{i(\mu^2+1)\sin p+2\mu\cos p}\quad .\ .\ .\ .\ .\ (14)$$

where $p=2\pi\mu d/\lambda$ and $q=2\pi d/\lambda$, d being the thickness of the lamina and μ its refractive index. We have only to substitute (13) and (14) in equations (11) and (12) and evaluate them numerically to find the intensity and state of polarization of the diffracted light in any direction.

Consider first a lamina so thin that we may put $\sin p=p$, $\sin q=q$, $\cos p=\cos q=1$. Since $p=\mu q$, we find readily on making these substitutions, that

$$1-(C_s+iD_s)=1-(C_p+iD_p)=A_s+iB_s=A_p+iB_p$$

and that

$$F_s+iG_s=\frac{\pi(\mu^2-1)d}{\lambda}\left[\frac{1}{\cos\frac{\varphi-\varphi_0}{2}}-\frac{1}{\cos\frac{\varphi+\varphi_0}{2}}\right]\quad .\ .\ .\ .\ .\ (15)$$

$$F_p+iG_p=\frac{\pi(\mu^2-1)d}{\lambda}\left[\frac{1}{\cos\frac{\varphi-\varphi_0}{2}}+\frac{1}{\cos\frac{\varphi+\varphi_0}{2}}\right]\quad .\ .\ .\ .\ .\ (16)$$

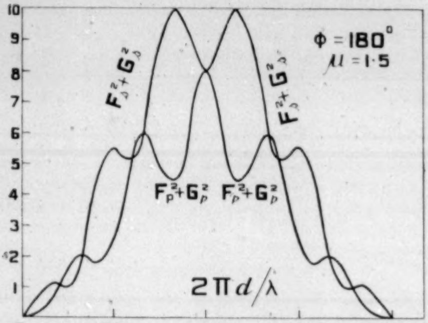
From (15) and (16), it follows at once that along the surface of the screen ($\varphi=0$ and $\varphi=2\pi$), the $\|^l$ component of the diffracted light would be zero, and the $\perp^r$ component would be finite, thus giving complete polarization. In the opposite direction

($\varphi=\pi$), however, the $\|^l$ component of the diffracted light would be finite, and the $\perp^r$ component would be zero, thus again giving complete polarization but in a perpendicular plane. The intensity of the diffracted light would, for such thin laminæ, be proportional to the square of the thickness.

IV. THICKER LAMINÆ: COLOUR AND ELLIPTIC POLARIZATION.

If we assume that formulæ (11) and (12) remain valid for greater thicknesses of the lamina, at least as an approximation, it is evident from these expressions taken together with (13) and (14) that the intensity and state of polarization of the diffracted light would be functions of the wavelength, the thickness of the lamina, and the angle of diffraction. Hence, in white light, the diffracted radiation from the edge would exhibit colour.

If we assume that the incident light is unpolarized, the diffracted light from the edge would be partially polarized to an extent depending essentially on the contribution proportional to (A_s+iB_s) or (A_p+iB_p) which arises from the reflected wave and appears with a negative sign in (11) and with a positive sign in (12). It follows that the degree of polarization should vary with sin p in a periodic manner, being zero when the thickness of the lamina is such that the reflected light vanishes and maximum for intermediate thicknesses. The intensity of the diffracted light, however, depends chiefly on the resultant of the contributions from the incident and transmitted waves which are proportional to $(1-C_s-iD_s)$ or $(1-C_p-iD_p)$. It is easily shown that this part depends on sin $(p-q)$ and that its intensity is a maximum when the relative retardation on the two sides of the boundary is half a wavelength, and a minimum when the relative retardation is a complete wavelength, and so on. Two sets of periodic variations in intensity thus occur giving rise to corresponding colour effects in the diffracted light. One set of variations affects both components of the vibration in the same way, while the other set affects the two components in opposite senses. Hence, the colour of the diffracted light would, to some extent, depend on the azimuth of polarization, and the two images of the diffracting edge seen through a double-image prism may in favourable cases actually exhibit complementary colours.



Fig·1

INTENSITIES OF COMPONENTS OF DIFFRACTED LIGHT AS FUNCTIONS OF THICKNESS.

Fig. 1 represents the values of $F_s^2+G_s^2$ and $F_p^2+G_p^2$ corresponding to an angle of diffraction of 90° ($\varphi=180^\circ$) for different thicknesses of the lamina, the refractive index being assumed to be 1·5. The periodic variations of the $\|^l$ and $\perp^r$ components of the intensity of the diffracted light are clearly seen. The polarization, which is given by the ratio of the two components, is almost complete for small thicknesses of the lamina; it is zero for thicknesses given by $2\pi d/\lambda = 78^\circ$, 120°, 218°, 240° and 360°. For intermediate thicknesses, the polarization fluctuates.

From (11), (12), (13) and (14) it is obvious that, when the incident light is plane-polarized in any azimuth, the diffracted light would in general be elliptically polarized. The phases of the $\perp^r$ and $\parallel^l$ components as calculated from the values of $\tan^{-1}(G_p/F_p)$ and $\tan^{-1}(G_s/F_s)$ respectively are shown in Fig. 2 for the same cases as those considered in Fig. 1. It is evident from the figure that, in certain cases, particularly for small thicknesses of the lamina, the phase differences are large and should easily be observed.

V. Supplementary Remarks.

The theory set out above is an approximation to the truth which is strictly applicable only to the thinnest laminæ. It succeeds in explaining the polarization effects which would otherwise be unintelligible. With laminæ of greater thickness, however, complications arise which are not here taken account of. In reality, we have not one edge, but two edges to deal with, namely, those relating to the front and rear surfaces of the lamina, at which the waves passing on either side of the boundary are diffracted and, diverging with a difference of path, interfere with each other.

It is readily seen that the path-difference between the interfering rays from the two edges would alter continually with increasing deviation of the diffracted ray, but in different ways in the two portions of the field to the right and the left of the direction of the incident pencil. Since the conditions on the two sides of the boundary between the incident and transmitted waves are thus dissimilar, the diffraction effects observed should be asymmetrical in intensity with reference to this boundary. Experimentally this is actually observed to be the case. Further, owing to the interference of the diffracted waves from the front and rear edges occurring under varying difference of path in different directions, the colour and intensity of the diffracted light should vary periodically with increasing deviation of the diffracted ray. This again is actually observed in experiment. It is not unlikely that a modified treatment which takes account of the complications referred to here may be successfully worked out. To enter into them more fully, however, or to make detailed comparisons between theory and observation would lie beyond the scope of the present Paper.

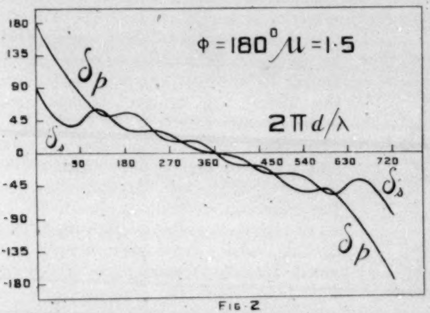


FIG. 2

Phases of Components of Diffracted Light as Functions of Thickness.

XLI.—LECTURE ON THE ECLIPSE OF THE SUN, JUNE 29, 1927.

Delivered by E. H. Rayner, *M.A.*, *Sc.D.*, on June 3, 1927.

Extracts from Lecture.

THE initial partial phase and totality take place before 6 a.m. G.M.T. In such circumstances the diurnal rotation causes England to have a small component opposite to the eastward motion of the moon's shadow. The final (fourth) contact will occur after 6 a.m. over England, and the partial phase at commencement will be some 3½ minutes shorter than that after totality. The times mentioned in this lecture are G.M.T. The centre of the shadow cone makes first contact with the earth at 5 H. 20·1 M. G.M.T., only 3 minutes before it reaches Cardigan Bay. At this point the diameter of the cone where it intersects the earth is about 13·7 miles, and the distance of intersection from the imaginary point is about 1,500 miles. In the two succeeding minutes, when the intersection is near Hartlepool, the diameter increases to about 16 miles, corresponding to about 1,700 miles from the point. Owing to the low altitude of the sun, the intersection of the shadow cone with the earth's surface is a long ellipse. The minor axis of the ellipse is only about half of the 30 mile width of the shadow track, since the major axis of the ellipse is inclined at about 23° to its direction of motion. The conditions at 5.25, which have been kindly provided by Dr. Comrie, are given in Fig. 1, in which the dimensions are in miles. One result of these circumstances is that over the northern two-thirds of the shadow track the approach of the shadow will appear to be from the E. of S. and not from the S.W. The direction at Richmond will be about 15° E. of S. The apparent velocity will be considerably less than the velocity of travel.

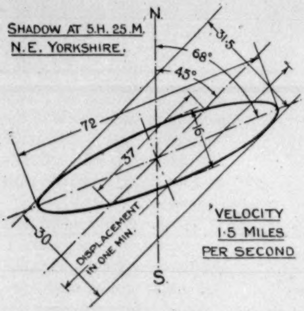


Fig. 1.

Particulars for Richmond, Yorks.

The progress of the eclipse has been computed for the Race Course at Richmond, Yorkshire. The information includes the times of the four contacts and their position angles. The moon will first be seen to encroach on the sun at 4 H. 30 M. 37 S. at an angle of 293° from the vertex, that is, from the highest point of the sun as seen at that time and place (Fig. 2). The angles are computed starting from North through East, to the left. The point is more conveniently found by considering it as 67° from the vertex counting towards the right.

The passage of the leading edge of the moon across the sun's disc takes about 54 minutes, the second contact, which is the commencement of totality, occurring at 5 H.

24 M. 39·9 S. Totality is computed to last 22·8 seconds ending with third contact at 5 H. 25 M. 3·7 S. The fourth contact, when the sun is finally clear of the moon is about 57·5 minutes later. The various times, position angles from the vertex and the direction of the North point of the sun are shown in the diagram.

ESTIMATION OF THE MOMENT OF TOTALITY BY MEASUREMENT OF THE SOLAR CRESCENT.

It is desirable to have an observational method of some accuracy for estimating the moment when totality is to commence a few seconds before it actually does so. When the moon has nearly covered the sun a diminishing crescent of the solar disc is visible. If the angular dimension or chord of this cusp has been computed for a number of instants before totality, it enables the desired time interval to be obtained by a direct measurement of the cusp. This measurement can be made by projecting the image of the sun, produced by a stand telescope, on to a white card, using the eye-piece as a magnifier in the usual manner. An image of the sun two or more inches in diameter can usually be obtained by this means. On the card pairs of parallel lines can be drawn, each pair being separated by a distance corresponding to the length of the chord of the cusp at a known interval before totality. The card is oriented so that the lines are parallel to the moon's apparent motion.

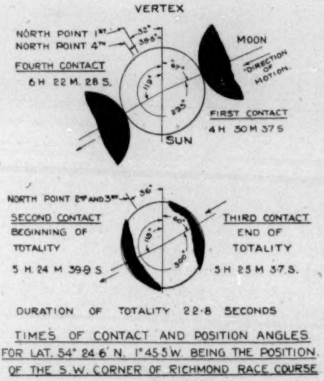

VERTEX
NORTH POINT 1ST
NORTH POINT 4TH
32°
38·5°
MOON
DIRECTION OF MOTION.
FOURTH CONTACT
6 H 22 M. 28 S.
87°
119°
235°
FIRST CONTACT
4 H 30 M 37 S
SUN
NORTH POINT 2ND AND 3RD
36°
SECOND CONTACT
BEGINNING OF TOTALITY
5 H. 24 M 39·9 S
60°
115°
300°
THIRD CONTACT
END OF TOTALITY
5 H 25 M 3·7 S.
DURATION OF TOTALITY 22·8 SECONDS
TIMES OF CONTACT AND POSITION ANGLES FOR LAT. 54° 24·6′ N. 1°45·5 W. BEING THE POSITION. OF THE S.W. CORNER OF RICHMOND RACE COURSE

FIG. 2.

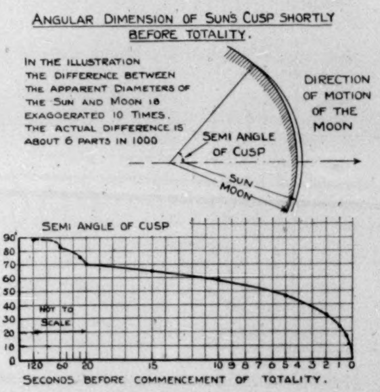


FIG. 3.

Fig. 3 shows the dimensions of the semiangle of the cusp in degrees, from which the corresponding chord can be computed. The chord will be D sin C, where D is the diameter of the image of the sun and C the semiangle of the cusp. In Fig. 4 the diameter of the sun is indicated by that of the semicircle, and the relative lengths of the chords from 20 to 0·5 seconds before (and after) totality are given by the corresponding numbers. The computation has been made on the assumption that the semi-diameter of the sun is 943·9 seconds, that the apparent semi-diameter of the moon is 950·1 seconds, and that the time of totality is 22 seconds. The relative motion assumed has been that of the centre of one disc moving along a diameter of the other. A rigid computation for the circumstances of Richmond, computed by Dr. Comrie, has confirmed these results very closely. The diagram may be used for places along the line of totality and for about five miles on each side with sufficient accuracy for the purpose intended.

CHROMOSPHERE.

The most striking part of the chromosphere may be expected to be the hydrogen prominences. The predominant visual light in the hydrogen spectrum is the red line at wavelength 6563. There are also other lines of shorter wavelength in the green and blue. These will tend to render less distinctive the red line, which is likely to be masked also by the general brightness of the corona. It may, therefore, be well worth while to try, for a second or two, even if not very successful, some method of enhancing the effect of this red hydrogen spectrum line.

A red glass of a common commercial quality cuts out practically all the blue and green of the spectrum and lets through only the orange and red. If the density of the colour is such as to absorb nine-tenths of ordinary daylight, the fraction of the line transmitted of the wavelength 6563 will be about one-half. There will be, therefore, a very considerable enhancement of the appearance of the hydrogen prominences, as seen in the light of this wavelength, compared with the general brightness of the corona. Two such glasses in series would cut down the red hydrogen line to a quarter of its intensity, while on the basis of average daylight it would only transmit 1 per cent.

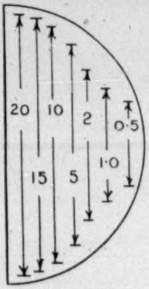


FIG. 4.

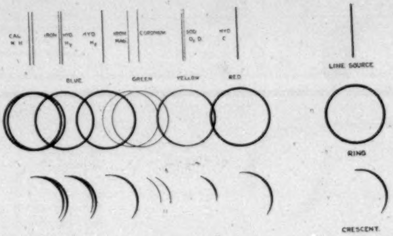


FIG. 5 (NEGATIVE).

A number of such glasses 3 in. by 2 in. in size are available, and it is proposed to distribute them to the Fellows of the Society and their friends on the journey to Richmond. The colour is similar to that of glass for photographic dark-room illumination, and to that used for the red light of railway signals.

A more analytical method of observing the chromosphere is by the use of a prism or grating to separate the light of different wavelengths.

The method is described in dealing with the "Flash" spectrum. The diagram Fig. 5 shows the appearance of light of the principal chromospheric lines as viewed through a straight slit, a circular slit and a crescent-shaped slit. It is not unlikely that the hydrogen lines, and possibly others, may be continuous all round the sun, or nearly so. The bright lines, green and red, due to the corona material, only visible during a total eclipse, may be expected to be visible as complete rings, not so bright as the chromosphere, and more diffuse because more extensive.

FLASH SPECTRUM.

Professor A. Fowler has suggested that, owing to the small difference between the apparent sizes of the sun and moon and to the probable brightness of the corona, there is a risk of observers not being aware that totality has commenced. The observation of the " flash " spectrum affords an accurate method of determining the limits of totality. At the base of the chromosphere, and for a distance of about 500 miles above the surface of the photosphere, there is a fairly sharply defined layer of metallic vapours as seen from the earth. These normally absorb some of the intense solar radiation of their own frequency passing through them, and give rise to the dark lines in the solar spectrum. At the moment of totality these gases in the region just clear of the apparent edge of the sun, not having its radiant surface behind them, do not give rise to an absorption spectrum; but, being at a sufficiently high temperature, they emit their own characteristic radiation, giving a spectrum of bright lines, the positions of which are the same as the corresponding lines of the absorption spectrum. This layer is comparatively thin, and the moon, traversing the sun's disc at a rate of 280 miles a second, soon covers it; so that the bright-line " flash " spectrum may only appear for a couple of seconds, though it may be visible longer in regions of activity. A plain or direct-vision prism is suggested for the observation of it and of the chromosphere, with or without any optical aid such as a field glass. For further information and suggestions as to what to look for, reference should be made to the Paper by Professor Fowler in the Journal of the British Astronomical Association, Vol. 37, No. 6, p. 205, price 3s., Eyre and Spottiswoode.

If accurate measurements are made of the time of appearance of the " flash " spectrum at the beginning and end of totality, referred preferably to the radio time signals, they may prove of distinct value. Any records should state whether the beginning or end of the " flash " is the moment recorded, and an estimate or measurement of its duration should be attempted.

SHADOW BANDS.

These appear to be a common feature of total eclipses. They have the appearance of alternate light and dark bands which are visible on the ground and other objects just before totality and after it. They are described as being in movement, but the movement does not appear to be too rapid to distinguish them. Their separation seems to be a matter of inches rather than feet. It is not known whether they have been photographed. It seems likely that they are a diffraction phenomenon caused by the last part of the solar disc being hidden by the moon, modified by atmospheric unsteadiness. The roughness of the moon's edge may be an important factor in their production. Preparations have been made to photograph them at one previous eclipse at least, which failed owing to clouds. A camera was focussed on white surfaces, one vertical facing the sun, and the other in front of it on the ground. An instantaneous photograph was to be attempted. A similar arrangement using a cinematograph camera or cinekodak would give an opportunity for determining not only the direction in space of these bands, but also their direction of progress. It is believed that any information of this character could not fail to lead to new knowledge. The ratio of the brightness of the light from the dark and bright parts is not known, nor the approximate absolute brightness, but it is hoped that a photograph would give a recognisable record. A lens of large aperture would be necessary, as the general intensity of the light will be very low.

GENERAL NOTES.

No one can fail to be impressed and have but an imperfect appreciation of the lapse of time during an eclipse. It is not unlikely that the diminution of daylight up to about a minute before totality may appear to many to be unexpectedly slight. The eye is so used to intensities varying many fold in a few minutes on ordinary days, and is naturally so adaptable, that it is possible that only the last few seconds may be conspicuous in this respect. The general tone of the illumination is likely to be very striking for some time before totality.

If attempts are made to do any timekeeping during totality for photographic or other observational purposes, or even for the purpose of making the most of the opportunities for visual observation, it is well to count seconds backwards. Counting in this manner avoids attempting to do mental arithmetic in order to find out how many seconds yet remain. A metronome should be used or some other audible signal, so as to avoid the use of the eye for this purpose. It is well to aim at underestimating the time of totality, and for a duration of 22 or 23 seconds it would be well to start counting from 20.

For the protection of the eyes while directly observing the sun some dark glass or similar material must be used. A transmission of about a hundred thousandth of the incident light is sufficient. A photographic film given an exposure to the sky in a camera of about the normal time and then developed to give a good dense deposit will transmit too much light if used singly. Two, three or four thicknesses can be superposed, a suitable number being found by trial. Process plates will give a denser deposit, and the use of glass will avoid the diffuseness due to the use of several thicknesses of celluloid. For information concerning photography of the corona, etc., reference should be made to page 208 of the Journal of the B.A.A., previously mentioned.

RADIO TIME SIGNALS.

Provision for the reception of radio time signals will be desirable, and it would be well for anyone providing such equipment to have a megaphone, in places where a considerable number of observers are expected.

It is specially desired that only the stations advertised as providing a public service should be tuned to, as delicate radio experiments are to be made on a number of other wavelengths. These may be seriously interfered with, if there is any attempt to tune to their wavelength. There will be nothing to listen to, as there will be no modulation. The radio experiments are extending over several days, during the early morning. The programme of the B.B.C. as regards time signals is as follows :—

TRANSMISSION OF GREENWICH SIGNALS FROM DAVENTRY ON 1,600 METRES ON JUNE 29, SUMMER TIME.

(1) Six dot seconds at 6.0, 6.15, 6.20, and 6.30 a.m.

(2) Single seconds continuously from 6.22 to 6.26 a.m., missing each 29th and 59th second with an announcer calling the 5th, 10th, 15th, etc.

DEMONSTRATION OF THE PRODUCTION OF SPLASHES BY ELECTRIC DISCHARGE.

By G. L. Addenbrooke.

THE experiments shown were an extension of those shown by Mr. Addenbrooke at the Physical Society's Exhibitions in 1926 and 1927, and have been described by him in The Philosophical Magazine, May, 1927.

A point electrode is mounted about 1 inch above a liquid dielectric (oil) and charged from an induction coil giving a 1-inch spark. For the purposes of optical projection the liquid is contained in a dish with metal rim and glass bottom. When the point is positively electrified, elongated radial splashes in the form of indentations spread out over the liquid and then sink into it. Similar results are obtained with a negatively charged electrode mounted nearer to the surface, the splashes covering a smaller area and being more definitely radial in this case. When a sphere of 1·5 cm. radius is substituted for the point, a circular hollow with a raised edge, from which the rays shoot out, is formed under the sphere.

DEMONSTRATION OF THE SCHÖNHERR-HESSBURGH NITROGEN FIXATION ARC.

By Capt. G. I. Finch, *M.B.E.*

THE combustion of free nitrogen to nitric oxide is an endothermic process, and therefore involves absorption of energy. In the arc nitrogen fixation processes this energy is furnished by means of an electric discharge. The suitability of an arc for this purpose depends in the main upon the extent to which the following requirements are fulfilled: (i) The arc must burn satisfactorily on an alternating-current supply of the usual frequency, (ii) without giving rise to excessive load fluctuations, and (iii) it should be possible to maintain a high rate of flow of air through the arc.

In the Birkeland-Eyde arc these conditions are sufficiently realised by magnetic, assisted to some extent by blast, quenching; and in the Pauling Brothers' arc by blast quenching alone. Both arcs are burnt between water-cooled copper tube electrodes. Thus immediately on striking at the beginning of each half-cycle (suitable inductance being included in the circuit to cause the voltage to lead to the necessary extent) the arc is distended—i.e., the arc gap virtually widened—and the heavy load fluctuations, which would otherwise occur owing to the characteristic of the discharge being negative, are suppressed. Further, since the origins of such quenched arcs are never stationary, but always receding rapidly from the gap, the cathode remains cool and the cathodic fall of potential is consequently of the order of 430 to 450 volts. Thus the Birkeland-Eyde and Pauling arcs are both cold cathode—i.e., true high-tension arcs.

The Schönherr-Hessberger arc differs fundamentally from either of those previously mentioned. This arc is not quenched; when once it has been struck and fully extended to its normal working length the latter remains constant. Further, the arc origins are stationary, and, although the iron rods upon which they are seated are water-cooled, such cooling does not suffice to prevent the formation of highly heated oxide layers at which the cathodic potential fall is of the order of only about 10 volts. Thus the Schönherr is a hot cathode—i.e., low-tension arc.*

The Schönherr-Hessberger furnace consists essentially of a long (30 ft., or even more) vertical iron tube, at either end of which are situated axially the two iron rod electrodes, of which the upper one and the tube are earthed. Air is blown through jets let in tangentially at the bottom of the furnace. This arrangement results in a helical movement being imparted to the air flowing up the tube, and thus preserves a practically stationary vertical column of air between the electrodes. On first striking an arc is formed between the bottom electrode and an adjacent point on the iron tube; but the arc origin on the latter is immediately carried up the tube by the swirling air current, until it finally lodges on the upper central electrode. Owing to the comparatively undisturbed state of the central air column now traversed by the discharge, the temperature and highly ionised condition of the gases therein persist sufficiently to enable the arc to re-strike almost immediately on reversal of the current at the close of each half-cycle.

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* The terms "low" and "high tension" as applied to the arc are apt to be somewhat misleading. It would certainly give rise to less confusion if the classification were based on the temperature of the cathode alone. For whilst the *minimum* striking potential of a hot cathode arc between, for example, carbon electrodes in air is only about 36 volts, and as much as from 450 to 470 volts when the cathode is cold, conditions can easily be realised in which the total potential drop across a hot cathode, "low tension" arc exceeds that across a cold cathode, "high tension" arc. A "high tension" arc with stationary origins, even on water-cooled but oxidisable metal electrodes, invariably breaks down into one of the hot cathode variety, unless, indeed, the current passed by the discharge be exceedingly small.

The Schönherr arc can be effectively demonstrated on a small scale in the following manner: A 1½ inch bore, 18 inches long, transparent quartz tube is clamped vertically and provided with a 3 inches long electrode of ¼ inch iron rod secured centrally by means of a rubber bung inserted into the lower end of the tube. Two glass tubes are passed through this bung, and their ends inside the quartz tube bent over at right angles, thus forming tangential blast jets. An upper ¼ inch thick iron rod electrode can be moved up or down along the axis of the tube, being guided in a glass tube held centrally in a bung in the upper end of the quartz tube. The secondary of a 1 kilowatt transformer working off a supply of the ordinary frequency and with a secondary output voltage of between 3,000 and 20,000 volts, is connected to the electrodes. Should the transformer secondary be centre-earthed, it will be necessary to insulate the upper electrode; otherwise it may be earthed in the interests of both convenience and safety. The movable electrode is now lowered towards the bottom one until the arc strikes, whereupon it is rapidly withdrawn to practically the full extent of the quartz tube. The arc thus formed is of strikingly beautiful appearance, brilliant golden white in colour and about ¾ inch in diameter. It moves sinuously and slowly about inside, but without ever actually touching the walls of the tube. On turning on the blast the discharge shrinks somewhat in diameter, and is steadied up until practically no motion can be detected in it. The gases leaving the upper end of the tube contain about 2 per cent. of nitric oxides and are strongly coloured.

DEMONSTRATION WITH MAGNETRON.

By D. Owen, *B.A., D.Sc.*

THE magnetron is a diode valve with cylindrical anode of non-magnetic material, along the axis of which lies the filament. A uniform magnetic field can be supplied parallel to the axis. A. W. Hull showed* that the device can be used to measure the ratio e/m of an electron. When a positive potential is applied to the anode electrons move in straight paths perpendicular to the axis. On applying the magnetic field the paths become curved, without, however, affecting the magnitude of the anode current, provided the electrons ultimately reach the anode. But as the magnetic field is gradually increased a critical value is reached at which the electrons just fail to reach the anode, passing tangentially to it and returning to the filament in a path similar to that of approach. This path can be shown to be a cardioid if the effect of the magnetic field on the distribution of space charge is neglected. If V denotes the potential-fall from anode to filament, H the critical magnetic field intensity, and R the internal radius of the anode, it can be shown that

$$e/m = 8V/H^2R^2$$

In the demonstration a diode in a bulb of the type used in the ordinary receiving valve is used, the anode being of copper. (This valve was supplied by the General Electric Research Laboratories, by courtesy of the Director, Mr. C. C. Paterson.) The valve is placed between the parallel faces of the pole-pieces of an electromagnet in series with battery, ammeter, carbon rheostat and switch. In the anode circuit of the valve is a shunted mirror galvanometer, the deflections of which are indicated by a disc of light moving over a scale. The value $V=100$ volts was found suitable. R was 0·5 cm., and the critical fall of anode current was reached at $H=130$ gausses.

The current through the electromagnet having been increased to the critical value, the switch is opened and the galvanometer spot allowed to regain its steady maximum deflection. On now once more closing the circuit of the electromagnet, no effect on the position of the spot of light is observed for an appreciable interval of time (some two or three seconds with the electromagnet used), after which the spot makes a sudden descent towards zero. The magnetron thus serves to show in a striking manner the finite time required for the electric current to build up in an inductive circuit to which a steady electromotive force is applied.

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* Physical Review (1921).

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